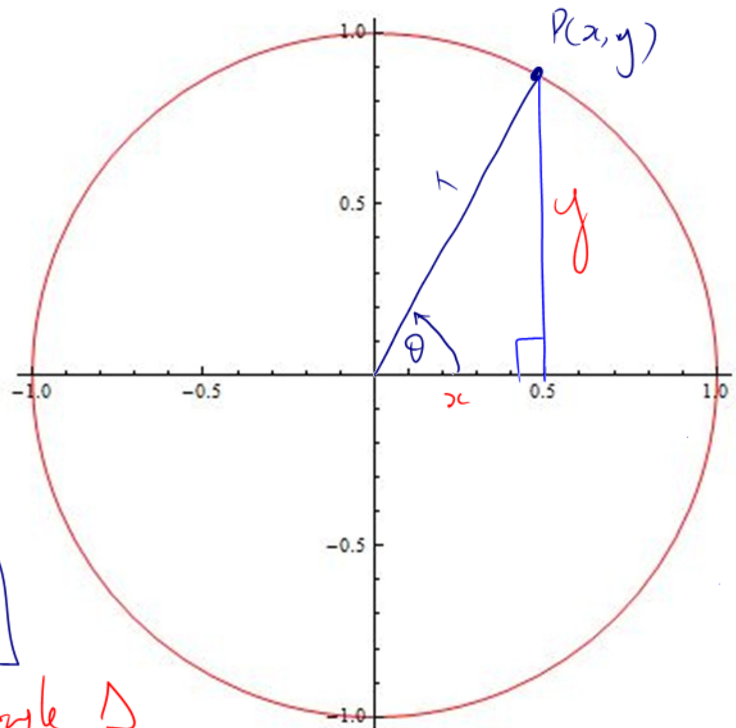
5.2 Trigonometric Ratios and Special Triangles (Part 1)

Consider the circle of radius 1:

Let θ be an angle of rotation

Consider a general point $P(x,y)$

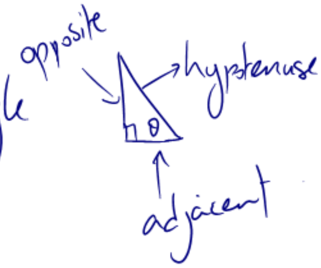


Dropping a $\perp$ from $P(x,y)$ to THE POLAR AXIS creates a right angle Δ

Now, your friend (and mine) Pythagoras tells us

$$x^2 + y^2 = 1 \quad (*)$$

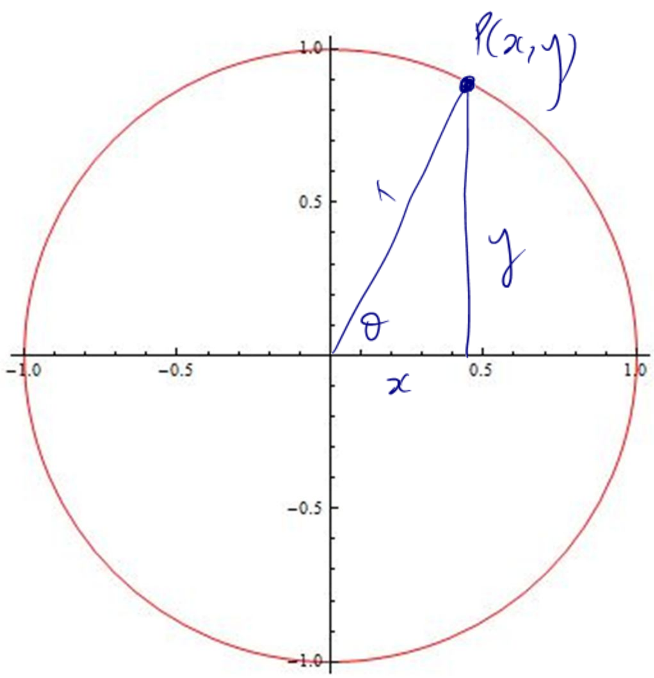
Given a right angle



Recall the six main Trigonometric Ratios:

Primary Trig Ratios		Reciprocal Trig Ratios	
$\sin(\theta) = \frac{opp}{hyp}$	SOH	$\frac{1}{\sin(\theta)} = \csc(\theta) = \frac{hyp}{opp}$ (cosecant)	CHO
$\cos(\theta) = \frac{adj}{hyp}$	CAH	$\frac{1}{\cos(\theta)} = \sec(\theta) = \frac{hyp}{adj}$	SHA
$\tan(\theta) = \frac{opp}{adj}$	TOA	$\frac{1}{\tan(\theta)} = \cot(\theta) = \frac{adj}{opp}$	CAO

