ADVANCED FUNCTIONS

Chapter 6 – Trigonometric Identities and Equations

(*Material adapted from Chapter 7 of your text*)

Chapter 6 – Trigonometric Identities and Equations

Contents with suggested problems from the Nelson Textbook (Chapter 7)

6.1 Basic Trigonometric Equivalencies

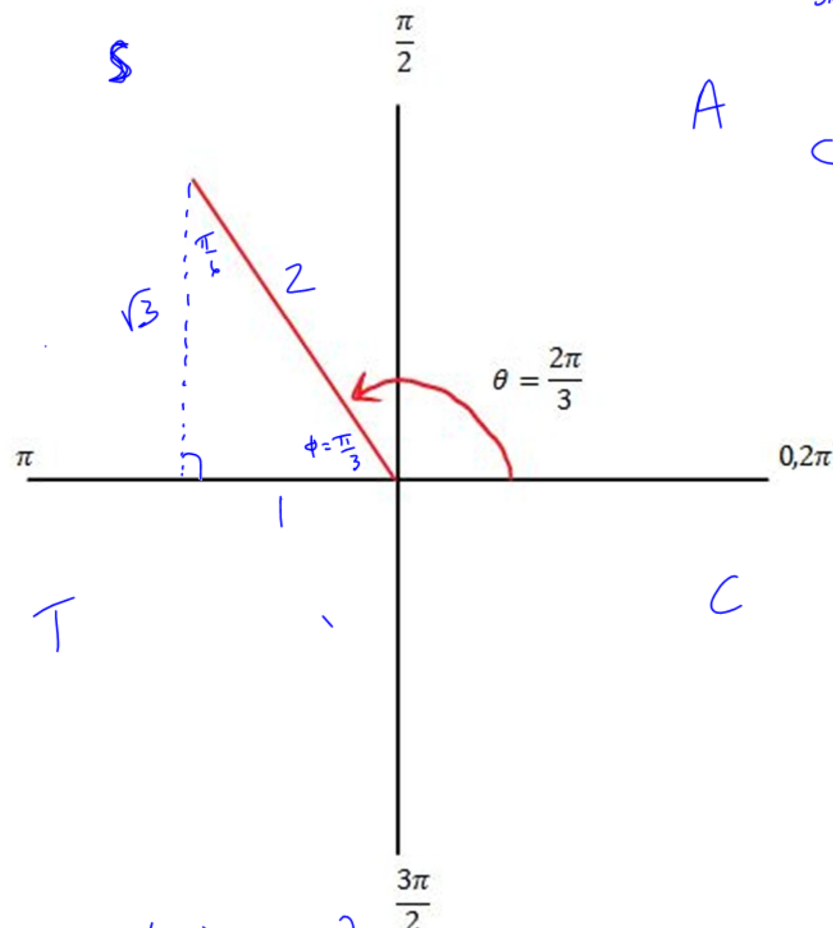
We have already seen a very basic trigonometric equivalency when we considered angles of rotation. For example, consider the angle of rotation for $\theta = \frac{2\pi}{3}$:

$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\cos\left(\frac{\pi}{3}\right) = +\frac{1}{2}$

