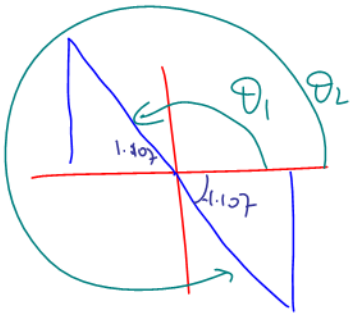
Hwk Check

Pg 427
7f)

$8 - \tan(\theta) = 10$ $\quad 0 \le x \le 2\pi$



$\tan(\theta) = -2$

$\theta = \tan^{-1}(-2)$ $\quad$ Q2, Q4

$\theta = -1.107$

$(-63.4°)$

$\theta_1 = \pi - 1.107$ $\Rightarrow$ $180 - 63.4$

$= 2.03$ rad $\quad = 116.6°$

$\theta_2 = 2\pi - 1.107$ $\Rightarrow$ $360 - 63.4$

$= 5.19$ rad. $\quad = 296.6°$

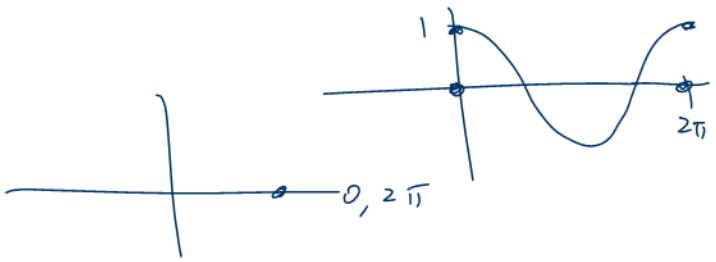
9c) $7\sec(x) = 7$

$\Rightarrow \sec(x) = 1$

$\Rightarrow \cos(x) = 1$

$\therefore x = \cos^{-1}(1)$

$= 0.$ or 2π



6.6 Quadratic Trigonometric Equations

Before moving on to Quadratic Trigonometric Equations, we need to consider a mind stretching problem, because it's good stretch from time to time.

Example 6.6.1

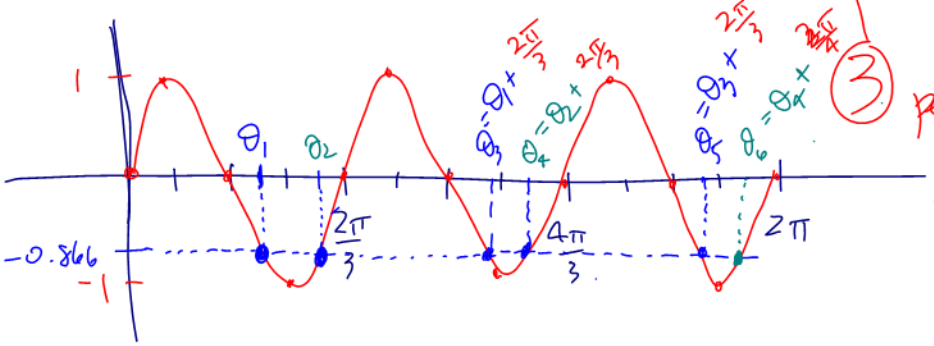
Solve $\sin(3x) = -\frac{\sqrt{3}}{2}$ exactly on $x \in [0, 2\pi]$

$= -0.866$

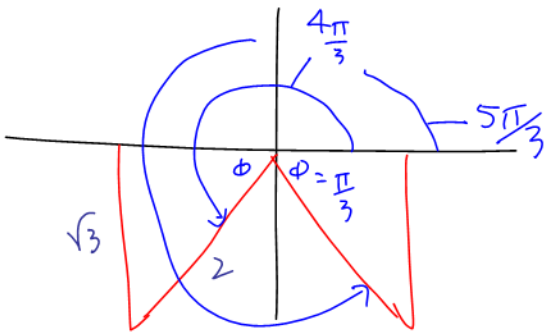
Don't be afraid of the **3**! (though it does give one some concern…)

