5.6 Transformations of Trigonometric Functions

By this point in your illustrious High School careers, you have a solid understanding of Transformations of Functions in general. In terms of the trig functions Sine and Cosine in particular, the concepts are as you expect, but the transformations have specific meanings relating to nature of the sinusoidal "wave".

General Form of the Sine and Cosine Functions

$$f(\theta) = a\sin\left(k(\theta - d)\right) + c \qquad g(\theta) = a\cos\left(k(\theta - d)\right) + c$$

$|a| =$ Amplitude (vertical stretch) $\quad a = \dfrac{\max - \min}{2}$ → distance from central axis to a max or min.

$k =$ Period Factor (horizontal stretch) $\quad$ Period $= P = \dfrac{2\pi}{k}$ ← "Amount" of the domain for one cycle

$d =$ Phase Shift (horizontal shift) $\quad$ Note: To determine d you MUST isolate the angle θ.

$c =$ central axis (vertical shift) $\quad c = \dfrac{\max + \min}{2}$ ← exactly $\frac{1}{2}$ way between the max & min values.

"$y = c$" is the eqn of the central axis — horizontal line around which the fn "bounces"

Example 5.6.1

Determine the amplitude, period, phase shift and the equation of the central axis for:

a) $f(\theta) = 2\sin\left(\theta + \frac{\pi}{3}\right) + 1$

$|a| = 2 \Rightarrow$ Amplitude $= 2$

C.A. : $y = 1$

Period : $k = 1$: $P = \frac{2\pi}{1} = 2\pi$

Phase Shift : $\frac{\pi}{3}$ left

b) $g(\theta) = 3\cos\left(2\theta - \frac{\pi}{2}\right)$ Isolate θ

$g(\theta) = 3\cos\left(2\left(\theta - \frac{\pi}{4}\right)\right)$

$|a| = 3$

C.A: $y = 0$

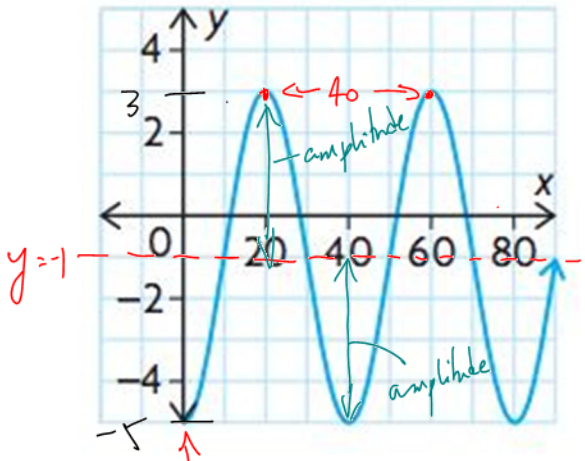
Period : $k = 2 \Rightarrow P = \frac{2\pi}{2} = \pi$

Phase Shift : $\frac{\pi}{4}$ right

Example 5.6.2

From your text: Pg. 346 #14c

Determine a ***sinusoidal*** function for the given sketch of a graph



min at vert axis ⇒ suggests a flip (for cos)

Amplitude: $|a| = \frac{\max - \min}{2} = \frac{3-(-5)}{2} = \frac{8}{2} = 4$

Period: $P = 40 \Rightarrow k = \frac{2\pi}{P} = \frac{2\pi}{40} = \frac{\pi}{20}$

Phase Shift: As Cosine starts at max (or a min if "flipped") 20 right OR $a = -4$ with no phase shift