$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$



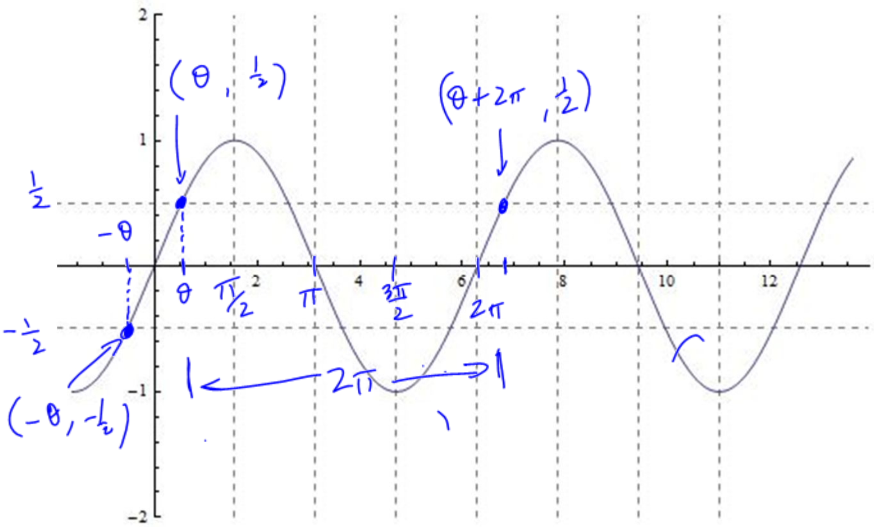
$$\left.\begin{aligned}\sin\left(\frac{2\pi}{3}\right) &= \sin\left(\frac{\pi}{3}\right)\\ \cos\left(\frac{2\pi}{3}\right) &= -\cos\left(\frac{\pi}{3}\right)\end{aligned}\right\}$$ basic trig equivalencies

Periodic Equivalencies

Sine is "2π - Periodic"

