A∞Ω
MATH@TD

MHF4U: Chapter Four Practice Test

SOLUTIONS

1. State the vertical asymptotes of $f(x) = \dfrac{1}{x^2 - 4x - 12}$.

   a. $x = 6, x = -2$
   b. $x = 6, x = 2$
   c. $x = -6, x = -2$
   d. $x = -6, x = 2$

$f(x) = \dfrac{1}{(x-6)(x+2)}$

$\Rightarrow x \neq 6,\ x \neq -2$

2. The graph of which of the following rational functions has a hole?

   a. $y = \dfrac{2x-6}{x-6}$
   b. $y = \dfrac{2x-3}{x-3}$
   c. $y = \dfrac{2x-6}{2x}$
   d. $y = \dfrac{2x-6}{x-3}$

d) $y = \dfrac{2(x-3)}{x-3}$

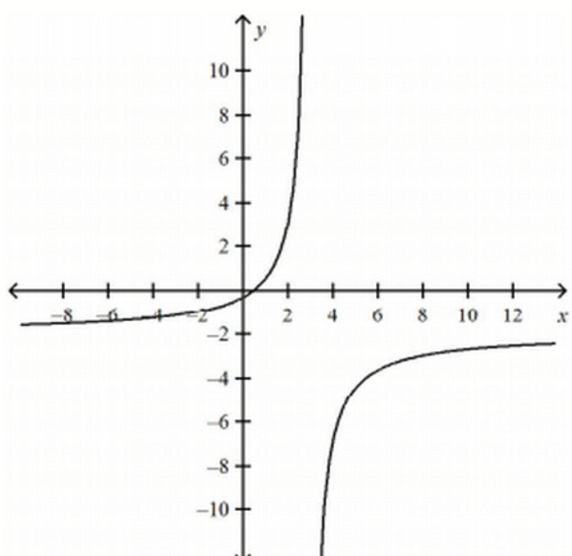
"$x-3$" can be cancelled $\Rightarrow x=3$ is a hole

3. Identify the vertical and horizontal asymptotes of $f(x) = \dfrac{x-4}{2x+1}$.

H.A. $\Rightarrow y = \frac{1}{2}$
V.A. $x = -\frac{1}{2}$

   a. vertical: $x = 4$, horizontal: $y = -\dfrac{1}{2}$
   b. vertical: $x = 4$, horizontal: $y = \dfrac{1}{2}$
   c. vertical: $x = -\dfrac{1}{2}$, horizontal: $y = \dfrac{1}{2}$
   d. vertical: $x = \dfrac{1}{2}$, horizontal: $y = -\dfrac{1}{2}$

4. State the equation of the rational function that this graph represents.



H.A $y = -2$ = "$\frac{-2x}{x}$"
V.A. $x = 3$
↳ "$x-3$" in denom.

   a. $y = \dfrac{-2x+1}{x-3}$
   b. $y = \dfrac{2x+1}{x-3}$
   c. $y = \dfrac{-2x+1}{x+3}$
   d. $y = \dfrac{2x+1}{x+3}$

5. Solve $\frac{4x}{x-2} = \frac{3x-2}{x-2}$ for x.

$x \neq 2 \Rightarrow 4x = 3x - 2$

$\Rightarrow x = -2$

a. $x = -\frac{1}{2}$

b. $x = -2, 2$

c. $x = -2$

d. $x = 2$

6. If $x \neq 3$ which of the following is equivalent to $\frac{5x}{x-3} = 8 + \frac{12}{x-3}$

$x \neq 3$

common denom

$x-3$

a. $\frac{5x}{x-3} = \frac{20}{x-3}$

b. $5x = 8 + 12$

c. $\frac{5}{-3} = 8 + \frac{4}{x-1}$

d. $5x = 8(x-3) + 12$

$$(x-3)\frac{5x}{x-3} = (x-3)\left(8 + \frac{12}{x-3}\right)$$

$$\Rightarrow 5x = 8(x-3) + 12 \quad d$$

7. The harmonic mean of two numbers, a and b, is a number m such that the reciprocal of m is the average of the reciprocals of a and b. Which of the following is a formula for the harmonic mean of a and b?

