5.3 Trigonometric Ratios and Special Triangles
(Part 2 – Exact Values)

Recall the "Unit Circle" from yesterday:

radius 1

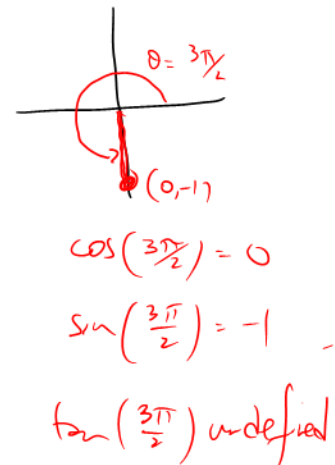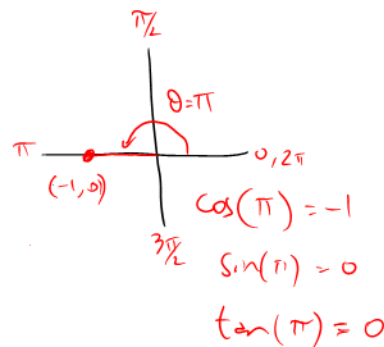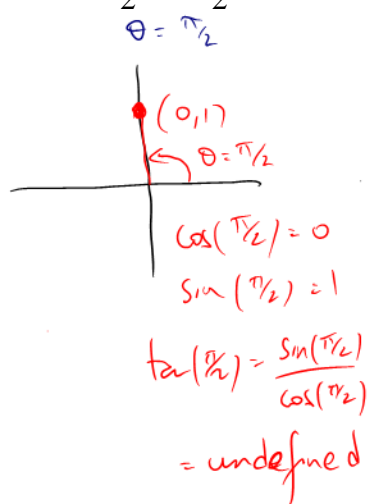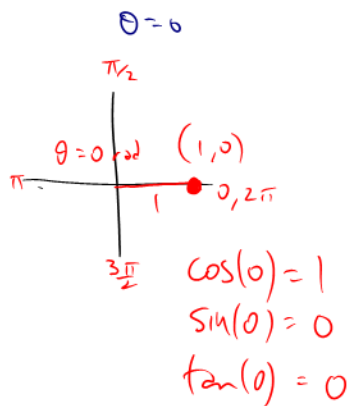$P(x,y) = P\left(\cos(\theta), \sin(\theta)\right)$

$\sin(\theta) = y$

$\cos(\theta) = x$

$\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$
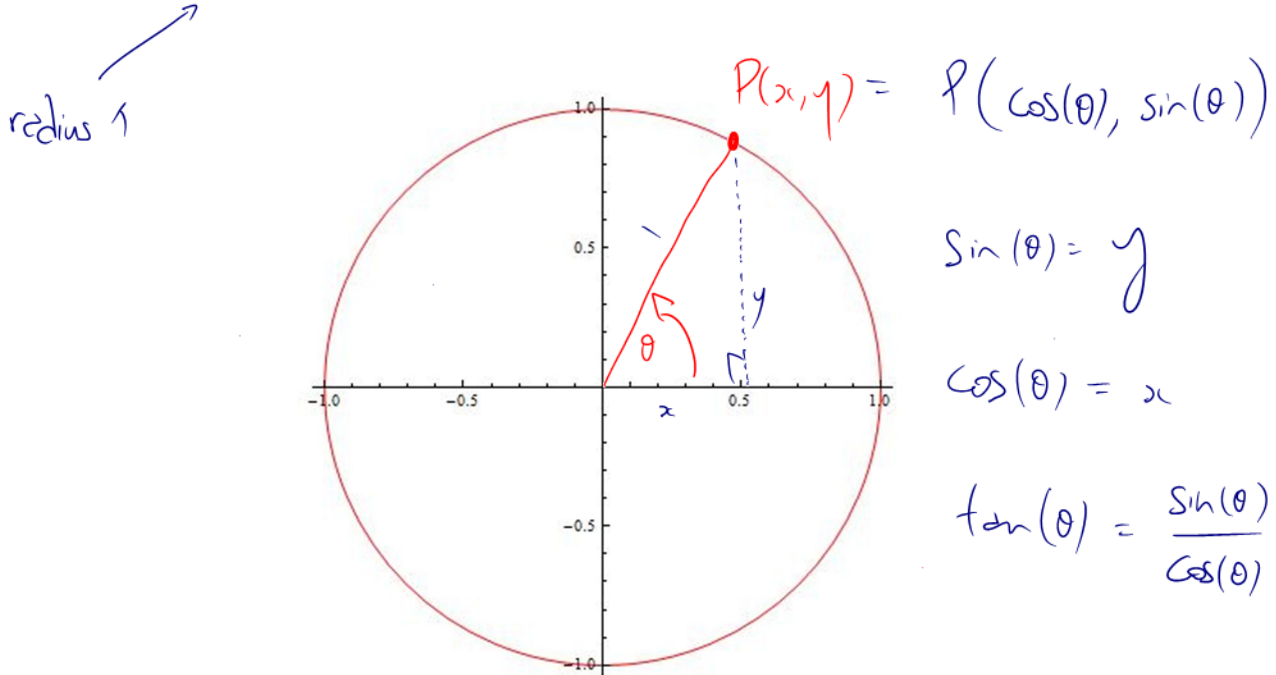
With this circle (and without a calculator!) we can evaluate EXACTLY the trig ratios for the angles (in radians) $\theta = 0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}, 2\pi$ radians.