Consider again the circle of radius 1 (but now keeping in mind SOH CAH TOA)



$$\sin(\theta) = \frac{y}{1} = y$$

$$\cos(\theta) = x$$

$$\tan(\theta) = \frac{y}{x}$$

Note: We can 'rename' $P(x, y)$ as $\boxed{P(\cos(\theta), \sin(\theta))}$ given an angle of rotation back on page 104 we saw $x^2 + y^2 = 1$

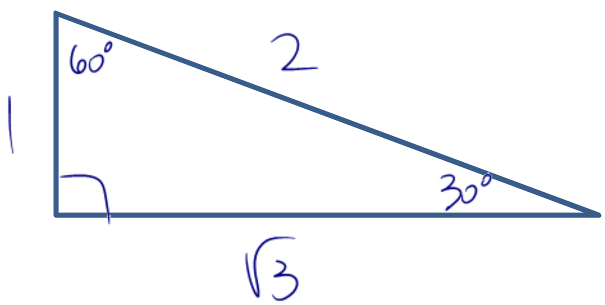
The TRIGONOMETRIC Pythagorean Identity

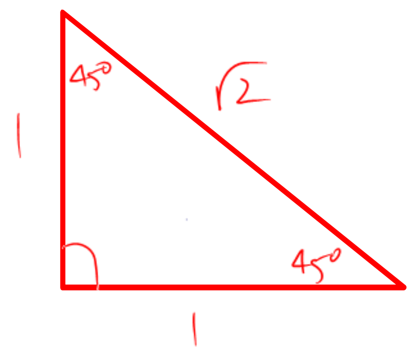
with $x = \cos(\theta)$ $y = \sin(\theta)$ we can write $(\cos(\theta))^2 + (\sin(\theta))^2 = 1$

usual notation: $\boxed{\sin^2(\theta) + \cos^2(\theta) = 1}$

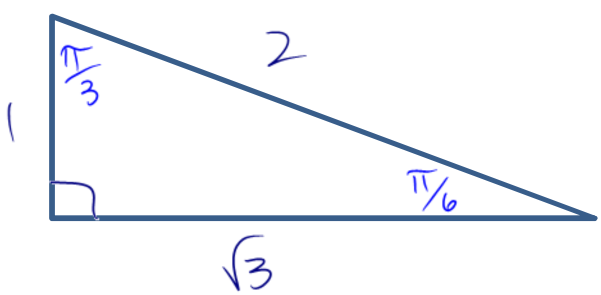
Special Triangles in Radians

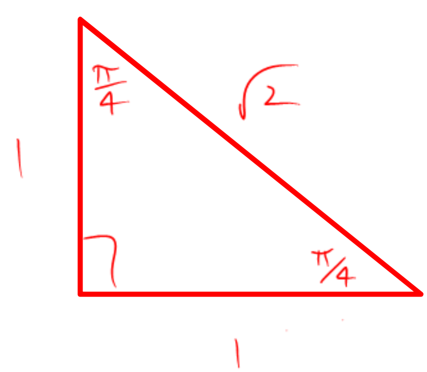
Recall: We have two "Special Triangles". In **degrees** they are:





In radians we have $30^\circ\left(\frac{\pi}{180}\right) = \frac{\pi}{6}$



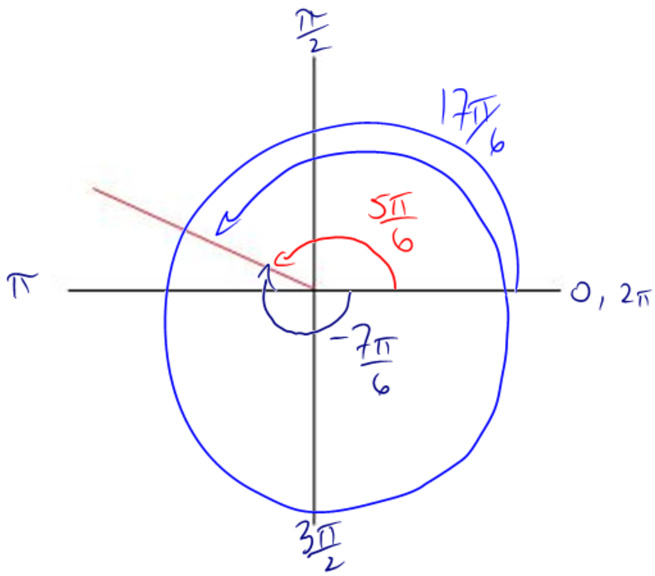


MEMORIZE THESE!

your life will be better for it.