Example 6.1.1

Consider the sketch of the function $f(\theta) = \sin(\theta)$



$\sin(\theta) = \sin(2\pi + \theta)$

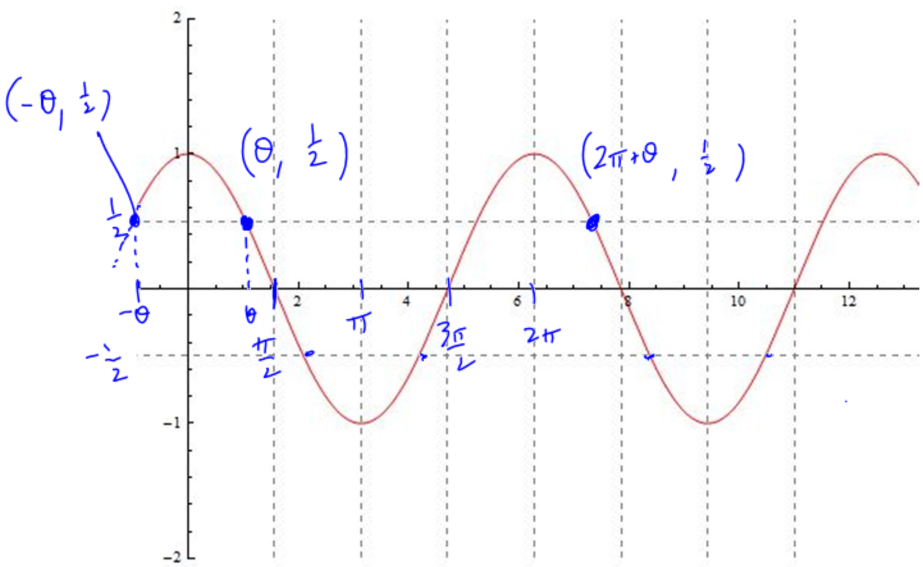
$\sin(\theta) = -\sin(-\theta)$

Sin is an odd fn

Example 6.1.2

Consider $g(x) = \cos(x)$

Cosine is 2π - periodic



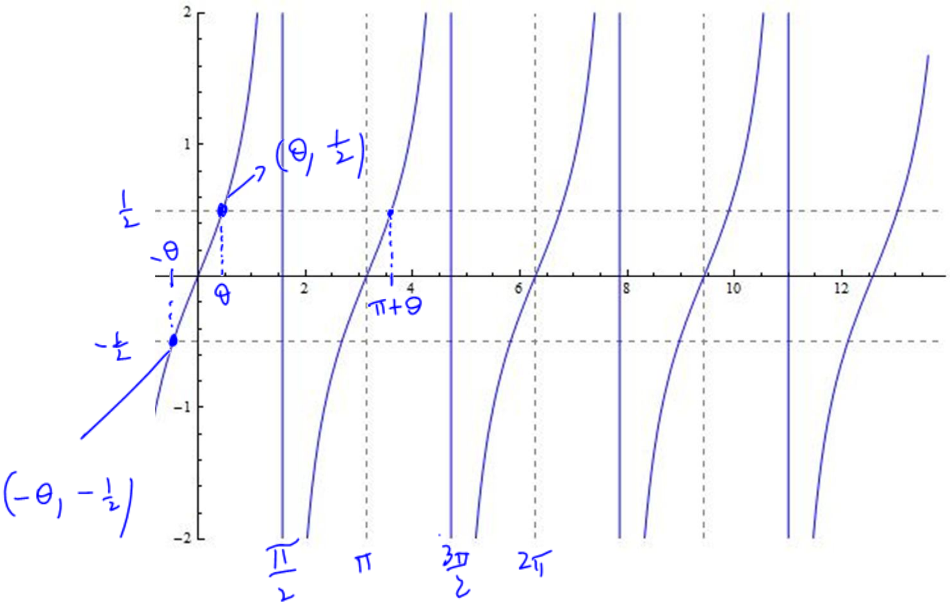
$\cos(\theta) = \cos(2\pi + \theta)$

$\cos(\theta) = \cos(-\theta)$

$\therefore$ $\cos\theta$ is even

Example 6.1.3

Consider $h(\theta)=\tan(\theta)$



tangent is π-periodic

$\Rightarrow \tan(\theta) = \tan(\pi+\theta)$

$\tan(\theta) = -\tan(-\theta)$

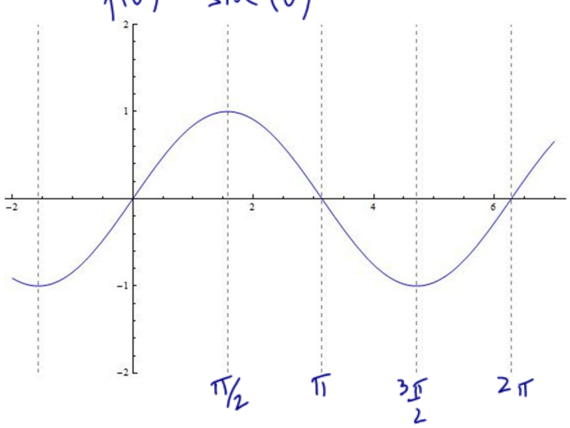
$\Rightarrow \tan(\theta)$ is odd.

Shift Equivalencies

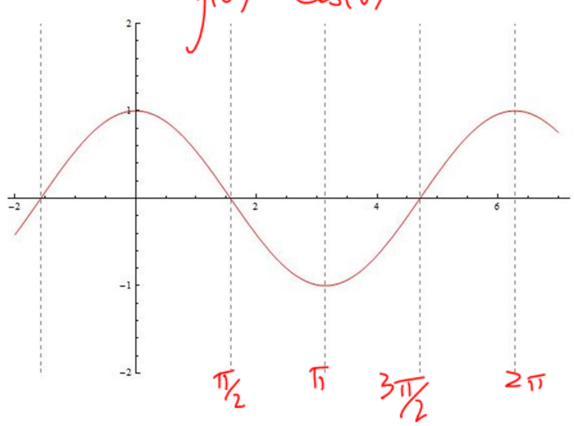
Example 6.1.4

Consider the sketches of the graphs for $f(\theta)=\sin(\theta)$ and $g(\theta)=\cos(\theta)$