As Sine starts at central 10 right

Equation of Central Axis: $y = \frac{\max + \min}{2} = \frac{3+(-5)}{2} = -1$

$y = -1$

Equation as a Cosine Wave

$f(x) = 4\cos\left(\frac{\pi}{20}(x - 20)\right) - 1$

OR

$f(x) = -4\cos\left(\frac{\pi}{20}x\right) - 1$

Equation as a Sine Wave

$g(x) = 4\sin\left(\frac{\pi}{20}(x - 10)\right) - 1$

$$f(x) = a\sin\left(k(\theta - d)\right) + c$$

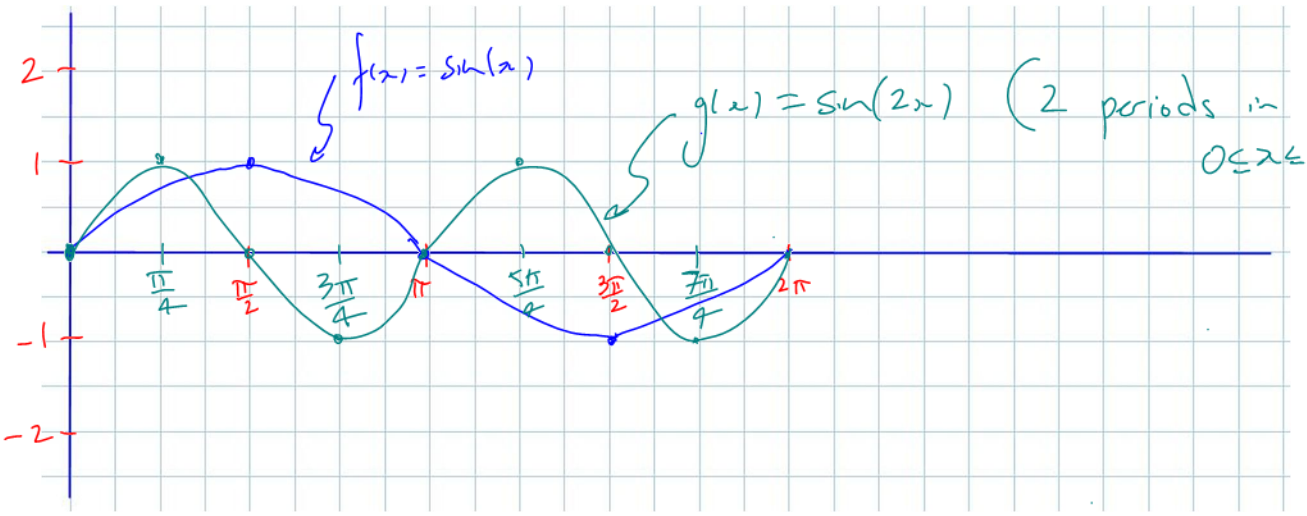
Example 5.6.3

Sketch $f(x) = \sin(x)$ and $g(x) = \sin(2x)$ for $0 \le x \le 2\pi$ on the same set of axes.

period is 2π

Period is. $P = \frac{2\pi}{2} = \pi$ — split 'P' into quadrants

one "division" is $\frac{\pi}{4}$



$-2\left(\frac{1}{2}\right) + 1 = 0$

use your calculator!

$f(2\pi) =$

Example 5.6.4

flips cos ⇒ starts at a min

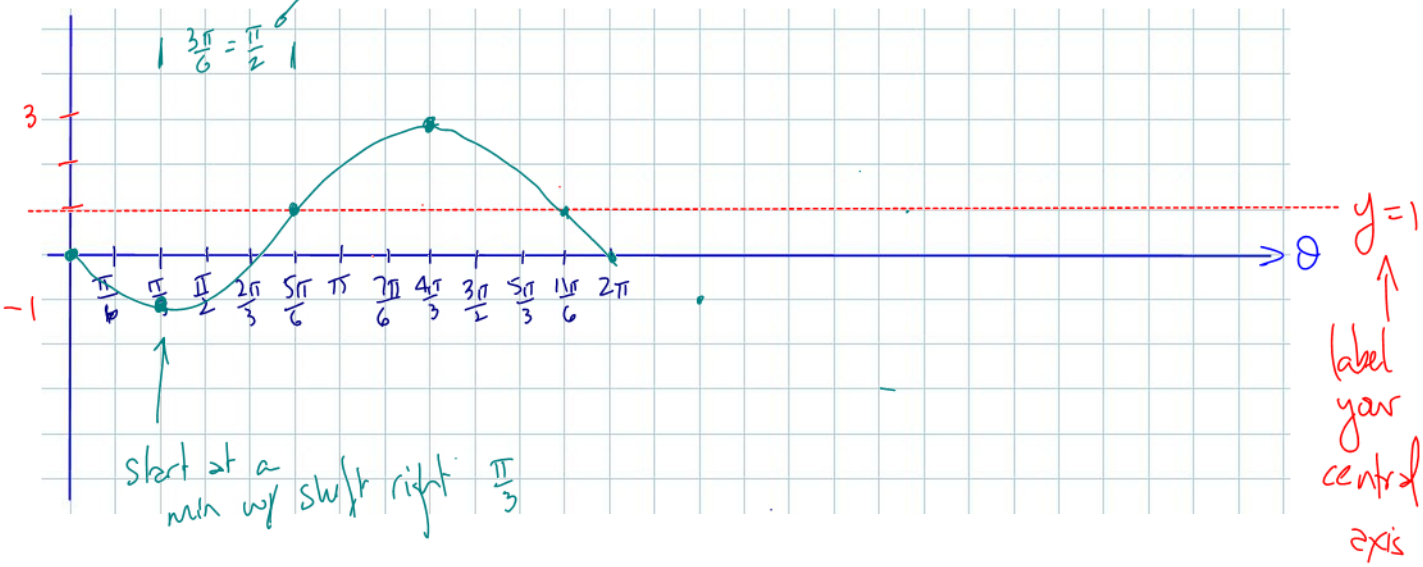
Sketch $f(\theta) = -2\cos\left(\theta - \frac{\pi}{3}\right) + 1$ on $0 \le \theta \le 2\pi$

