Hwk Check

Pg 418
10e Prove $\sin\left(\frac{\pi}{4}+x\right)+\sin\left(\frac{\pi}{4}-x\right)=\sqrt{2}\cos(x)$

LHS $\sin\left(\frac{\pi}{4}+x\right)+\sin\left(\frac{\pi}{4}-x\right)$

$=\sin\left(\frac{\pi}{4}\right)\cos(x)+\sin(x)\cos\left(\frac{\pi}{4}\right)+\sin\left(\frac{\pi}{4}\right)\cos(x)-\sin(x)\cos\left(\frac{\pi}{4}\right)$

$=2\sin\left(\frac{\pi}{4}\right)\cos(x)$

$=2\left(\frac{1}{\sqrt{2}}\right)\cos(x)$

$=\frac{\sqrt{2}}{\sqrt{2}}\times\frac{2}{\sqrt{2}}\cos(x)$

$=\frac{2\sqrt{2}}{2}\cos(x)$

$=\sqrt{2}\cos(x)$

$=$ RHS $\square$

d) $\tan^2\beta+\cos^2\beta+\sin^2\beta=\frac{1}{\cos^2\beta}$ β = beta

LHS $=\tan^2\beta+\cos^2\beta+\sin^2\beta$

$=\tan^2\beta+1=\frac{\sin^2\beta}{\cos^2\beta}+1=\frac{\sin^2\beta+\cos^2\beta}{\cos^2\beta}$

$=\frac{1}{\cos^2\beta}$

$=\sec^2\beta$

$=\frac{1}{\cos^2\beta}=$ RHS $\square$

9d) Prove

$$\frac{1}{1+\cos\theta} + \frac{1}{1-\cos(\theta)} = \frac{2}{\sin^2\theta}$$

$$LHS = \frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} \qquad \text{C.D. } (1+\cos(\theta))(1-\cos\theta)$$

$$= \frac{(1-\cos\theta) + (1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{2}{1-\cos^2\theta}$$

$$= \frac{2}{\sin^2\theta} = RHS \ \square$$

11 h $\csc(2x) + \cot(2x) = \cot(x)$

$$LHS = \csc(2x) + \cot(2x)$$

$$= \frac{1}{\sin(2x)} + \frac{1}{\tan(2x)} \qquad \tan(2x) = \frac{2\tan(x)}{1-\tan^2 x}$$

$$= \frac{1}{2\sin(x)\cos(x)} + \frac{1-\tan^2 x}{2\tan(x)}$$

$$= \frac{1}{2\sin x\cos x} + \frac{1-\frac{\sin^2 x}{\cos^2 x}}{2\frac{\sin x}{\cos x}}$$

$$= \frac{1}{2\sin x\cos x} + \frac{\left(1-\frac{\sin^2 x}{\cos^2 x}\right)\cdot\cos(x)}{2\sin(x)}$$

$$= \frac{1}{2\sin x\cos x} + \frac{\left(1-\frac{\sin^2 x}{\cos^2 x}\right)\cdot\cos^2 x}{2\sin x\cos x}$$

$$= \frac{1+(\cos^2 x - \sin^2 x)}{2\sin x\cos x}$$

$$= \frac{2\cos^2 x}{2\sin x\cos(x)}$$

$$= \cot x = RHS \ \square$$

6.5 Linear Trigonometric Equations

By this time, asking you to solve a "linear equation" is almost an insult to your intelligence. BUT it is never an insult to ask you to solve problems with math. Instead it is a special treat to be able to spend time thinking mathematically. And so, you're very welcome.

e.g. Solve the linear equation

$3x-4=9$

$\Rightarrow 3x = 9+4$

$\Rightarrow 3x = 13$

$x = \frac{13}{3}$

$\sin(\theta)$

Example 6.5.1

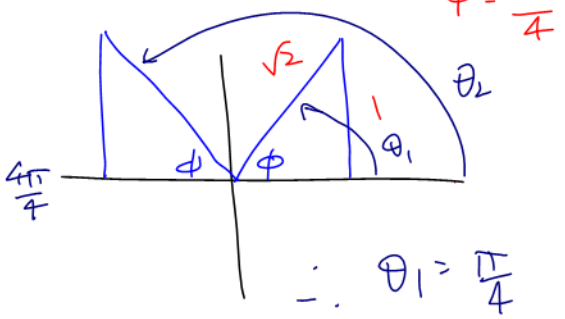
From your text: Pg. ~~427 #6~~ → RADIANS

For $\theta \in [0, 2\pi]$, solve the linear trigonometric equation

a) $\sin(\theta) = \frac{1}{\sqrt{2}}$ exactly, and using a calculator.

EXACT

Q1, Q2



$\phi = \frac{\pi}{4}$

$\therefore \theta_1 = \frac{\pi}{4}$

$\theta_2 = \frac{3\pi}{4}$

CALCULATOR

Notes: ① Your calculators give only one angle ⇒ to get the solns, sketch a diagram

② for $\sin(\theta) = x$ CALC GIVES

$\Rightarrow \theta = \sin^{-1}(x)$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\cos(\theta) = x$

$\Rightarrow \theta = \cos^{-1}(x)$, $0 \le \theta \le \pi$

$\tan(\theta) = x$

$\Rightarrow \theta = \tan^{-1}(x)$ $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

Q1, Q2 $\sin(\theta) = \frac{1}{\sqrt{2}}$

$\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\theta = 0.785$

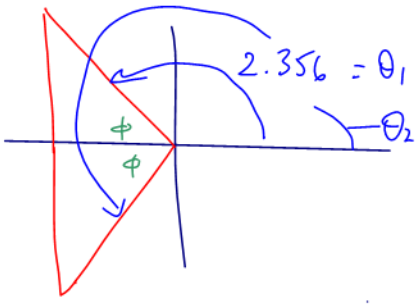
$\therefore \theta_1 = 0.785$ rad. $\theta_2 = \pi - 0.785 = 2.356$ rad

e) $\cos(\theta) = -\frac{1}{\sqrt{2}}$ exactly and using a calculator.