a. $m = \frac{2ab}{a+b}$

b. $m = \frac{a+b}{2ab}$

c. $m = \frac{a+b}{2}$

d. $m = \frac{2}{a+b}$

$$\frac{1}{m} = \frac{\frac{1}{a} + \frac{1}{b}}{2}$$

$$\Rightarrow \frac{1}{m} = \frac{\frac{a+b}{ab}}{2} \Rightarrow \frac{1}{m} = \frac{a+b}{2ab} \Rightarrow m = \frac{2ab}{a+b} \quad a$$

8. The inequality $3x - 2 < \frac{x+4}{x-2}$ is equivalent to which of the following?

a. $3x^2 - 9x + 8 < 0$

b. $\frac{3x^2 - 9x + 8}{x-2} < 0$

c. $3x^2 - 9x > 0$

d. $\frac{3x^2 - 9x}{x-2} < 0$

$$\Rightarrow 3x - 2 - \frac{x+4}{x-2} < 0$$

$$\Rightarrow \frac{(3x-2)(x-2) - (x+4)}{x-2} < 0$$

$$\Rightarrow \frac{3x^2 - 8x + 4 - x - 4}{x-2} < 0$$

$$\Rightarrow \frac{3x^2 - 9x}{x-2} < 0$$

d!

9. Which inequality is equivalent to $\frac{-2x}{x+5} < \frac{-18}{x+5}$?

a. $\frac{2x-18}{x+5} < 0$

b. $\frac{18-2x}{x+5} > 0$

c. $\frac{x-9}{x+5} < 0$

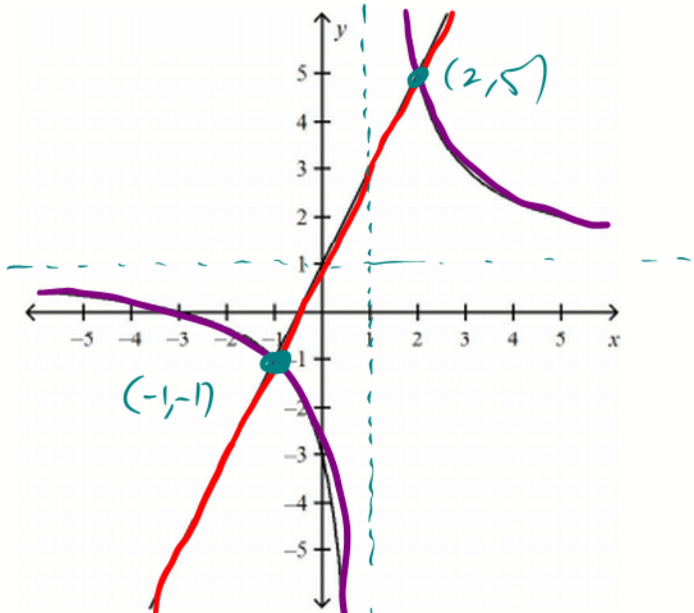
d. $\frac{x-9}{x+5} > 0$

$\Rightarrow \frac{-2x}{x+5} - \frac{-18}{x+5} < 0$

$\div -2$ on both sides

$\Rightarrow \frac{-2x+18}{x+5} < 0 \Rightarrow \frac{-2(x-9)}{x+5} < 0 \Rightarrow \frac{x-9}{x+5} > 0$

10. Use this graph to determine which of the following is a part of the solution set of $2x+1 > \frac{x+3}{x-1}$.



We want the red line to be above the purple curve. This occurs for

$\boxed{x \in (-1,1) \cup (2,\infty)}$

a. $x < -1$

b. $-1 < x < 2$

c. $x \geq 2$

d. $x > 2$

$\Longleftarrow$ all incorrect

11. Write a rational function in the form of $f(x) = \frac{ax+b}{cx+d}$ that has a zero at $x = -1$, a vertical asymptote at $x = 0$, and a horizontal asymptote at $y = 1$.

