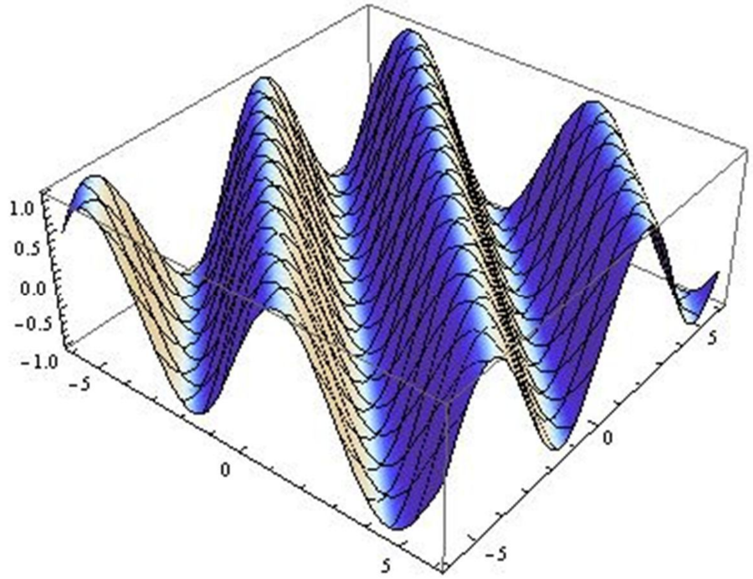
ADVANCED FUNCTIONS

Chapter 5 – Trigonometric Functions

(*Material adapted from Chapter 6 of your text*)



Chapter 5 – Trigonometric Functions

Contents with suggested problems from the Nelson Textbook (Chapter 5)

5.1 Radian Measure and Arc Length

Radian Measure

We are familiar with measuring angles using "degrees", and now we will turn to another measure for angles: **Radians**.

↳ arise in the concept of Arc Length.

Before getting to the notion of radians, we need to learn some notation.

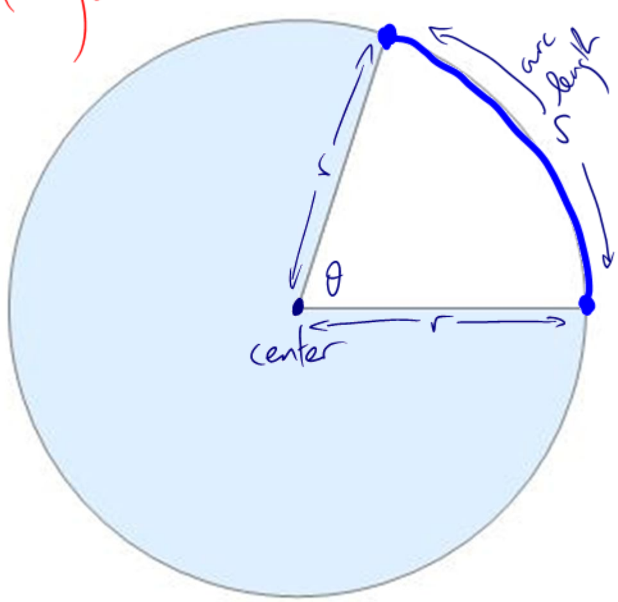
Picture

θ is called a central angle

r is the radius

s is an arc

(a section of a circumference)



There is a DIRECT RELATIONSHIP between s and r & θ

ARC LENGTH FORMULA

$$\boxed{s = r\theta}$$

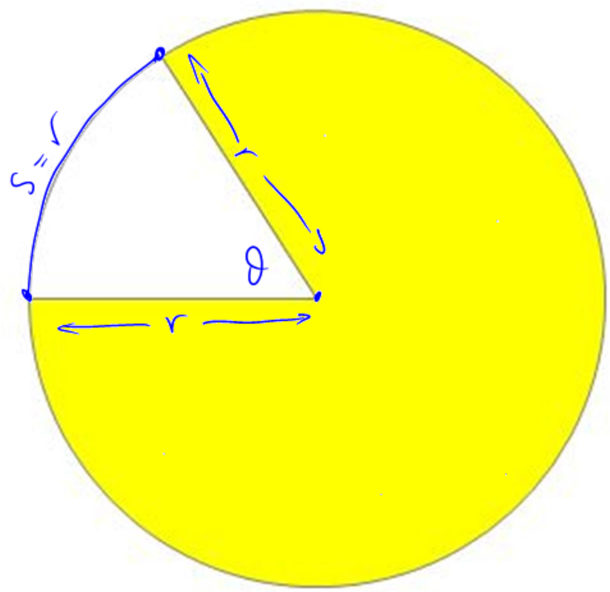
where θ MUST BE IN RADIANS

We say the arc, s, is subtended by the central angle θ

Definition 5.1.1

In a circle of radius r, a central angle θ subtending an arc of length $s = r$ measures 1 radian.

