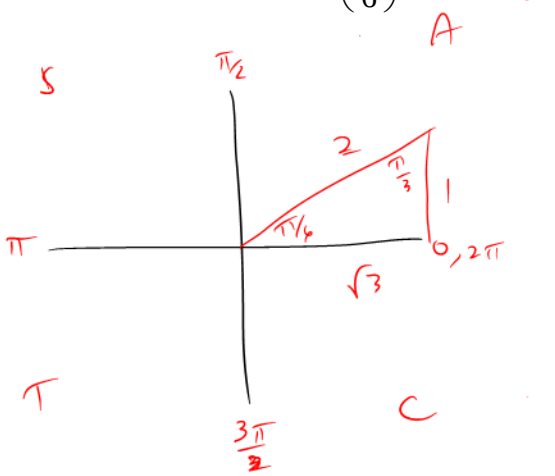
5.4 Trigonometric Ratios and Special Triangles (Part 3 – Getting the Angles)

We have been looking at **evaluating exact values** for trigonometric ratios using special triangles and CAST, given an **angle of rotation**. We now turn our attention to the **inverse operation** – determining **angles of rotation** given a **trig ratio**.

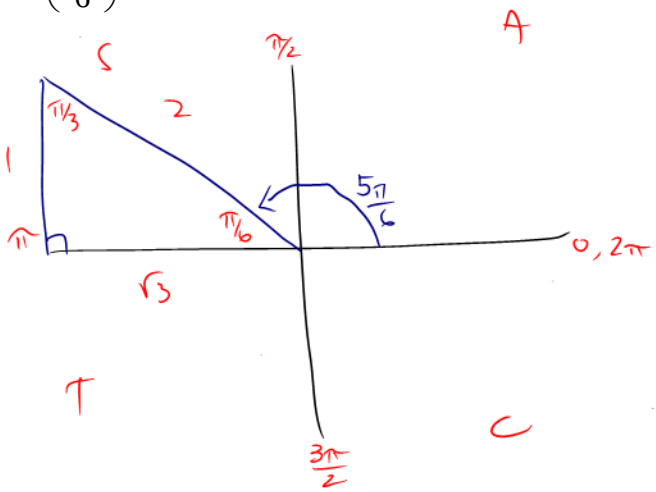
Example 5.4.1

Determine exactly:

a) $\sin\left(\frac{\pi}{6}\right)$ = $\frac{1}{2}$



b) $\sin\left(\frac{5\pi}{6}\right)$ = $\frac{1}{2}$



Note: There are TWO angles which have the same trig ratio

⇒ for problems like: $\sin(\theta) = \frac{1}{2}$, there will be two answers for θ

Example 5.4.2

Determine BOTH angles of rotation, θ, for $0 \le \theta \le 2\pi$ given

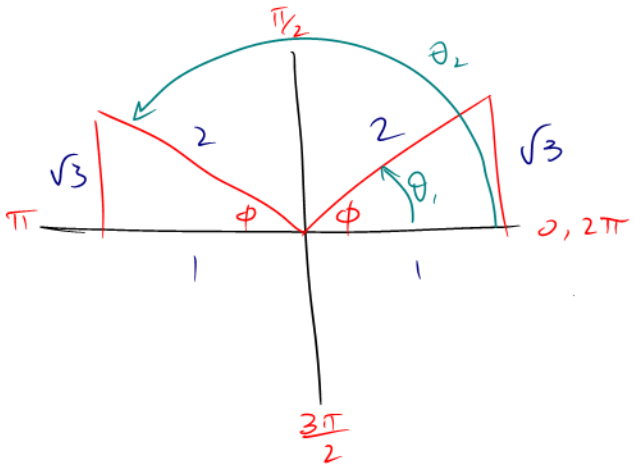
a) $\sin(\theta) = \frac{\sqrt{3}}{2}$

↑ the trig ratio is positive

sin is positive in Q1 & Q2

Procedure

1) Determine the quadrant**s** θ is in.
2) Draw the angle**s** of rotation.
3) Determine the related acute angle φ and construct the appropriate special triangle**s**.
4) Determine the angle**s** of rotation.



$\phi = \frac{\pi}{3}$

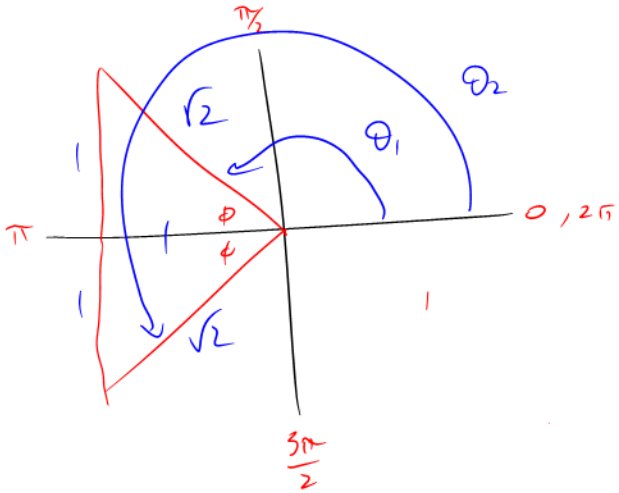
$\therefore \theta_1 = \frac{\pi}{3}, \theta_2 = \frac{2\pi}{3}$

b) $\cos(\theta) = -\frac{1}{\sqrt{2}}$

Cos is neg in Q2, Q3

$\phi = \frac{\pi}{4}$

$\therefore \theta_1 = \frac{3\pi}{4}, \theta_2 = \frac{5\pi}{4}$



← reciprocal trig ratio. Convert to a primary trig ratio by "flipping"

