6.4 Trigonometric Identities

Proving Trigonometric Identities is so much fun, it's plainly ridiculous. I should be paid extra for letting you play with these proofs! We will be using **ALGEBRA** (remember the rules?). Inside our algebra we will be using the following tools:

Reciprocal Identities

e.g. $\csc(\theta) = \dfrac{1}{\sin(\theta)}$

Quotient Identities

e.g. $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$, or $\cot(x) = \dfrac{\cos(x)}{\sin(x)}$

The Pythagorean Trig Identities

$\sin^2(\theta) + \cos^2(\theta) = 1$

$\Rightarrow \sin^2(\theta) = 1 - \cos^2(\theta)$

or $\cos^2(\theta) = 1 - \sin^2(\theta)$

The Compound Angle Formulae

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \sin(B)\cos(A)$$

$$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$$

$$\tan(A \pm B) = \frac{\tan(A) \pm \tan(B)}{1 \mp \tan(A)\tan(B)}$$

The Double Angle Formulae

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = \begin{cases} 2\cos^2\theta - 1 \\ 1 - 2\sin^2\theta \end{cases}$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$$

(And let's not forget our friends, "The Trig Equivalencies" such as the Cofunction Identities!)

A General Rule of Thumb

Convert

~~Covert~~ everything to sine and cosine

(use the side with the "most going on")

Example 6.4.1 LHS RHS

Prove $1 + \tan^2(x) = \sec^2(x)$

$$\begin{aligned} LHS &= 1 + \tan^2(x) \\ &= 1 + \frac{\sin^2(x)}{\cos^2(x)} \quad \text{common denom} \\ &= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} \\ &= \frac{1}{\cos^2(x)} \\ &= \sec^2(x) \\ &= RHS \quad \square \end{aligned}$$

Example 6.4.2

Prove $\sin(x+y)\cdot\sin(x-y)=\cos^2(y)-\cos^2(x)$

LHS: $\sin(x+y)\cdot\sin(x-y)$

$= \left(\sin(x)\cos(y)+\sin(y)\cos(x)\right)\left(\sin(x)\cdot\cos(y)-\sin(y)\cdot\cos(x)\right)$

$= \sin^2x\cos^2y - \sin(x)\cos(y)\cdot\sin(y)\cos(x) + \sin(y)\cos(x)\cdot\sin(x)\cos(y) - \sin^2y\cos^2x$

$= \sin^2x\cos^2y - \sin^2y\cos^2x$

$= (1-\cos^2x)\cos^2y - (1-\cos^2y)\cos^2x$

$= \cos^2y - \cos^2x\cos^2y - \cos^2x + \cos^2y\cos^2x$

$= \cos^2y - \cos^2x$

$= \text{RHS}$ $\square$

Example 6.4.3

Prove $\sin(\theta)\cdot\tan(\theta)=\sec(\theta)-\cos(\theta)$

RHS $= \sec(\theta)-\cos(\theta)$

$= \dfrac{1}{\cos(\theta)} - \cos(\theta)$

$= \dfrac{1}{\cos(\theta)} - \dfrac{\cos^2(\theta)}{\cos(\theta)}$

$= \dfrac{1-\cos^2(\theta)}{\cos(\theta)}$

$= \dfrac{\sin^2(\theta)}{\cos(\theta)}$

LHS $= \sin(\theta)\cdot\tan(\theta)$

$= \dfrac{\sin(\theta)}{1}\cdot\dfrac{\sin(\theta)}{\cos(\theta)}$

$= \dfrac{\sin^2(\theta)}{\cos(\theta)}$

$= \text{RHS}$ $\square$

Example 6.4.4

Prove $\tan(x)\cdot\tan(y)=\dfrac{\tan(x)+\tan(y)}{\cot(x)+\cot(y)}$

$$\text{RHS} = \frac{\tan(x)+\tan(y)}{\cot(x)+\cot(y)}$$

common denom $\tan(x)\cdot\tan(y)$

$$= \frac{\tan(x)+\tan(y)}{\frac{1}{\tan(x)}+\frac{1}{\tan(y)}}$$

$$= \frac{\tan(x)+\tan(y)}{\frac{\tan(y)+\tan(x)}{\tan(x)\cdot\tan(y)}}$$

$$= \frac{\cancel{\tan(x)+\tan(y)}}{1} \times \frac{\tan(x)\cdot\tan(y)}{\cancel{\tan(y)+\tan(x)}}$$

$$= \tan(x)\cdot\tan(y)$$

$$= \text{LHS} \quad \square$$

Example 6.4.5

From your text: Pg. 417 #9a

Prove $\dfrac{\cos^2(\theta)-\sin^2(\theta)}{\cos^2(\theta)+\sin(\theta)\cos(\theta)}=1-\tan(\theta)$

$$LHS = \frac{\cos^2(\theta) - \sin^2(\theta)}{\cos^2(\theta) + \sin(\theta)\cos(\theta)}$$

$$= \frac{(\cos(\theta) - \sin(\theta))\cancel{(\cos(\theta) + \sin(\theta))}}{\cos(\theta)\cancel{(\cos(\theta) + \sin(\theta))}}$$

$$= \frac{\cos(\theta) - \sin(\theta)}{\cos(\theta)}$$

$$= \frac{\cos(\theta)}{\cos(\theta)} - \frac{\sin(\theta)}{\cos(\theta)}$$

$$= 1 - \tan(\theta)$$

$$= RHS$$

$\square$

Class/Homework for Section 6.4

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