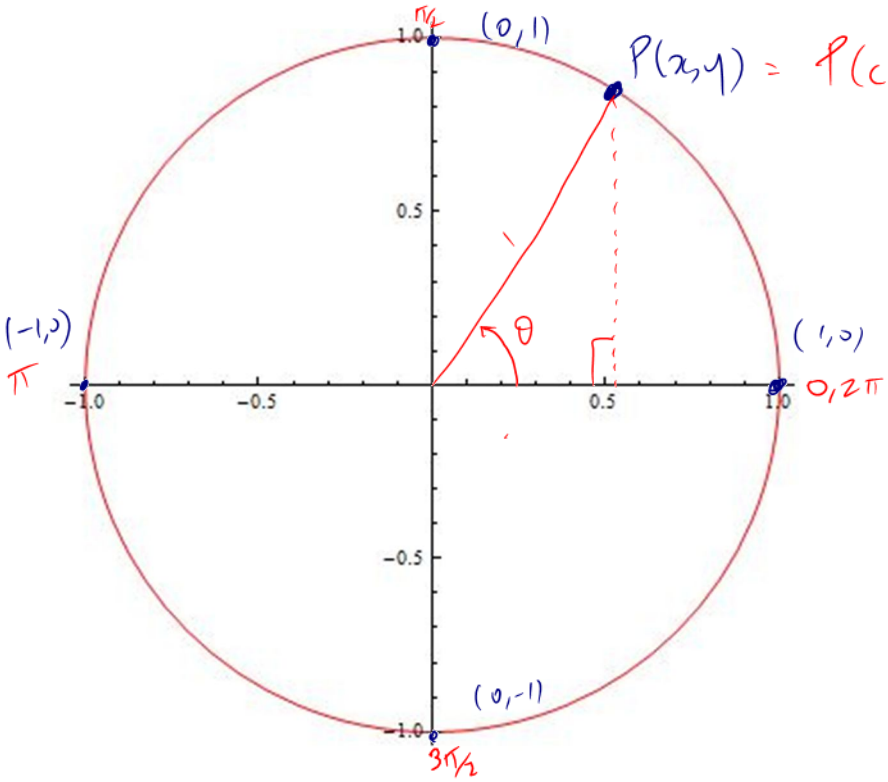
5.5 Sketching the Trigonometric Functions

Before beginning the sketches, recall the diagram of the unit circle that we have been using to explore the basic ideas in trigonometry:



$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$

$\sin(0) = 0$ $\quad \cos(0) = 1$ $\quad \tan(0) = 0$

$\sin\left(\frac{\pi}{2}\right) = 1$ $\quad \cos\left(\frac{\pi}{2}\right) = 0$ $\quad \tan\left(\frac{\pi}{2}\right) = \text{undef.}$

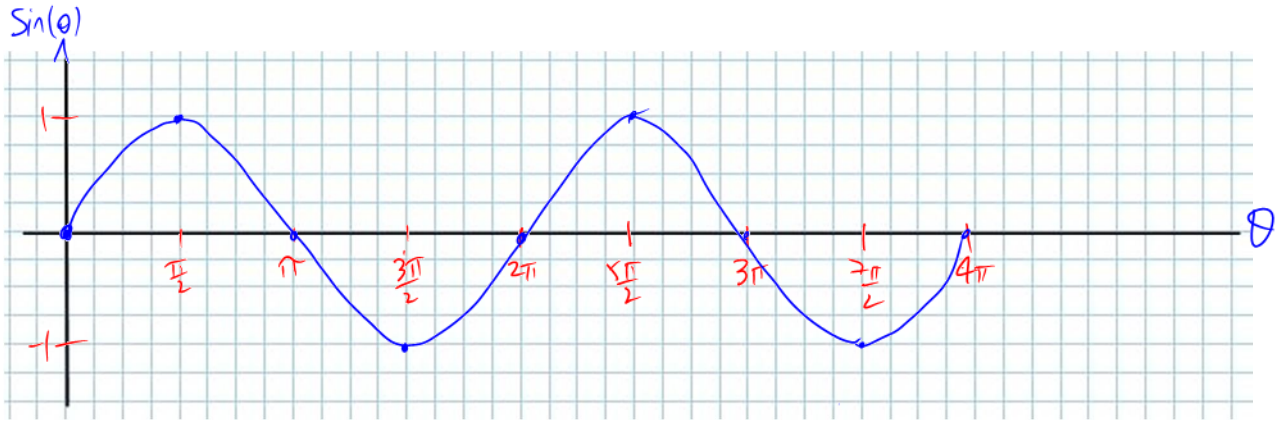
$\sin(\pi) = 0$ $\quad \cos(\pi) = -1$ $\quad \tan(\pi) = 0$