$\rightarrow$ denominator has a factor of "x"

numerator & denominator are the same order, with the same lead coefficient

$\hookrightarrow$ numerator has factor $(x+1)$

$$\boxed{f(x) = \frac{x+1}{x}}$$

Simplest fn satisfying the conditions

12. Determine all asymptotes and/or hole discontinuities for the functions:

To FIND ASYMPTOTES AND HOLES YOU MUST FACTOR.

a) $f(x) = \dfrac{3x^2 - 7x + 2}{x^2 - 4}$

$\Rightarrow f(x) = \dfrac{(3x-1)(x-2)}{(x-2)(x+2)}$

Hole: $x = 2$ (because "$x-2$" can be cancelled)

V.A. $x = -2$ (because "$x+2$" cannot be cancelled)

denominator is zero

H.A.: $y = 3$ $\left(\dfrac{\text{order } 2}{\text{order } 2} \Rightarrow\right.$ the H.A. is the ratio of lead coefficients)

b) $g(x) = \dfrac{2x^2 + 3x + 1}{x + 4}$

$\Rightarrow g(x) = \dfrac{(2x+1)(x+1)}{x+4}$

Holes: NONE (no cancelling)

V.A. $x = -4$

H.A. NONE $\left(\dfrac{\text{order } 2}{\text{order } 1} \Rightarrow\right.$ but there IS an O.A.

O.A. (oblique) $\Rightarrow$ divide.

$$\begin{array}{r|rrr} -4 & 2 & 3 & 1 \\ & & -8 & 20 \\ \hline & 2 & -5 & 21 \end{array}$$

$\boxed{y = 2x - 5}$ is O.A.

13. Sketch the graph of $f(x) = \dfrac{2x-3}{x-2}$. State the domain and range of the function. State where the function is increasing or decreasing, and where the function is positive.

$f(x) = \dfrac{2x-3}{x-2}$

zeros: $x = \frac{3}{2}$

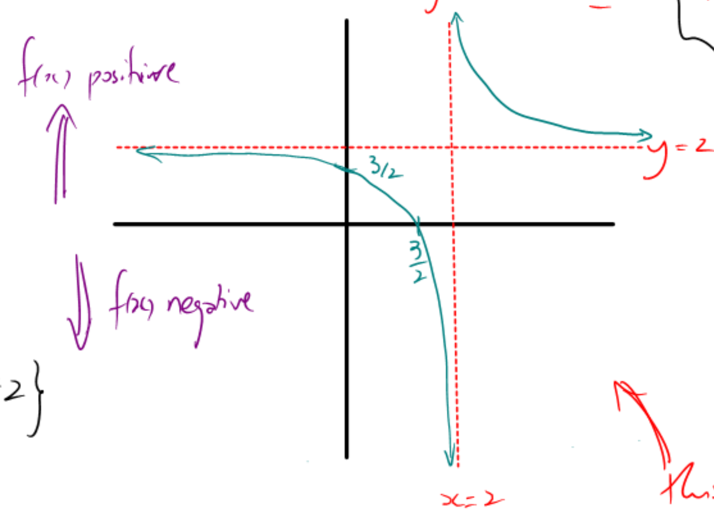
y-int: $f(0) = +\frac{3}{2}$

V.A. $x = 2$

H.A. $y = 2$

remember: these fns are either always inc. or dec

COMMUNICATE: LABEL YOUR ASYMPTOTES, LABEL INTERCEPTS



$D_f: \{x \in \mathbb{R} \mid x \neq 2\}$

$R_f: \{f(x) \in \mathbb{R} \mid f(x) \neq 2\}$

this graph is beautiful

$f(x)$ is always decreasing

$\Rightarrow f(x)$ is decreasing on $(-\infty, 2) \cup (2, \infty)$

$f(x)$ is positive on $(-\infty, \frac{3}{2}) \cup (2, \infty)$

14. Solve $\dfrac{x}{3x+2} - 1 = \dfrac{5}{x-1}$.

equations: multiply both sides by common denom.

restrictions: $x \neq -\frac{2}{3}$, $x \neq 1$

Common denom: $(3x+2)(x-1)$

$$\Rightarrow (3x+2)(x-1)\left(\frac{x}{3x+2} - 1\right) = (3x+2)(x-1)\left(\frac{5}{x-1}\right)$$