Consider a sketch of $f(x) = \sin(3x)$ on $x \in [0, 2\pi]$

Note: $f(x) = \sin(3x)$ has a period of $P = \frac{2\pi}{3}$



"3x" is in Q3 or Q4



$\therefore 3x = \frac{4\pi}{3}, \frac{5\pi}{3}$

$\Rightarrow x = \frac{4\pi}{9}, \frac{5\pi}{9}$

add a period

$\therefore x_1 = \frac{4\pi}{9}, \; x_3 = x_1 + \frac{2\pi}{3} = \frac{4\pi}{9} + \frac{6\pi}{9} = \frac{10\pi}{9}, \; x_5 = \frac{10\pi}{9} + \frac{6\pi}{9} = \frac{16\pi}{9}$

$x_2 = \frac{5\pi}{9}, \; x_4 = x_2 + \frac{2\pi}{3} = \frac{5\pi}{9} + \frac{6\pi}{9} = \frac{11\pi}{9}, \; x_6 = \frac{11\pi}{9} + \frac{6\pi}{9} = \frac{17\pi}{9}$

$\therefore x = \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$

In Quadratic Trigonometric Functions the highest power on the trig 'factor' will be 2.

Example 6.6.2

From your text: Pg. 436 #4: Solve, to the nearest degree, for $0^\circ \le \theta \le 360^\circ$

b) $\cos^2(\theta) = 1$

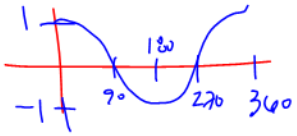
degrees

we need to find θ

$\Rightarrow \cos(\theta) = \pm 1$

$x^2 = 1$

$x = \pm 1$



$\cos(\theta) = 1$ $\quad \cos(\theta) = -1$

$\theta = 0, 360 \quad \theta = 180$

$\theta = 0^\circ, 180^\circ, 360^\circ$

f) $2\sin^2(\theta) = 1$

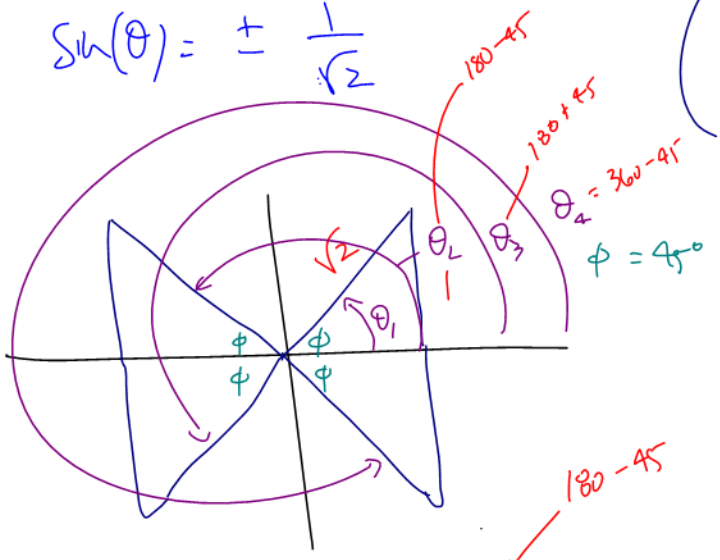
$\sin^2(\theta) = \frac{1}{2}$

$\sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\sin(\theta) = \pm \frac{1}{\sqrt{2}}$

$\left(\sin(\theta) = +\frac{1}{\sqrt{2}} \Rightarrow Q1, Q2 \right)$

$\left(\sin(\theta) = -\frac{1}{\sqrt{2}} \Rightarrow Q3, Q4 \right)$



180 - 45 $\quad$ 180 + 45 $\quad$ 360 - 45

$\therefore \theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$

Example 6.6.3

From your text: Pg. 436 #5: Solve for $0^\circ \le x \le 360^\circ$

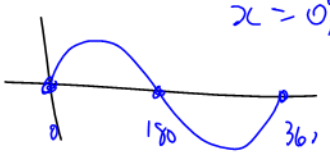
b) $(\sin(x))(\cos(x)-1)=0$

$a \cdot b = 0$

$\Rightarrow a = 0$ or $b = 0$

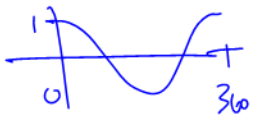
$\therefore \sin(x) = 0$ or $\cos(x) - 1 = 0$

