Review of Chapter 4 ***(note: all the assigned homework is good review too!!)***

In Chapter 4 (5 in your text) we studied a new kind of function: Rational Functions. Rational Functions are related to Polynomial Functions and so share some "polynomial behaviour", but Rational Functions have their own unique characteristics. We began with the **definition of a Rational Function, which is a function of the form**

$$R(x) = \frac{P(x)}{Q(x)},\ \ Q(x) \neq 0 \text{ where both } P(x), \text{ and } Q(x) \text{ are Polynomial Functions.}$$

In Section 4.1 we examined Domain and Asymptotes of Rational Functions.

A Rational Function $f(x) = \dfrac{p(x)}{q(x)}$ will have:

- Vertical Asymptotes where $q(x) = 0$ if the "factor" of $q(x)$ cannot be "cancelled away".

  e.g. $f(x) = \dfrac{2x-1}{x+1}$ has a V.A. $x = -1$ since $x+1$ cannot be cancelled.

- A Hole Discontinuity where $q(x) = 0$ if the "factor" of $q(x)$ can be "cancelled away".

  e.g. $g(x) = \dfrac{x-1}{x^2-1}$ has a hole at $x = 1$ and a V.A. $x = -1$

- A Horizontal Asymptote IF the numerator and denominator have the same order.

  e.g. $f(x) = \dfrac{3x^2-5x+1}{5x^2+3x-4}$ has a H.A. $y = \dfrac{3}{5}$ (the ratio of lead coeeficiants!)

- A Horizontal Asymptote $y = 0$ IF the order of the numerator is smaller than the order of the denominator.

  e.g. $h(x) = \dfrac{2x+3}{x^3-4x^2+3x-1}$ has a H.A. $y = 0$.

- An Oblique Asymptote IF the order of the numerator is 1 more than the order of the denominator. We find the O.A. (which is a line with equation $y = mx + b$) using polynomial division.

  e.g. $f(x) = \dfrac{x^2+2x-5}{x-1}$ has an O.A. $y = x+3$ check my work!)

- If the order of the numerator is larger than the order of the denominator, then the Rational Function has no H.A.

In Section 4.2 we sketched the graphs of simple Rational Functions of the form $f(x)=\dfrac{ax+b}{cx+d}$.

These Rational Functions have:

- A V.A. $x=-\dfrac{d}{c}$ (unless the function looks something like $f(x)=\dfrac{3x-6}{x-2}$ which has a hole at $x=2$)
- A H.A. $y=\dfrac{a}{c}$
- A functional intercept ("*y* intercept") $f(0)=\dfrac{b}{d}$
- A zero $x=-\dfrac{b}{a}$

We sketched a few of these functions, and looked at analyzing functional behavior "near" V.A.'s.

In Section 4.4 we solved Rational Equations, using a modified version of "cross multiplication".

> e.g. For the equation $\dfrac{x}{x-2}+\dfrac{3}{4}=\dfrac{2}{x-1}$ **we multiply every term by the common denominator of the entire equation**, which is $4(x-2)(x-1)$. Multiplying through the equation by this Common Denominator eliminates all "fractions". We then solve the equation in the same manner we solve polynomial equations. However, we MUST keep in mind the restrictions for the rational equation. In this case, the restrictions are $x \neq 2,\ x \neq 1$. We may need to use graphing technology if our rational expression cannot be factored.

In Section 4.5 we solved Rational Inequalities.

We saw that we **DO NOT use cross multiplication in any form**. The goal is to get a single rational expression on one side of the inequality, and then use an interval chart to determine the Solution Set. We may need to use graphing technology if our rational expression cannot be factored.

Practice: pg 308: # 3 – 11, pg 310: #1, 3, 5, 6

Further Examples (*Solutions to be posted Monday*):

1. Determine all asymptotes of the following:

a) $f(x) = \dfrac{2x^2+1}{3x^2+1}$

a) V.A. $\Rightarrow 3x^2+1=0$

NONE $\quad x^2 = -\frac{1}{3}$ impossible

H.A. $y = \frac{2}{3}$

b) $g(x) = \dfrac{2x-1}{2x^2-7x+3}$ 01 / 02

$g(x) = \dfrac{2x-1}{(2x-1)(x-3)}$

VA. : $x = 3$

HA. $y = 0$

Hole $x = \frac{1}{2}$

c) $h(x) = \dfrac{2x^2-x+4}{x+1}$ 02 / 01

VA. $x = -1$

H.A. NONE

OA.