$\sin\left(\frac{3\pi}{2}\right) = -1$ $\quad \cos\left(\frac{3\pi}{2}\right) = 0$ $\quad \tan\left(\frac{3\pi}{2}\right) = \text{undef}$

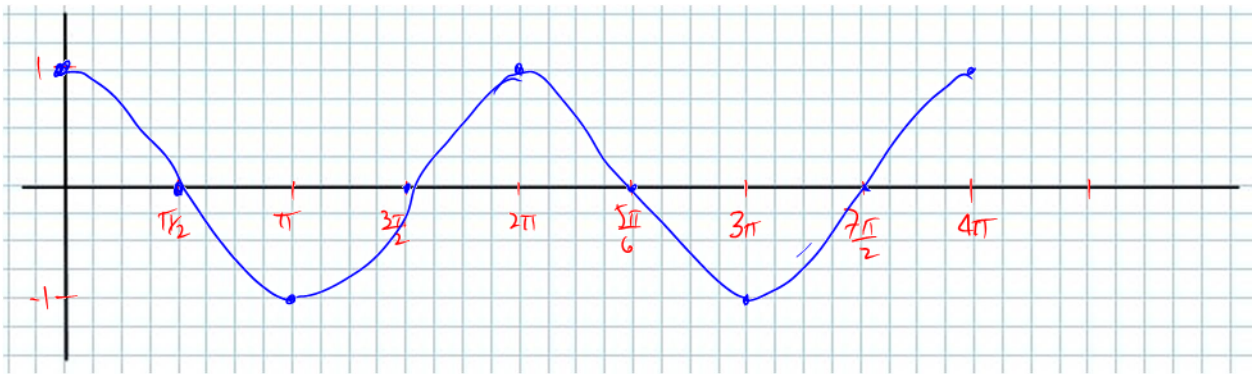
$\sin(2\pi) = 0$ $\quad \cos(2\pi) = 1$ $\quad \tan(2\pi) = 0$

The Primary Trigonometric Functions

$f(\theta) = \sin(\theta), \quad \theta \in [0, 4\pi]$



$g(\theta) = \cos(\theta)$

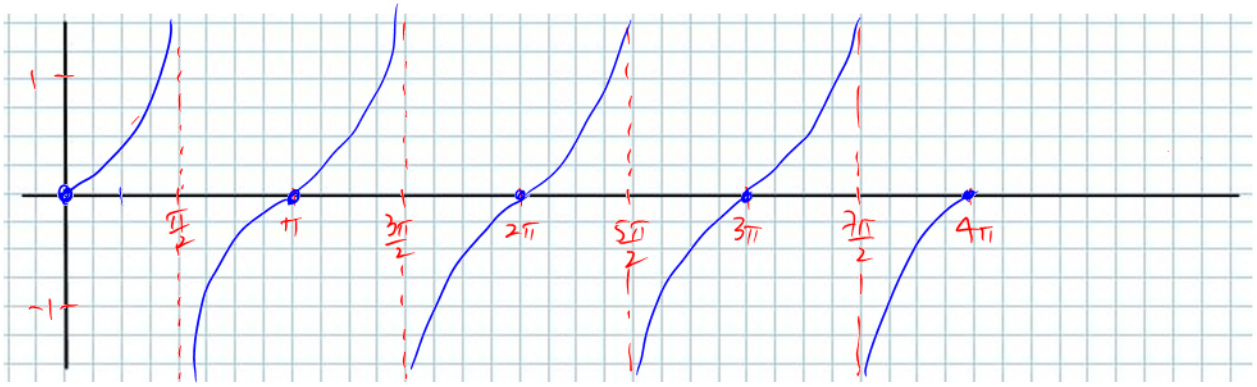


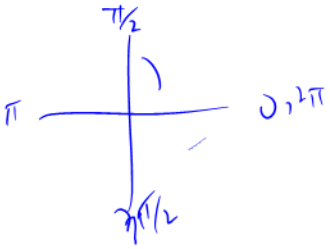
$h(\theta) = \tan(\theta)$

because $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)} \Rightarrow \tan(\theta)$ has a V.A. when

$\cos(\theta) = 0 \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\tan(\theta) = 0$ when $\sin(\theta) = 0 \Rightarrow \theta = 0, \pi, 2\pi, 3\pi, 4\pi$