$|a| = 2$

$k = 1 \Rightarrow P = \frac{2\pi}{1}$ division is $\frac{2\pi}{4} = \frac{\pi}{2}$

P.S.: $\frac{\pi}{3}$ right ca. $y = 1$

$f(0) = -2\cos\left(-\frac{\pi}{3}\right) + 1$
$= 0$



Note: to get a "good" scale find the common denominator between the "division" of the period and the phase shift (quads)

$\frac{\pi}{2}, \frac{\pi}{3}$

125

scale $\frac{\pi}{6}$

in $0 \le \theta \le 2\pi$

$\frac{2\pi}{3} \div 2 = \frac{2\pi}{3} \times \frac{1}{2}$.

Example 5.6.5

Sketch $f(\theta) = 3\sin\left(2\theta + \frac{2\pi}{3}\right) - 1$ $\Rightarrow$ $f(\theta) = 3\sin\left(2\left(\theta + \frac{\pi}{3}\right)\right) - 1$

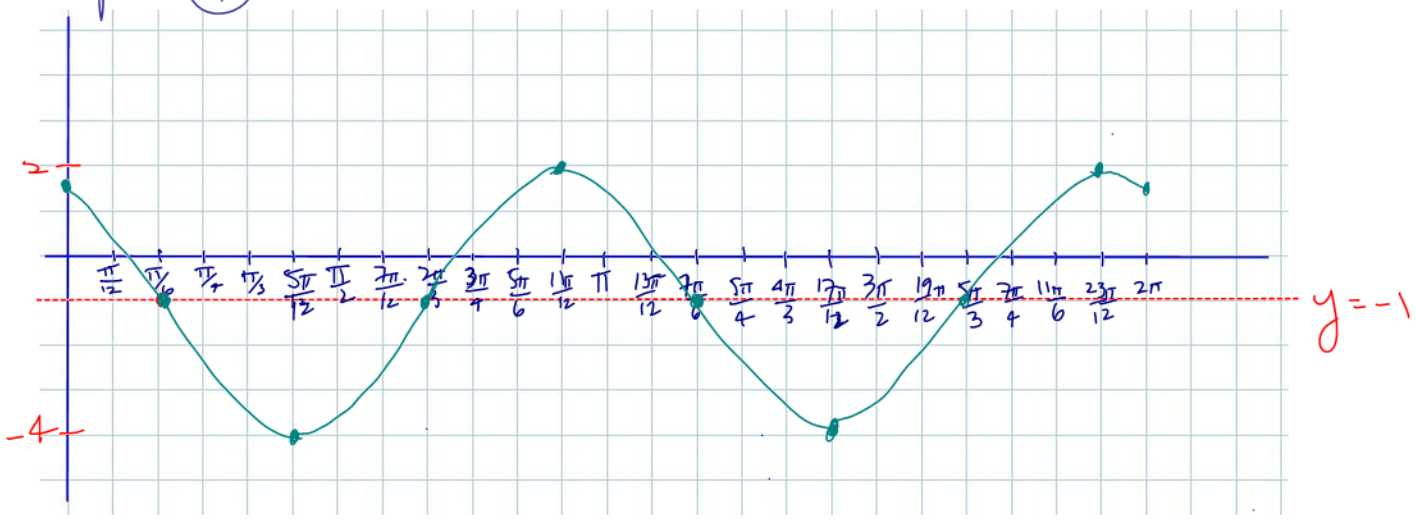
$|a| = 3$

$k = 2 \Rightarrow P = \frac{2\pi}{2} = \pi$

Phase shift : $\frac{\pi}{3}$ left

CA. : $y = -1$

div into quad. $\frac{\pi}{4}$



Scale: common denom between the $d\theta$ $\frac{\pi}{4}$ and the phase shift $\Rightarrow \frac{\pi}{3}$

$\frac{\pi}{12}$

We need

$f(0) = 1.6$, $f(2\pi) = 1.6$

Class/Homework for Section 5.6

Pg. 343 - 345 #1, 4, 6 – 8, 13, 14ab