$\theta = 0$

$\theta = 0$ rad $(1,0)$ $0, 2\pi$

$\cos(0) = 1$
$\sin(0) = 0$
$\tan(0) = 0$

$\theta = 2\pi$ $(1,0)$

$\sin(2\pi) = 0$, $\cos(2\pi) = 1$, $\tan(2\pi) = 0$

$\theta = \frac{\pi}{2}$

$(0,1)$ $\theta = \pi/2$

$\cos\left(\frac{\pi}{2}\right) = 0$
$\sin\left(\frac{\pi}{2}\right) = 1$
$\tan\left(\frac{\pi}{2}\right) = \dfrac{\sin(\pi/2)}{\cos(\pi/2)}$
$= $ undefined

$\theta = \pi$ $0, 2\pi$
$(-1,0)$

$\cos(\pi) = -1$
$\sin(\pi) = 0$
$\tan(\pi) = 0$

$\theta = \frac{3\pi}{2}$

$(0,-1)$

$\cos\left(\frac{3\pi}{2}\right) = 0$
$\sin\left(\frac{3\pi}{2}\right) = -1$
$\tan\left(\frac{3\pi}{2}\right)$ undefined

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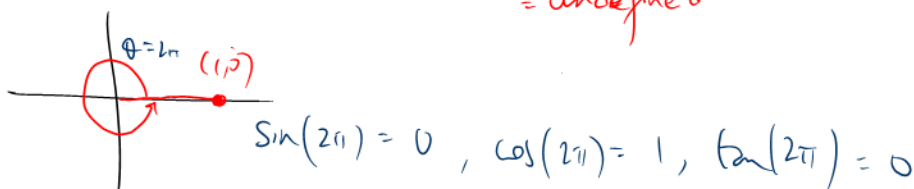
Now, using **Special Triangles**, and **CAST** we can evaluate **EXACTLY** trig ratios for "special angles".

Note: A trig ratio is a NUMBER.
Numbers have 2 qualities
1) value   (size)

2) Sign

Thus a trig ratio has a value

(which we e**valu**ate using the related acute angle and Special Triangles)

AND, a trig ratio has a Sign which we find using CAST

Example 5.3.1

Determine **Exactly** (i.e. the **use of a calculator** means **MARKS OFF**)