Picture



θ = 1 radian

(~60°)

tilde

Note: The circumference of a circle is given by $C = 2\pi r$

So, for a central angle of 360^{o}, ~~I~~ in a circle of radius $r = 1$, then (arc length s is the entire circumference)

$S = C$

$r\theta = 2\pi r$

$\Rightarrow \theta = 2\pi$

$\therefore 360^{\circ} = 2\pi$ radians

$\therefore \boxed{180^{\circ} = \pi \text{ rad}}$ ← the key for converting between degrees & radians.

$180 = \pi$ $\quad \frac{\pi}{180}$ is used to convert from deg to radians

$\frac{180}{\pi}$ is used to convert from rad to deg

Example 5.1.1

Convert the following to radians: EXACT FORM WHEN POSSIBLE

a) 30° b) 45° c) 120°

$\cancel{30}\left(\frac{\pi}{\cancel{180}_6}\right) = \frac{\pi}{6}$ rad

$\cancel{45}°\left(\frac{\pi}{\cancel{180}_4}\right) = \frac{\pi}{4}$ rad

$\cancel{120}°\left(\frac{\pi}{\cancel{180}_3}\right) = \frac{2\pi}{3}$ rad

d) 315° e) 161.3° ⇐ not exact ⇒ use calculator

$\cancel{315}\left(\frac{\pi}{\cancel{180}_4}\right) = \frac{7\pi}{4}$ rad

$161.3\left(\frac{\pi}{180}\right) = 2.8$ rad

exact form otherwise.

Example 5.1.2

Convert the following to degrees (round to two decimal places where necessary)

a) $\frac{7\pi}{12}$ rad b) $\frac{10\pi}{9}$ rad c) 2.5 rad ⇐ non exact

$\frac{7\cancel{\pi}}{\cancel{12}}\left(\frac{\cancel{180}^{15}}{\cancel{\pi}}\right) = 105°$

$\frac{10\cancel{\pi}}{\cancel{9}}\left(\frac{\cancel{180}^{20}}{\cancel{\pi}}\right) = 200°$

$2.5\left(\frac{180}{\pi}\right) = 143.24°$

d) $\frac{\pi}{2}$ rad e) $-\frac{\pi}{3}$ rad

$\frac{\cancel{\pi}}{\cancel{2}}\left(\frac{\cancel{180}^{90}}{\cancel{\pi}}\right) = 90°$

$-\frac{\cancel{\pi}}{\cancel{3}}\left(\frac{\cancel{180}^{60}}{\cancel{\pi}}\right) = -60°$

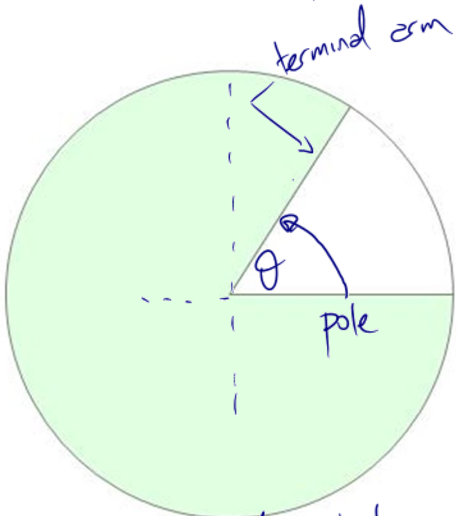
Q. What the rip is a negative degree?

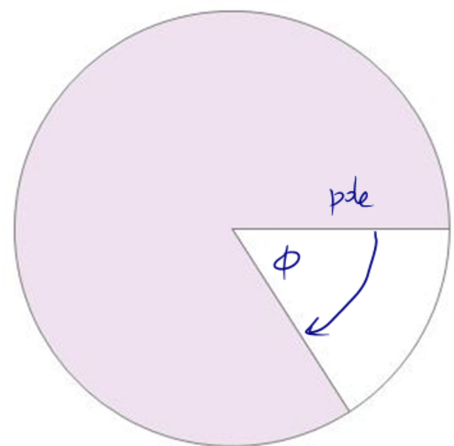
Angles of Rotation

The sign on an angle can be thought of as the direction of rotation (around a circle).

Pictures Angles of rotation always begin at pole

" stop at the terminal arm





ccw angles of rotation are considered positive angles

clock-wise rotations are considered negative angles

Example 5.1.3

Sketch the following angles of rotation:

a) $\frac{\pi}{6}$ rad b) $\frac{2\pi}{3}$ rad c) $-\frac{3\pi}{4}$ rad d) $\frac{7\pi}{6}$