$\theta \in Q2, Q3$

$\theta = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$= 2.356$



$\phi = \pi - 2.356$

$= 0.786$

$\therefore \theta_1 = 2.356$ rad

$\theta_2 = \pi + 0.786$

$= 3.928$ rad.

Example 6.5.2

From your text: Pg. 427 #7

Using a calculator, determine solutions for $0^\circ \le \theta \le 360^\circ$

a) $2\sin(\theta) = -1$

Note: Our Domain is in Degrees!!

we are in deg.

Isolate the trig fn

$\sin(\theta) = -\frac{1}{2}$ Q3, Q4

$\Rightarrow \theta = \sin^{-1}\left(-\frac{1}{2}\right)$

$= -30^\circ$

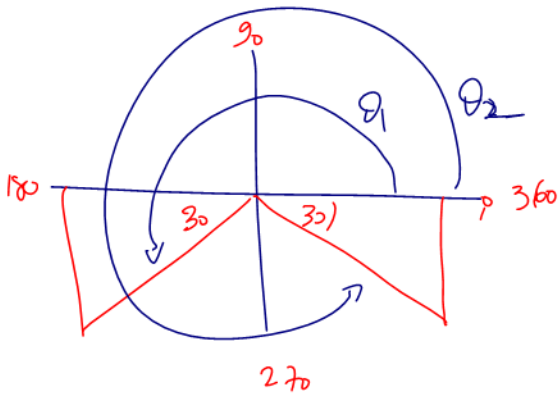
WAIT! But $0 \le \theta \le 360$

$\therefore \theta_1 = 180 + 30$

$= 210^\circ$

$\theta_2 = 360 - 30$

$= 330^\circ$



d) $-3\sin(\theta)-1=1$ (correct to one decimal place) (we are still in deg.

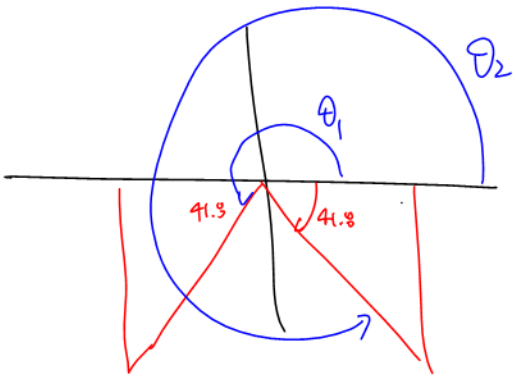
$\Rightarrow -3\sin(\theta) = 2$

$\sin(\theta) = -\frac{2}{3}$

$\Rightarrow \theta = \sin^{-1}\left(-\frac{2}{3}\right)$

$= -41.8$

Q3, Q4



$\therefore \theta_1 = 180^\circ + 41.8^\circ$ $\theta_2 = 360 - 41.8$

$= 221.8^\circ$ $= 318.2^\circ$

Example 6.5.3

From your text: Pg. 427 #8

Using a calculator determine solutions to the equations for $0 \le x \le 2\pi$.

a) $3\sin(x) = \sin(x)+1$ ↳ radians

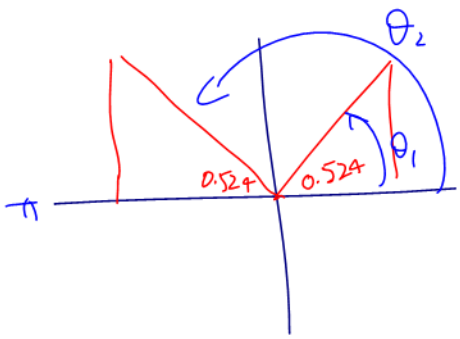
$2\sin(x) = 1$

$\sin(x) = \frac{1}{2}$

$\Rightarrow x = \sin^{-1}\left(\frac{1}{2}\right)$

$= 0.524$ rad

Q1, Q2



$\therefore \theta_1 = 0.524$ rad $\theta_2 = \pi - 0.524$

$= 2.618$ rad.

Example 6.5.4

From your text: Pg. 427 #9f

Solve for $x \in [0, 2\pi]$

$$8 + 4\cot(x) = 10$$

Note: your calculator has no reciprocal trig ratio buttons!

$\Rightarrow$ convert to primary trig ratios

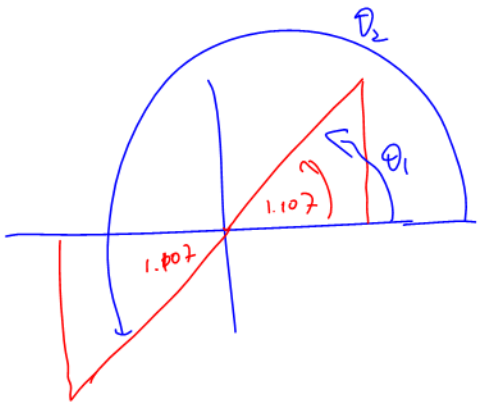
$\Rightarrow 4\cot(x) = 2$

$\Rightarrow \cot(x) = \frac{1}{2}$

$\Rightarrow \tan(x) = 2$ Q1, Q3

$\Rightarrow x = \tan^{-1}(2)$

$= 1.107$ rad



$\therefore \theta_1 = 1.107$ rad, $\theta_2 = \pi + 1.107$

$= 4.249$ rad

Class/Homework for Section 6.5

Pg. 427 – 428 #6, 7def, 8, 9abc