$x = 0, 180^\circ, 360^\circ$



$\Rightarrow \cos(x) = 1$

$x = 0, 360^\circ$

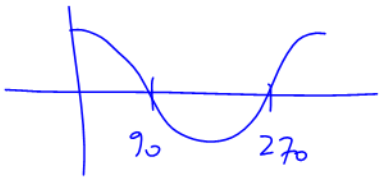


$\therefore x = 0^\circ, 180^\circ, 360^\circ$

d) $(\cos(x))(2\sin(x)-\sqrt{3})=0$

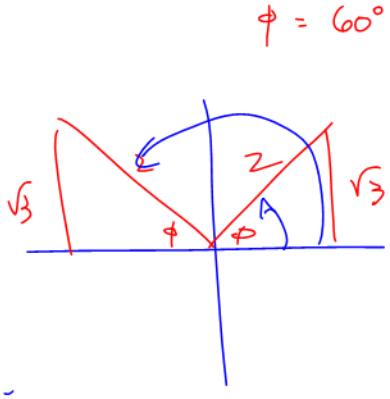
$\Rightarrow \cos(x) = 0$ or $2\sin(x) - \sqrt{3} = 0$

$x = 90^\circ, 270^\circ$



$\sin(x) = \frac{\sqrt{3}}{2}$

$\therefore x = 60^\circ, 120^\circ$



$\therefore x = 60^\circ, 90^\circ, 120^\circ, 270^\circ.$

Example 6.6.4

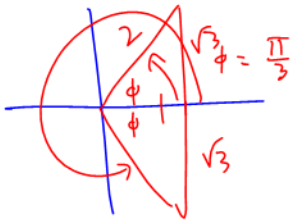
From your text: Pg. 436 #6: Solve for $0 \le x \le 2\pi$ radians

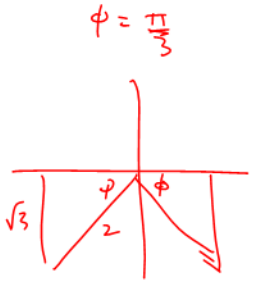
d) $(2\cos(x)-1)(2\sin(x)+\sqrt{3})=0$

$\therefore 2\cos x - 1 = 0$ or $2\sin(x) + \sqrt{3} = 0$

$\cos(x) = \frac{1}{2}$ | $\sin(x) = -\frac{\sqrt{3}}{2}$

$\therefore x = \frac{\pi}{3}, \frac{5\pi}{3}$ | $x = \frac{4\pi}{3}, \frac{5\pi}{3}$





$\therefore x = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

Example 6.6.5

From your text: Pg. 436 #7: Solve for $0 \le \theta \le 2\pi$ to the nearest hundredth (if necessary).

a) $2\cos^2(\theta)+\cos(\theta)-1=0$

$2x^2 + x - 1 = 0$

$(2x-1)(x+1)=0$

$(2\cos(\theta)-1)(\cos(\theta)+1) = 0$

$\Rightarrow \cos(\theta) = \frac{1}{2}$ | $\cos(\theta) = -1$ (already done)

$\theta = \pi/3, 5\pi/3$ | $\theta = \pi$

$\therefore \theta = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

e) $3\tan^2(\theta)-2\tan(\theta)=1$

$\Rightarrow 3\tan^2(\theta) - 2\tan\theta - 1 = 0$

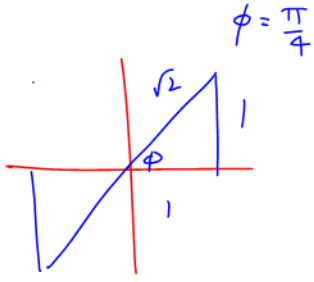
$(3\tan(\theta) + 1)(\tan(\theta) - 1) = 0$