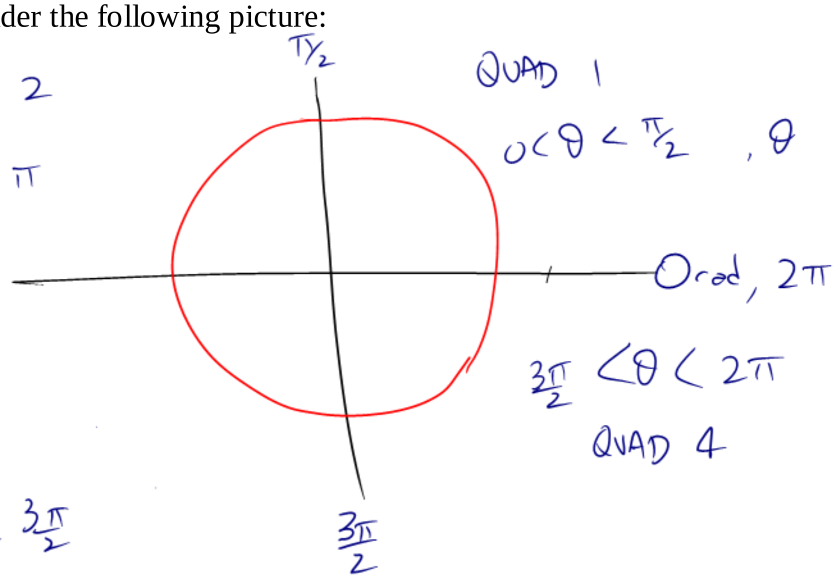
BUT FIRST: Consider the following picture:

QUAD 2

$\frac{\pi}{2} < \theta < \pi$

QUAD 1

$0 < \theta < \pi/2$, θ is an angle of rotation



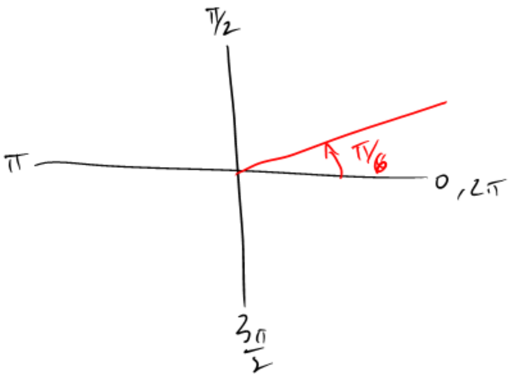
$\frac{3\pi}{2} < \theta < 2\pi$

QUAD 4

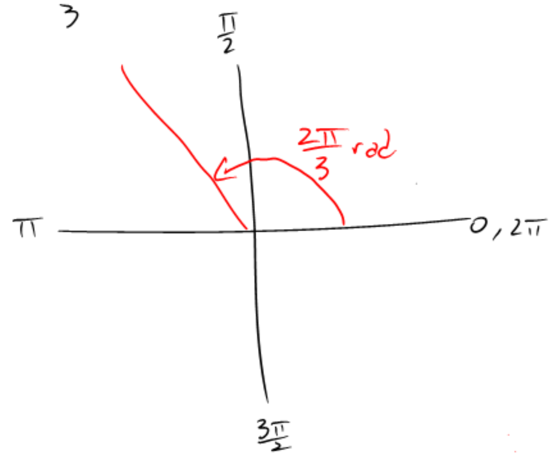
QUAD 3

$\pi < \theta < \frac{3\pi}{2}$

a) $\frac{\pi}{6}$ Q1

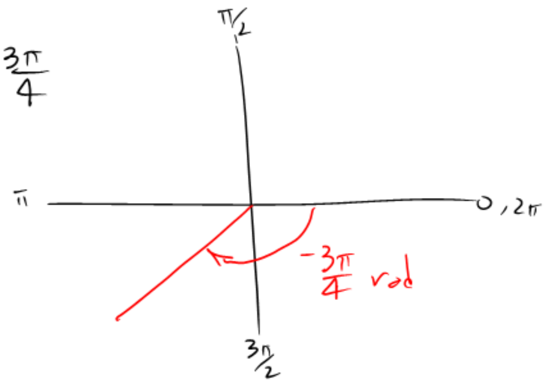


b) $\frac{2\pi}{3}$ Q2

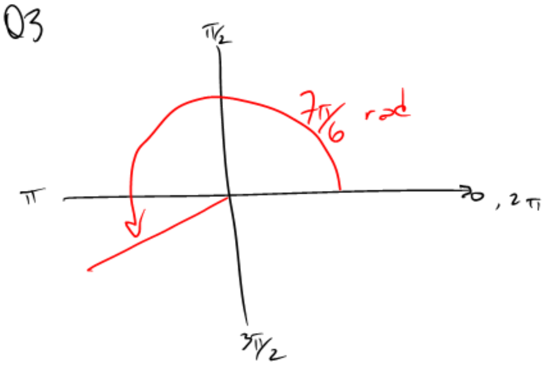


c) $-\frac{3\pi}{4}$

Q3



d) $\frac{7\pi}{6}$ Q3



Example 5.1.4

Determine the length of an arc, on a circle of radius $5cm$, subtended by an angle:

a) $\theta = 2.4$ rad

b) $\theta = 120°$

$S = r\theta$

$= (5)(2.4)$

$= 12$ cm.

must convert to rad.

$\cancel{120}^{2}\left(\frac{\pi}{\cancel{180}_{3}}\right) = \frac{2\pi}{3}$

$S = r\theta$

$= 5\left(\frac{2\pi}{3}\right)$

$= \frac{10\pi}{3}$ cm $(\sim 10.5 \text{ cm})$

> *Class/Homework for Section 5.1*
>
> *Pg. 321 #2edfh, 3 – 9*