Angles of Rotations and Trig Ratios

Consider the following sketch of the angle of rotation $\theta = \frac{5\pi}{6}$:



Problem!
Angles of rotation are not unique

$\frac{5\pi}{6}, -\frac{7\pi}{6}, \frac{17\pi}{6}$ all have the same terminal arm

Angles of rotation must begin at the pole.

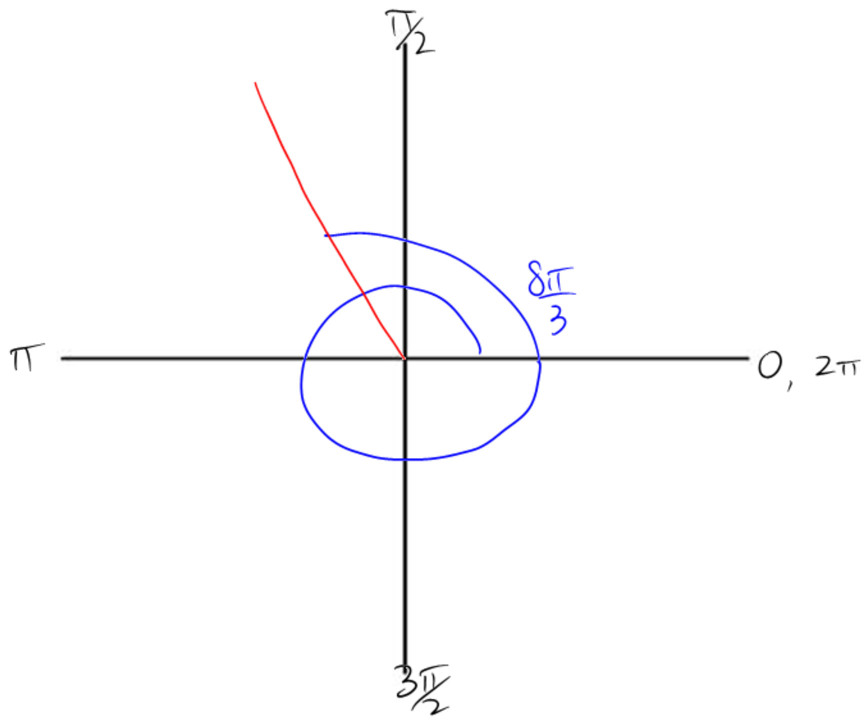
In this Example, we call $\theta = \frac{5\pi}{6}$ the **PRINCIPAL ANGLE**, or the angle in standard position.

We take this to mean, the smallest, positive angle of rotation to the given terminal arm.

Note: All principal angles $\theta \in [0, 2\pi]$

Example 5.2.1

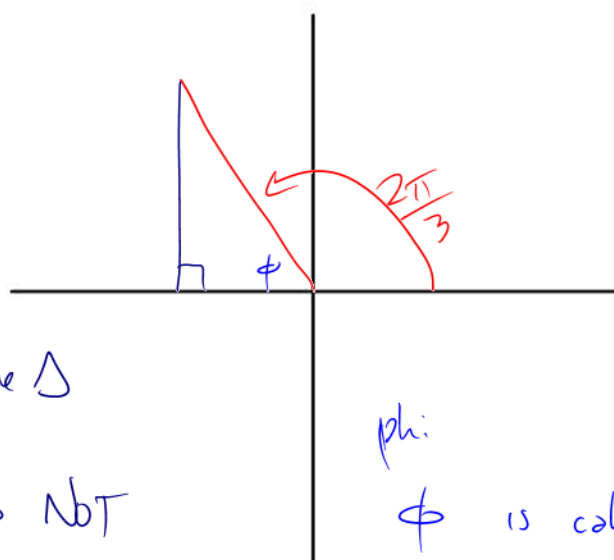
Sketch the angle of rotation $\theta = \frac{8\pi}{3}$ and determine the principal angle.



Principal Angle: $\frac{2\pi}{3}$

Dropping a ⊥ to the poles constructs a right angle Δ

BUT $\theta = \frac{2\pi}{3}$ is NOT in the Δ



Q. How can calculate the number $\sin\left(\frac{2\pi}{3}\right)$?

phi:

ϕ is called the related acute angle

ϕ is always between the pole and terminal arm.