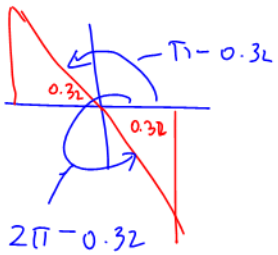
$\tan(\theta) = -\frac{1}{3}$ or $\tan(\theta) = 1$ Q1, Q3

calc Q2, Q4

$\Rightarrow \theta = \tan^{-1}(-\frac{1}{3})$ | $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

$= -0.32$





$\theta = 2.82, 5.96$

$\therefore \theta = \frac{\pi}{4}, 2.82, \frac{5\pi}{4}, 5.96$ rad

Example 6.6.6 (decimals are between the sixes!)

From your text: Pg. 436 #8: Solve for $x \in [0, 2\pi]$

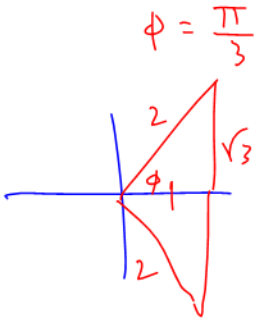
a) $\sec(x) \cdot \csc(x) - 2\csc(x) = 0$

$\Rightarrow \csc x \left(\sec(x) - 2 \right) = 0$

$\therefore \csc(x) = 0$ | $\sec(x) = 2$

$\Rightarrow \sin(x) = \frac{1}{0}$ | $\cos(x) = \frac{1}{2}$

No.



$x = \frac{\pi}{3}, \frac{5\pi}{3}$

c) $2\sin(x) \cdot \sec(x) - 2\sqrt{3}\sin(x) = 0$

$2\sin(x)\left(\sec x - \sqrt{3} \right) = 0$

$\Rightarrow 2\sin(x) = 0$

$\Rightarrow \sin(x) = 0$

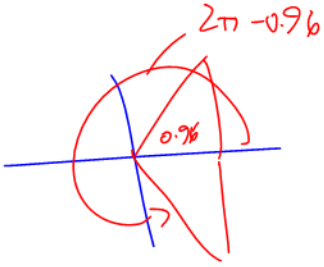
$\Rightarrow x = 0, \pi, 2\pi$

$\sec(x) = \sqrt{3}$

$\Rightarrow \cos(x) = \frac{1}{\sqrt{3}}$

$\Rightarrow x = \cos^{-1}\left(\frac{1}{\sqrt{3}} \right) = 0.96$

Q1, Q4



$\therefore x = 0, 0.96, \pi, 5.33, 2\pi$

Example 6.6.7

From your text: Pg. 437 #9: Solve for $x \in [0, 2\pi]$. Round to two decimal places.

a) $5\cos(2x) - \cos(x) + 3 = 0$

$\cos(2x) = 2\cos^2 x - 1$

$\Rightarrow 5(2\cos^2 x - 1) - \cos(x) + 3 = 0$

$10\cos^2(x) - \cos(x) - 2 = 0$

$\Rightarrow (5\cos(x) + 2)(2\cos(x) - 1) = 0$

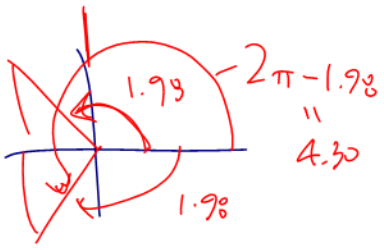
$\Rightarrow \cos(x) = -\frac{2}{5}$ or $\cos(x) = \frac{1}{2}$

Q2, 3

Calc

$x = \cos^{-1}(-0.4)$ $\Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$

$= 1.98$



$\therefore x = \frac{\pi}{3}, 1.98, 4.30, \frac{5\pi}{3}$.

Class/Homework for Section 6.6

Pg. 436 – 437 #4ade, 5acef, 6ac, 7 – 9