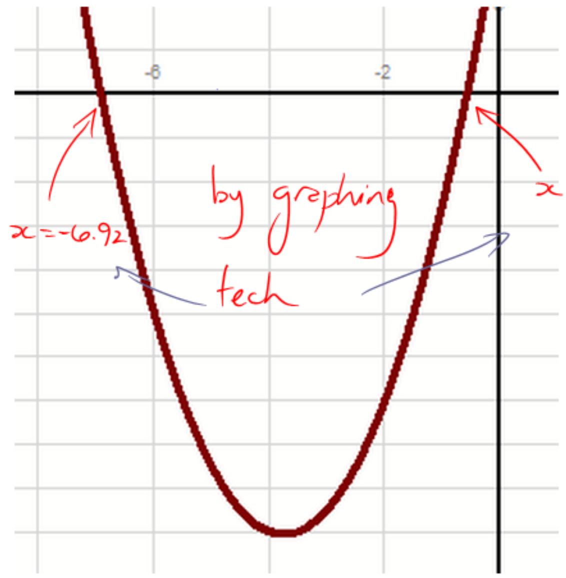
$$\Rightarrow x(x-1) - (3x+2)(x-1) = 5(3x+2)$$

$$\Rightarrow x^2 - x - 3x^2 + x + 2 = 15x + 10$$

$$\Rightarrow -2x^2 - 15x - 8 = 0$$

$$\Rightarrow 2x^2 + 15x + 8 = 0$$

$$\therefore\ x = -6.92 \text{ or } x = -0.58$$



15. Solve $\dfrac{x}{2x-1} \leq \dfrac{1}{x+4} + \dfrac{3x+7}{2x^2+7x-4}$.

inequalities - DO NOT multiply both sides by the common denom.

Goal: "single rational expression ≤ 0" ← whatever the case may be

INTERVAL CHART!!!

$$\Rightarrow \frac{x}{2x-1} \leq \frac{1}{x+4} + \frac{3x+7}{(2x-1)(x+4)}$$

restrictions: $x \neq \frac{1}{2}$, $x \neq -4$

$$\Rightarrow \underbrace{\frac{x}{2x-1}}_{\text{needs }(x+4)} - \underbrace{\frac{1}{x+4}}_{\text{needs }(2x-1)} - \underbrace{\frac{3x+7}{(2x-1)(x+4)}}_{\text{already has the denom.}} \leq 0$$

common denominator $(2x-1)(x+4)$

$$\Rightarrow \frac{x(x+4) - (2x-1) - (3x+7)}{(2x-1)(x+4)} \leq 0$$

be careful with signs!

$$\Rightarrow \frac{x^2 + 4x - 2x + 1 - 3x - 7}{(2x-1)(x+4)} \leq 0$$

(next page)

$$\Rightarrow \frac{x^2 - x - 6}{(2x-1)(x+4)} \leq 0$$

Note: Solving this inequality is EQUIVALENT to solving the original

$$\Rightarrow \frac{(x-3)(x+2)}{(2x-1)(x+4)} \leq 0$$

zeros : $x = 3$, $x = -2$

restrictions: $x \neq \frac{1}{2}$, $x \neq -4$

Intervals	$(-\infty, -4)$	$(-4, -2)$	$(-2, \frac{1}{2})$	$(\frac{1}{2}, 3)$	$(3, \infty)$
Test Val.	-5	-3	0	1	4
$x-3$	$-$ve	$-$ve	$-$ve	$-$ve	$+$ve
$x+2$	$-$ve	$-$ve	$+$ve	$+$ve	$+$ve
$2x-1$	$-$ve	$-$ve	$-$ve	$+$ve	$+$ve
$x+4$	$-$ve	$+$ve	$+$ve	$+$ve	$+$ve
$\frac{(x-3)(x+2)}{(2x-1)(x+4)}$	$+$ve	$-$ve	$+$ve	$-$ve	$+$ve

$$\therefore \frac{x}{2x-1} \leq \frac{1}{x+4} + \frac{3x+7}{2x^2+7x-4} \text{ on}$$

$$x \in (-4, 2] \cup (\tfrac{1}{2}, 3]$$

$x = -4$ is a restriction $\Rightarrow$ NOT included

$x = \frac{1}{2}$ is a restriction $\therefore$ not included.