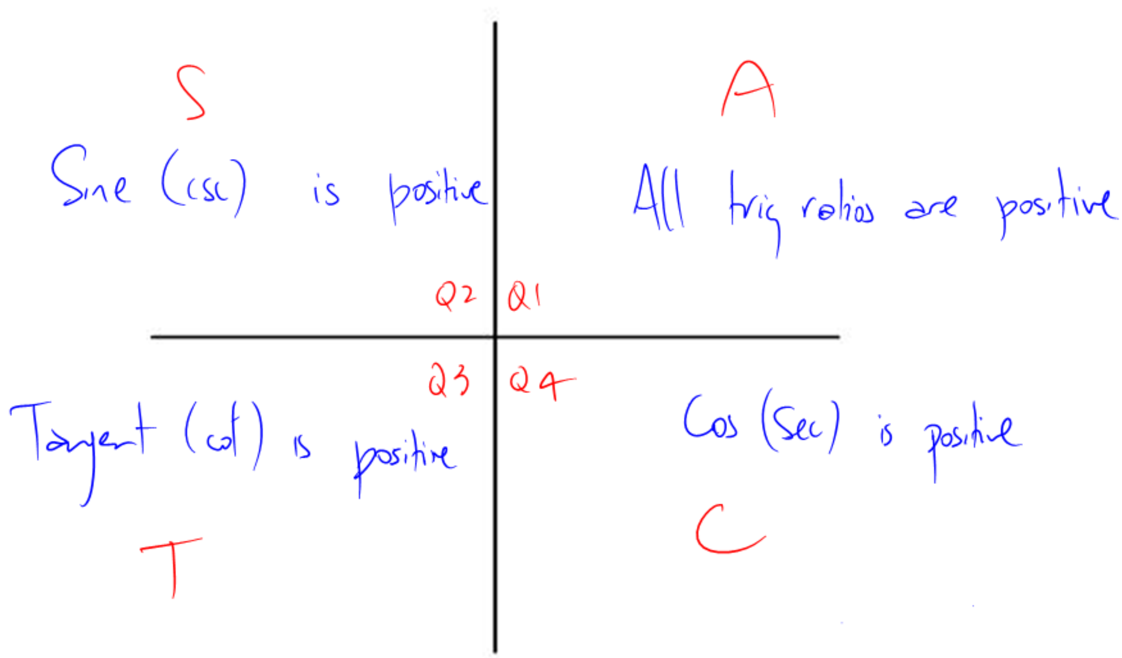
We now have enough tools to calculate the trigonometric ratios of any angle!

For any given angle θ (in radians from here on) we will:

1) Draw θ in standard position (i.e. draw the principal angle for θ)
2) Determine the related acute angle (between the terminal arm and the polar axis)
3) Use the related acute angle and the CAST RULE (and SOH CAH TOA) to determine the trig ratio in question

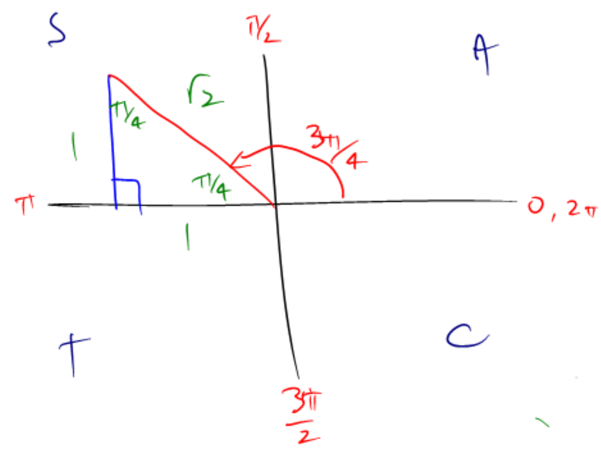
Recall the CAST RULE

Note: The CAST RULE determines the sign (+ or -) of the trig ratio



Example 5.2.2

Determine the trig ratio $\sin\left(\frac{3\pi}{4}\right)$



$$\sin\left(\frac{3\pi}{4}\right) = +\frac{1}{\sqrt{2}}$$

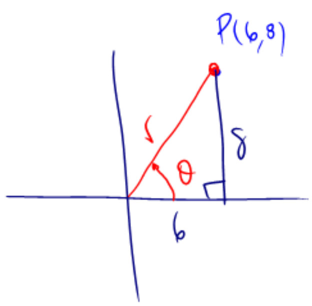
Example 5.2.3

The point $(6,8)$ lies on the terminal arm (of length *r*) of an angle of rotation. Sketch the angle of rotation.

Determine: a) the value of *r*

b) the primary trig ratios for the angle

c) the value of the angle of rotation in radians, to two decimal places



a) $r = \sqrt{6^2 + 8^2}$

$= 10$

b) $\sin(\theta) = \frac{8}{10} = \frac{4}{5}$

$\cos(\theta) = \frac{6}{10} = \frac{3}{5}$

$\tan(\theta) = \frac{8}{6} = \frac{4}{3}$

c) Using $\sin(\theta) = \frac{4}{5}$

$\theta = \sin^{-1}\left(\frac{4}{5}\right) = 0.93$ rad

Class/Homework for Section 5.2

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