a) $\sin\left(\dfrac{\pi}{3}\right)$

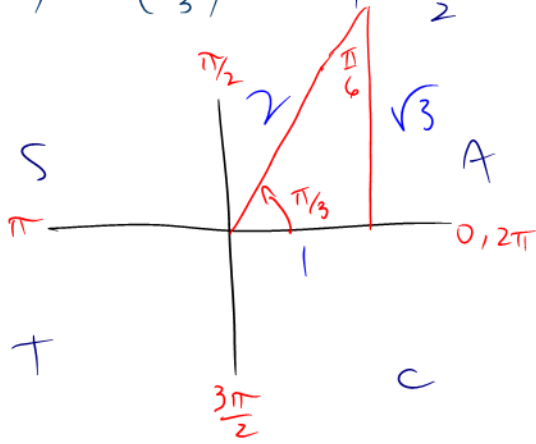b) $\cos\left(\dfrac{5\pi}{6}\right)$

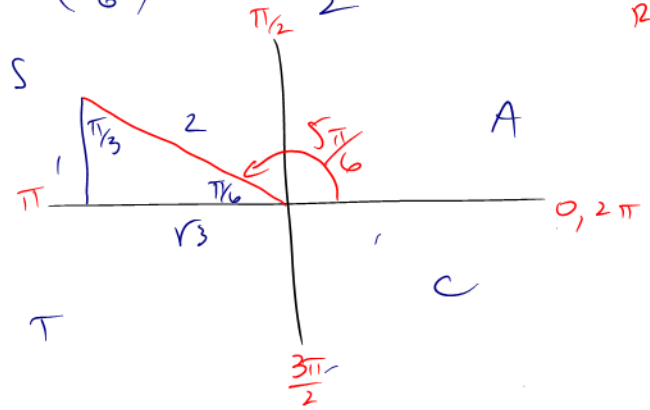c) $\tan\left(\dfrac{5\pi}{4}\right)$

d) $\sec\left(\dfrac{5\pi}{3}\right)$

e) $\tan\left(\dfrac{3\pi}{2}\right)$

f) $\csc(-\pi)$

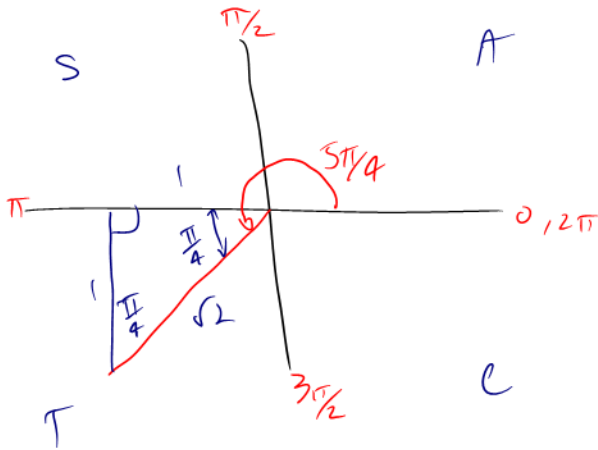**a)** $\sin\left(\frac{\pi}{3}\right) = \#\ \frac{\sqrt{3}}{2}$

$\frac{\pi}{2}$ 2 $\frac{\pi}{6}$ $\sqrt{3}$

S   A

π $\frac{\pi}{3}$ 1 $0, 2\pi$

T $\frac{3\pi}{2}$ C

b) $\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

$\frac{\pi}{2}$

S   2 $\frac{5\pi}{6}$ A

π 1 $\frac{\pi}{3}$ $\frac{\pi}{6}$ $0, 2\pi$

$\sqrt{3}$ 1   C

T $\frac{3\pi}{2}$

c) $\tan\left(\frac{5\pi}{4}\right) = +1$

$\frac{\pi}{2}$ A

S $\frac{5\pi}{4}$

π 1 $0, 2\pi$

$\frac{\pi}{4}$ b

1   $\frac{\pi}{4}$ $\sqrt{2}$ $\frac{3\pi}{2}$ C

T

d) $\sec\left(\frac{5\pi}{3}\right) = +2$

recip of cos. $\frac{hyp}{adj}$

$\frac{\pi}{2}$ A

S $\frac{5\pi}{3}$ 1

π $\frac{\pi}{3}$ $0, 2\pi$

2   $\sqrt{3}$

T $\frac{3\pi}{2}$ $\frac{\pi}{6}$

C

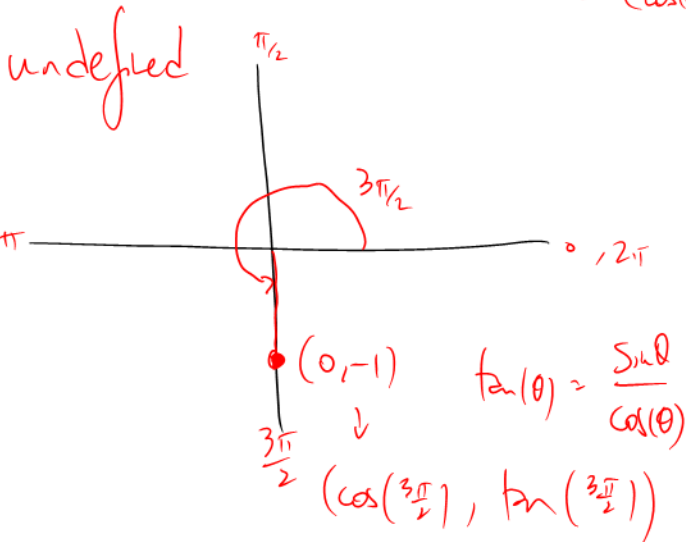**e)** $\tan\left(\frac{3\pi}{2}\right)$
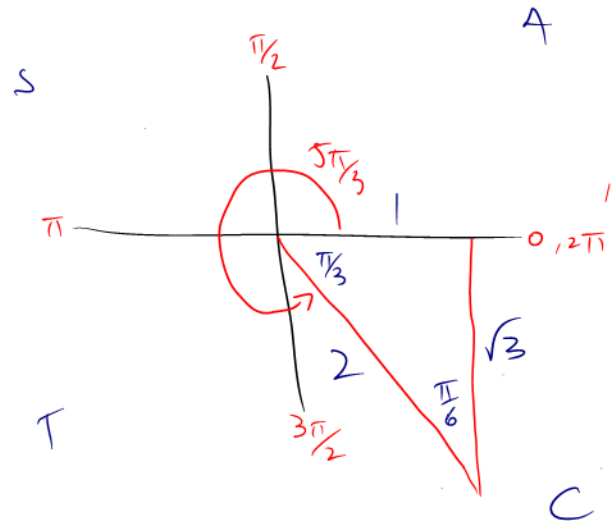
axis angle → use the coordinates of the "point" on the unit circle $(\cos(\theta), \sin(\theta))$