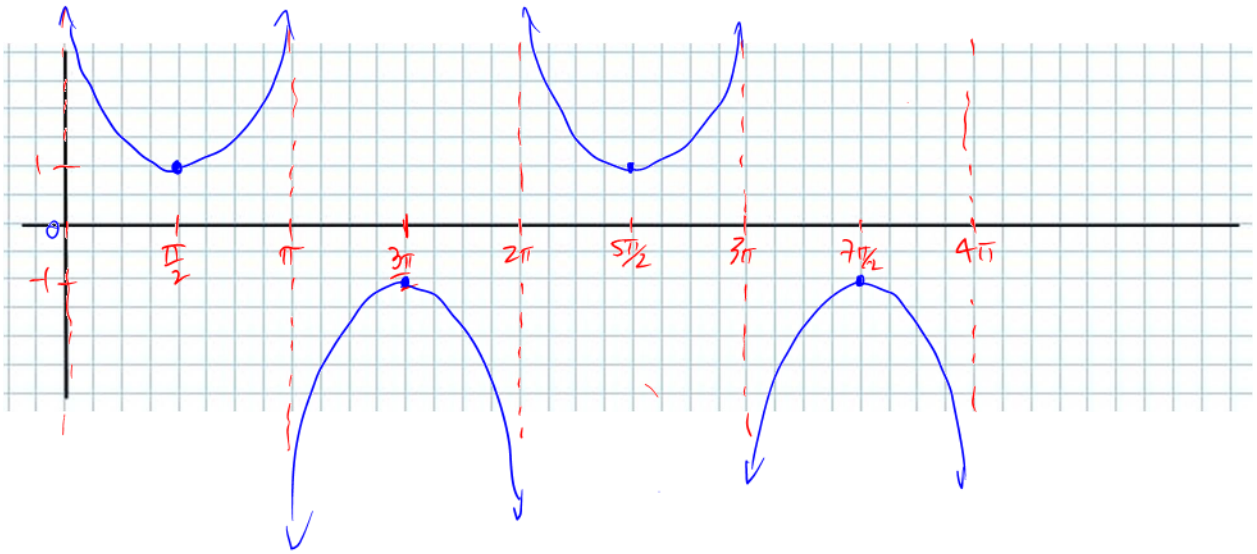




The Reciprocal Trig Functions

$f(\theta) = \csc(\theta)$ $\Rightarrow \csc(\theta) = \dfrac{1}{\sin(\theta)}$

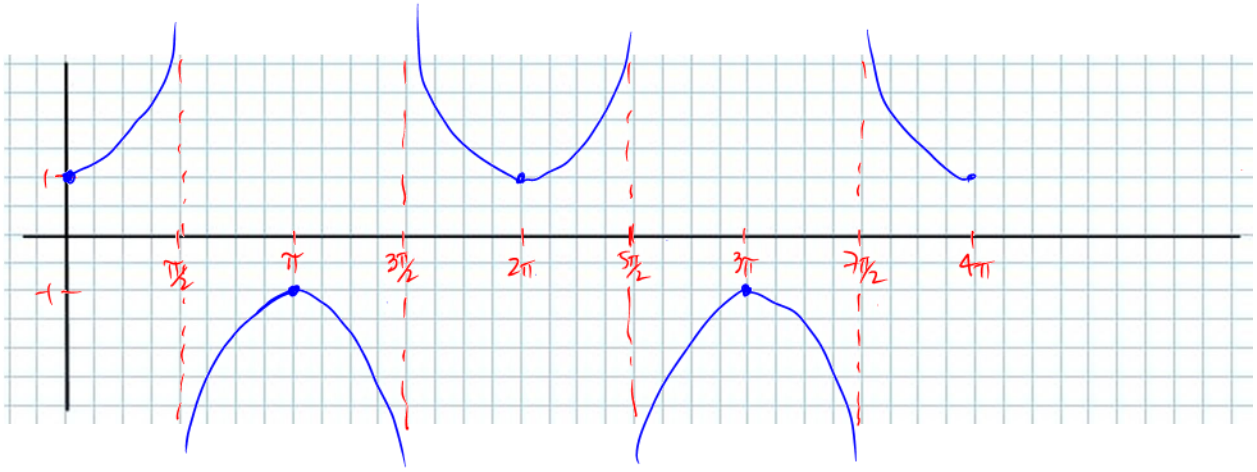
V.A when $\sin\theta = 0 \Rightarrow \theta = 0, \pi, 2\pi, 3\pi, 4\pi$



$g(\theta) = \sec(\theta)$

$\frac{1}{\cos\theta}$ ⇒ V.A when $\cos(\theta) = 0$

⇒ $\theta = \pi/2, 3\pi/2, 5\pi/2, 7\pi/2$



$h(\theta) = \cot(\theta)$

$\cot(\theta) = \frac{\cos\theta}{\sin\theta}$

$= \frac{1}{\tan\theta}$ ∴ V.A whenever $\tan(\theta) = 0 \Rightarrow \theta = 0, \pi, 2\pi, 3\pi, 4\pi$

Zero when $\cos(\theta) = 0 \Rightarrow \theta = \pi/2, 3\pi/2, 5\pi/2, 7\pi/2$