c) $\cot(\theta) = -\sqrt{3}$

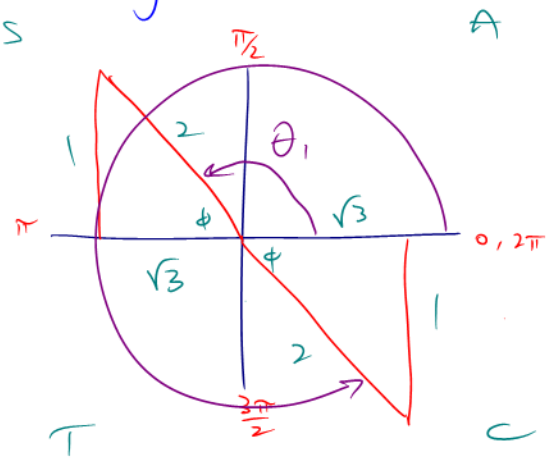
$\tan(\theta) = -\frac{1}{\sqrt{3}}$

Q2
Q.4

$\phi = \frac{\pi}{6}$

$\theta_1 = \frac{5\pi}{6}$

$\theta_2 = \frac{11\pi}{6}$



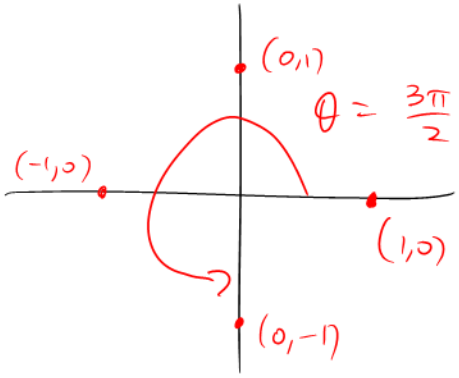
d) $\sin(\theta) = -1$

Note: $\sin(\theta) = \pm 1$ or $\sin\theta = 0$ ⇒ axis angle

$\cos(\theta) = \pm 1$ or $\cos\theta = 0$ ⇒ axis angle

$\tan(\theta) = 0$ ⇒ axis angle.

easiest technique is use the "points" $P(x,y) = P(\cos\theta, \sin\theta)$



Example 5.4.3

Determine θ where $\frac{3\pi}{2} \le \theta \le 2\pi$ for $\csc(\theta) = -2$

QA

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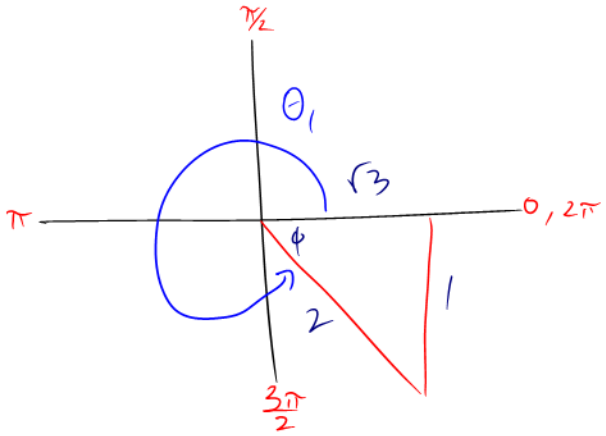
$\sin(\theta) = -\frac{1}{2}$



$\phi = \frac{\pi}{6}$

$\theta_1 = \frac{11\pi}{6}$.

Practice Problems

Determine the angles of rotation, θ, for $0 \le \theta \le 2\pi$:

a) $\sin(\theta) = -\frac{\sqrt{3}}{2}$

b) $\sec(\theta) = \sqrt{2}$

c) $\tan(\theta) = \frac{1}{\sqrt{3}}$

d) $\cot(\theta) = -1$

e) $\csc(\theta) = \frac{2}{\sqrt{3}}$

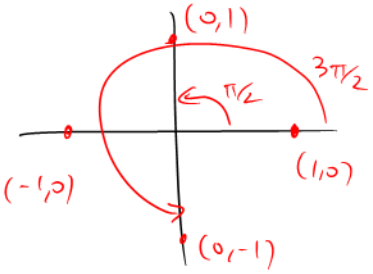
f) $\cos(\theta) = 0$

g) $\sin(\theta) = 1$

h) $\sqrt{3}\cos(\theta) - 2\cos(\theta)\cdot\sin(\theta) = 0$

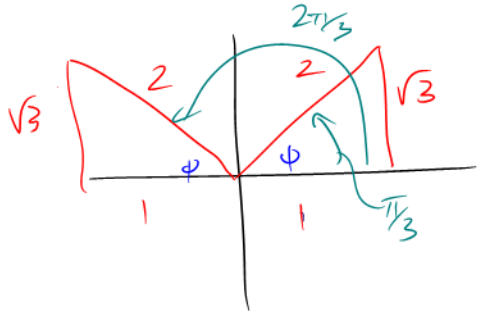
h) $\cos(\theta)\left(\sqrt{3} - 2\sin(\theta)\right) = 0$

$\Rightarrow \cos(\theta) = 0$ or $\sqrt{3} - 2\sin\theta = 0$



$\Rightarrow \sin(\theta) = \frac{\sqrt{3}}{2}$

$\phi = \frac{\pi}{3}$



$\therefore \theta_1 = \frac{\pi}{3}, \theta_2 = \frac{\pi}{2}, \theta_3 = \frac{2\pi}{3}, \theta_4 = \frac{3\pi}{2}$

Class/Homework for Section 5.4

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