$$\begin{array}{r|rrr} -1 & 2 & -1 & 4 \\ & & -2 & 3 \\ \hline & 2 & -3 & 7 \end{array}$$

$\therefore$ OA. $\boxed{y = 2x-3}$

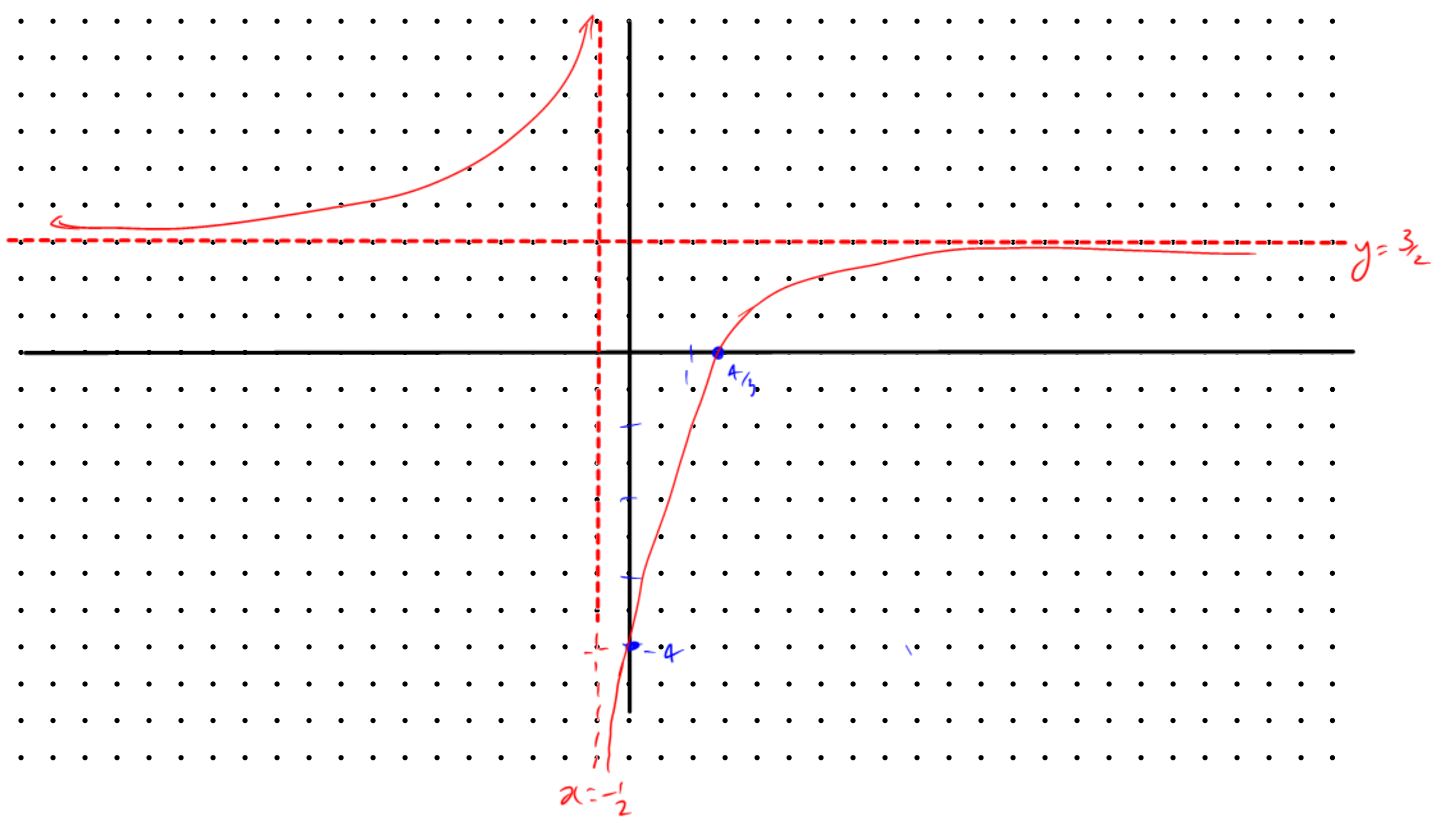
2. Sketch $f(x) = \dfrac{3x-4}{2x+1}$

zero: $3x-4=0 \Rightarrow x = \frac{4}{3}$

VA. $2x+1=0 \Rightarrow x = -\frac{1}{2}$

yint : $f(0) = -4$

HA. $y = \frac{3}{2}$



$f(x)$ is inc on $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$, $f(x) \leq 0$ on $x \in (-\frac{1}{2}, \frac{4}{3}]$