$f(\theta) = \sin(\theta)$



$g(\theta) = \cos(\theta)$



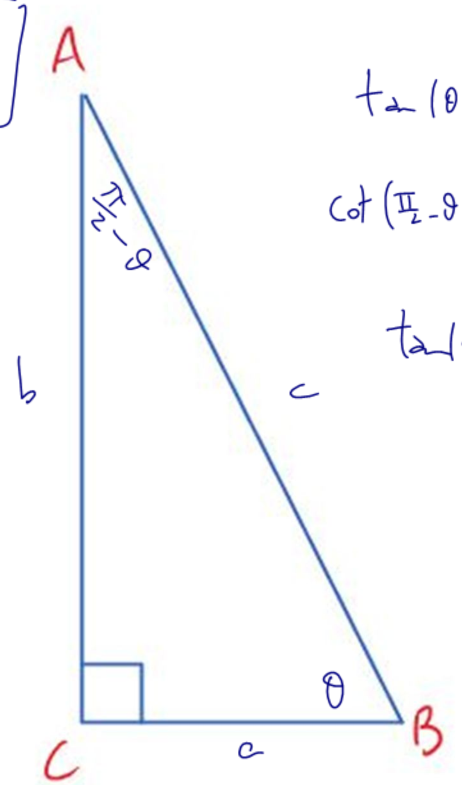
$\sin(\theta) = \cos(\theta - \pi/2)$

$\cos(\theta) = \sin(\theta + \frac{\pi}{2})$

$$\boxed{fn(\theta) = cofn\left(\frac{\pi}{2}-\theta\right)}$$

Cofunction Identities

Consider the right angle triangle



$\sin(\theta) = \frac{b}{c}$ but $\cos\left(\frac{\pi}{2}-\theta\right) = \frac{b}{c}$