undefined

$\frac{\pi}{2}$

$\frac{3\pi}{2}$

π $0, 2\pi$

$(0,-1)$

$\frac{3\pi}{2}$ $\tan(\theta) = \frac{\sin\theta}{\cos(\theta)}$

$\left(\cos\left(\frac{3\pi}{2}\right), \tan\left(\frac{3\pi}{2}\right)\right)$

f) $\csc(-\pi)$

recip of sin   axis angle

$\sin(-\pi) = 0$

$\therefore \csc(-\pi)$ is undefined
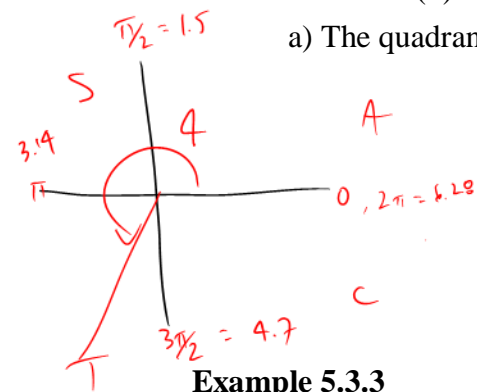
$(-1,0)$ $-\pi$

$(\cos(\theta), \sin(\theta))$

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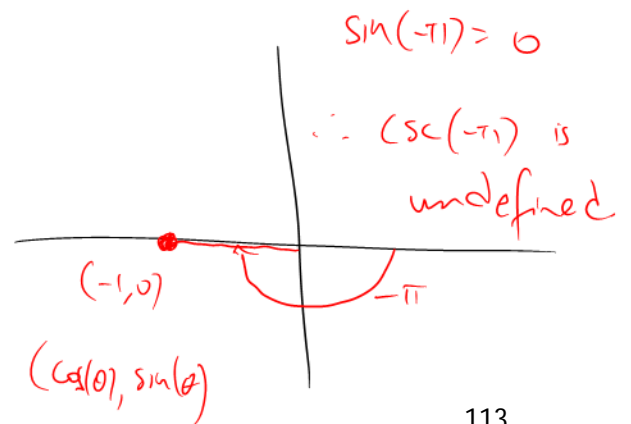
Example 5.3.2

Given $\sin(4)$ determine:

a) The quadrant $\theta = 4$ is in.

b) The sign of $\sin(4)$ (no calculators!)

$\frac{\pi}{2} = 1.5$

S

3.14

π

4

$0, 2\pi = 6.28$

$\frac{3\pi}{2} = 4.7$

T

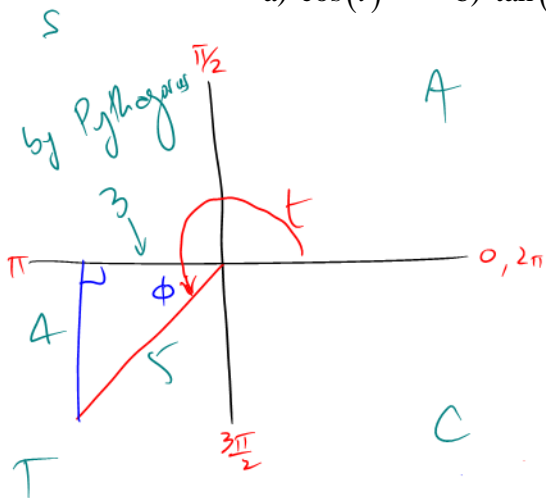A   QUADRANT

C

negative by CAST

Example 5.3.3

Q3

Given $\sin(t) = -\dfrac{4}{5}, \quad +\pi \leq t \leq \dfrac{3\pi}{2}$, determine

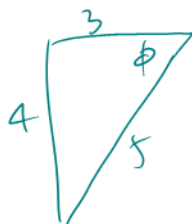a) $\cos(t)$ b) $\tan(t)$ c) t in radians, rounded to three decimal places.

S

by Pythagoras

$\frac{\pi}{2}$

A

3

ϕ

t

π

$0, 2\pi$

4

5

$\frac{3\pi}{2}$

C

T

a) $\cos(t) = -\dfrac{3}{5}$

b) $\tan(t) = \dfrac{4}{3}$

c) to get $t = \pi + \phi$ forget the sign!

calculate ϕ in the "given" $\triangle$

ϕ

3

4

5

$\phi = \sin^{-1}\left(\dfrac{4}{5}\right) = 0.927$

$\therefore \quad t = \pi + \phi$

$= 3.141 + 0.927 = 4.068 \text{ rad.}$

Class/Homework for Section 5.3

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