3. A model describing the concentration, $f(x)$, of a drug in the bloodstream, x hours after it is taken (as a pill) is given by:

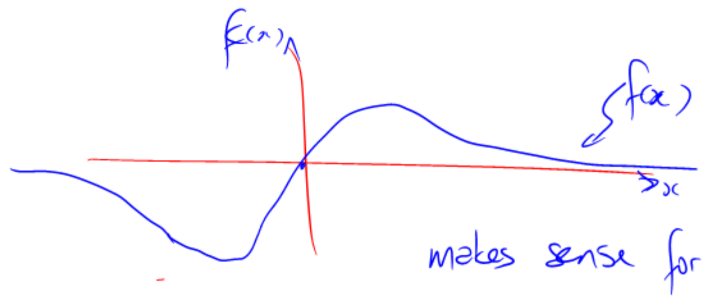
$$f(x)=\frac{7x}{x^2+2}$$

Note: $f(x)$ has no "domain restrictions"

$x^2+2=0$
$x^2=-2$
impossible

Determine D_f and sketch a graph of $f(x)$ using graphing tech. Does this model seem reasonable? Explain.

$D_f=(-\infty,\infty)$



makes sense for restricted domain

$D_{f_r}=[0,\infty)$

4. Solve $\dfrac{x}{x-2}+\dfrac{1}{x-4}=\dfrac{2}{x^2-6x+8}$

multiply by common denom

$\Rightarrow \dfrac{x}{x-2}+\dfrac{1}{x-4}=\dfrac{2}{(x-4)(x-2)}$

restrictions: $x\neq 4$, $x\neq 2$

CD: $(x-4)(x-2)$

$(x-4)(x-2)\left(\dfrac{x}{x-2}+\dfrac{1}{x-4}\right)=\cancel{(x-4)(x-2)}\left(\dfrac{2}{\cancel{(x-4)(x-2)}}\right)$

$x(x-4)+x-2=2$

$\Rightarrow x^2-3x-4=0$

$\Rightarrow (x-4)(x+1)=0$

$x=4$ or $x=-1$

inadmissible

5. Solve

a) $\dfrac{x-1}{x+5}\geq 0$

b) $\dfrac{x-8}{x}<3-x$

Do NOT multiply by the common denom.

zeros: $x=1$

restriction $x\neq -5$

intervals	$(-\infty,-5)$	$(-5,1)$	$(1,\infty)$
T.V.	-6	0	2
$x-1$	$-$ve	$-$ve	$+$ve
$x+5$	$-$ve	$+$ve	$+$ve
$\frac{x-1}{x+5}$	$+$ve	$-$ve	$+$ve

$\therefore \dfrac{x-1}{x+5}\geq 0$ on $x\in(-\infty,-5)\cup[1,\infty)$

$$\frac{x-8}{x} < 3-x$$

$$\Rightarrow \frac{x-8}{x} + x - 3 < 0$$

restriction: $x \neq 0$

CD x

$$\frac{x-8}{x} + \frac{x(x-3)}{x} < 0$$

$$\Rightarrow \frac{x-8+x^2-3x}{x} < 0$$

$$\Rightarrow \frac{x^2-2x-8}{x} < 0$$

$$\Rightarrow \frac{(x-4)(x+2)}{x} < 0$$

zeros: $x = 4$, $x = -2$

restriction: $x \neq 0$

intervals	$(-\infty, -2)$	$(-2, 0)$	$(0, 4)$	$(4, \infty)$
T.V.	-3	-1	1	5
$x-4$	$-$ve	$-$ve	$-$ve	$+$ve
$x+2$	$-$ve	$+$ve	$+$ve	$+$ve
x	$-$ve	$-$ve	$+$ve	$+$ve
$\frac{(x-4)(x+2)}{x} < 0$	$-$ve	$+$ve	$-$ve	$+$ve

$\therefore \frac{x-8}{x} < 3-x$ on $x \in (-\infty, -2) \cup (0, 4)$