$\therefore \sin(\theta) = \cos\left(\frac{\pi}{2}-\theta\right)$

$\cos(\theta) = \sin\left(\frac{\pi}{2}-\theta\right)$

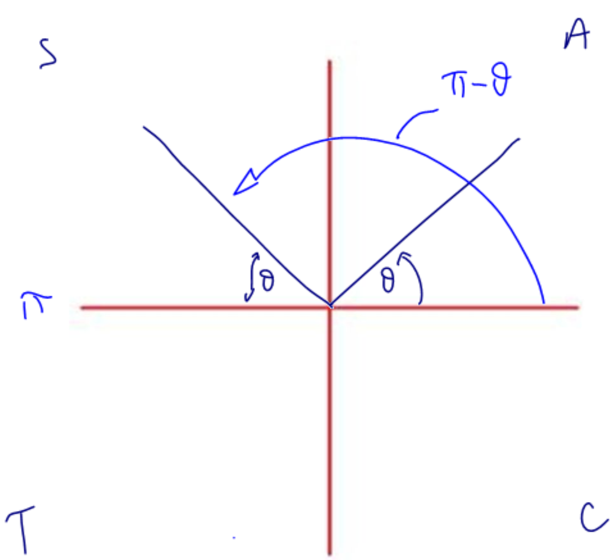
$\tan(\theta) = \frac{b}{a}$

$\cot\left(\frac{\pi}{2}-\theta\right) = \frac{b}{a}$

$\tan(\theta) = \cot\left(\frac{\pi}{2}-\theta\right)$

Using CAST, relating angles of rotation to π and 2π

Compare Q1 and Q2

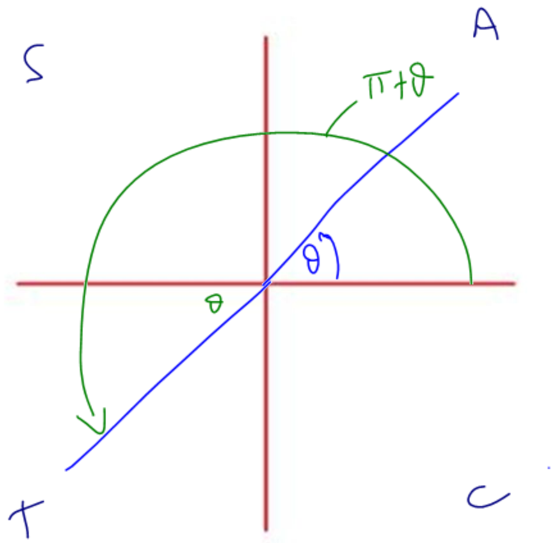


$\sin(\theta) = \sin(\pi-\theta)$

$\cos(\theta) = -\cos(\pi-\theta)$

$\tan(\theta) = -\tan(\pi-\theta)$

Compare Q1 and Q3

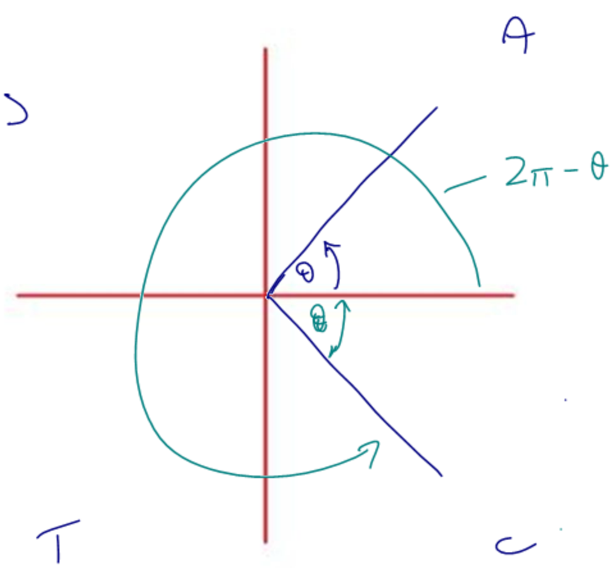


$$\sin(\theta) = -\sin(\pi+\theta)$$

$$\cos(\theta) = -\cos(\pi+\theta)$$

$$\tan(\theta) = +\tan(\pi+\theta)$$

Compare Q1 and Q4



$$\sin(\theta) = -\sin(2\pi-\theta)$$

$$\cos(\theta) = +\cos(2\pi-\theta)$$

$$\tan(\theta) = -\tan(2\pi-\theta)$$

$fn(\theta) = cofn\left(\frac{\pi}{2} - \theta\right)$

Example 6.1.5

From your text: Pg. 392 #3

Use a cofunction identity to find an equivalency:

a) $\sin\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{2} - \frac{\pi}{6}\right) = \cos\left(\frac{3\pi}{6} - \frac{\pi}{6}\right) = \cos\left(\frac{2\pi}{6}\right)$

$= \cos\left(\pi/3\right)$

θ

$\therefore \sin\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{3}\right)$

d) $\cos\left(\frac{5\pi}{16}\right) = \sin\left(\frac{\pi}{2} - \frac{5\pi}{16}\right)$

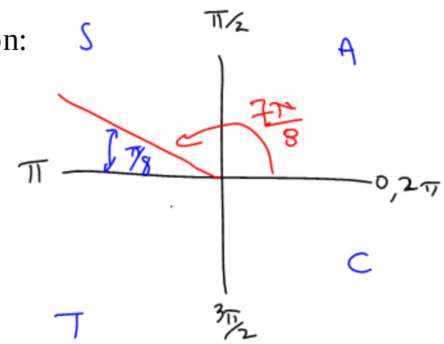
$= \sin\left(\frac{8\pi}{16} - \frac{5\pi}{16}\right) = \sin\left(\frac{3\pi}{16}\right)$

$\therefore \cos\left(\frac{5\pi}{16}\right) = \sin\left(\frac{3\pi}{16}\right)$

Example 6.1.6

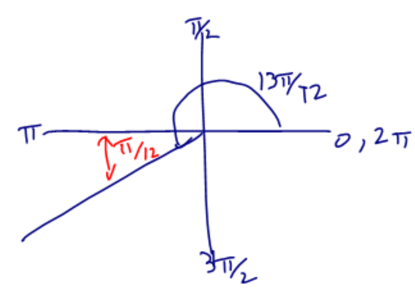
From your text: Pg. 393 #5

Using the related acute angle, find an equivalent expression:



a) $\sin\left(\frac{7\pi}{8}\right) = \sin\left(\frac{\pi}{8}\right)$

b) $\cos\left(\frac{13\pi}{12}\right) = -\cos\left(\frac{\pi}{12}\right)$



Class/Homework for Section 6.1

Pg. 392 – 393 #3cdef, 5cdef