6.3 Double Angle Formulae

This is a nice extension of the compound angle formulae from section 6.2.

Recall the compound Angle Formulae:

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \sin(B)\cos(A)$$

$$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$$

$$\tan(A \pm B) = \frac{\tan(A) \pm \tan(B)}{1 \mp \tan(A)\tan(B)}$$

The Double Angle Formulae

1) $\sin(2A) = \sin(A+A)$

$= \sin(A)\cos(A) + \sin(A)\cos(A)$

$$\boxed{\sin(2A) = 2\sin(A)\cos(A)}$$

2) $\cos(2A) = \cos(A+A)$

$= \cos(A)\cos(A) - \sin(A)\sin(A)$

$$\boxed{\cos(2A) = \cos^2(A) - \sin^2(A)}$$

By Pythagoras

$\cos^2 A = 1 - \sin^2 A$

or $\sin^2 A = 1 - \cos^2 A$

or

$$\cos(2A) = (1 - \sin^2(A)) - \sin^2(A)$$

$$\Rightarrow \boxed{\cos(2A) = 1 - 2\sin^2 A}$$

$\left(\Rightarrow \text{we can write } \sin^2(A) = \dfrac{1-\cos(2A)}{2} \right)$

OR

$$\cos(2A) = \cos^2(A) - (1 - \cos^2(A))$$

$$\Rightarrow \boxed{\cos(2A) = 2\cos^2(A) - 1}$$

$\left(\Rightarrow \text{we can write } \cos^2(A) = \dfrac{\cos(2A)+1}{2} \right)$

there are $\boxed{3}$ formulae for $\cos(2A)$

3) $\tan(2A) = \dfrac{\sin(2A)}{\cos(2A)} = \dfrac{2\sin(A)\cos(A)}{\cos^2(A) - \sin^2(A)}$ (not often used)

OR

$$\tan(2A) = \tan(A+A) = \frac{\tan(A) + \tan(A)}{1 - \tan(A)\cdot\tan(A)}$$

$$\rightarrow \boxed{\tan(2A) = \frac{2\tan(A)}{1-\tan^2(A)}}$$

Example 6.3.1

From your text: Pg. 407 #2ae

Express as a single trig ratio and evaluate:

a) $2\sin(45^\circ)\cos(45^\circ)$

$= \sin(2(45))$

$= \sin(90)$

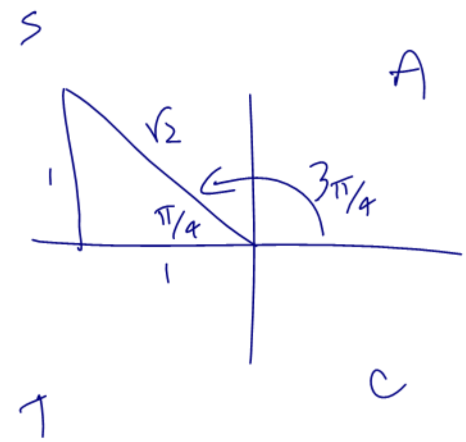
$= 1$

e) $1-2\sin^2\left(\frac{3\pi}{8}\right)$

$$= \cos\left(2\left(\frac{3\pi}{8}\right)\right)$$

$$= \cos\left(\frac{3\pi}{4}\right)$$

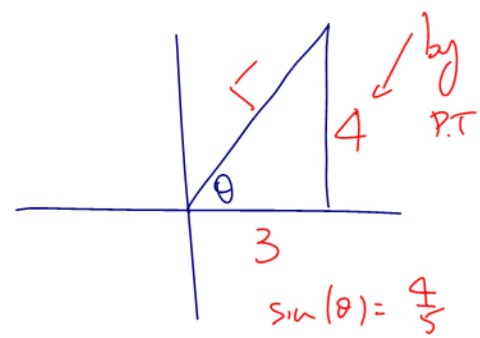
$$= -\frac{1}{\sqrt{2}}$$



Example 6.3.2

From your text: Pg. 407 #4

Determine the values of $\sin(2\theta)$, $\cos(2\theta)$, $\tan(2\theta)$ given $\cos(\theta)=\frac{3}{5}$, $0\le\theta\le\frac{\pi}{2}$



$\sin(\theta)=\frac{4}{5}$

$\cos(\theta)=\frac{3}{5}$

$\tan(\theta)=\frac{4}{3}$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$$

$$= \frac{24}{25}$$

$$\cos(2\theta) = 2\cos^2(\theta)-1$$

$$= 2\left(\frac{3}{5}\right)^2-1$$

$$= 2\left(\frac{9}{25}\right)-1$$

$$= -\frac{7}{25}$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$$

$$= \frac{2\left(\frac{4}{3}\right)}{1-\left(\frac{4}{3}\right)^2}$$

$$= \frac{\frac{8}{3}}{-\frac{7}{9}} = \frac{8}{\cancel{3}}\cdot\frac{-\cancel{9}^{\,3}}{7} \rightarrow -\frac{24}{7}$$

Example 6.3.3

From your text: Pg. 408 #12

Use the appropriate angle and double angle formulae to determine a formula for:

a) $\sin(3\theta)$

$= \sin(2\theta + \theta)$

$= \underbrace{\sin(2\theta)}\cos(\theta) + \sin(\theta)\underbrace{\cos(2\theta)}$

$= (2\sin\theta\cos\theta)\cos\theta + \sin(\theta)(1 - 2\sin^2\theta)$

$= 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$

Example 6.3.4

From your text: Pg. 407 #8

Determine the value of a in the following:

$$2\tan(x) - \tan(2x) + 2a = 1 - \tan(2x)\cdot\tan^2(x)$$

$2a = 1 - \tan(2x)\cdot\tan^2(x) - 2\tan(x) + \tan(2x)$

$\Rightarrow 2a = 1 - \tan(2x)\tan^2(x) + \tan(2x) - 2\tan(x)$

$\Rightarrow 2a = 1 - \tan(2x)(\tan^2(x) - 1) - 2\tan(x)$

$\Rightarrow 2a = 1 - \dfrac{2\tan(x)}{\cancel{1 - \tan^2 x}}\left(\dfrac{\cancel{\tan^2 - 1}}{1}\right) - 2\tan(x)$

$2a = 1 + 2\tan x - 2\tan(x)$

$\Rightarrow 2a = 1$

$\boxed{a = \frac{1}{2}}$

Class/Homework for Section 6.3

Pg. 407 – 408 Finish #2, 4, 12 – Do # 6, 7