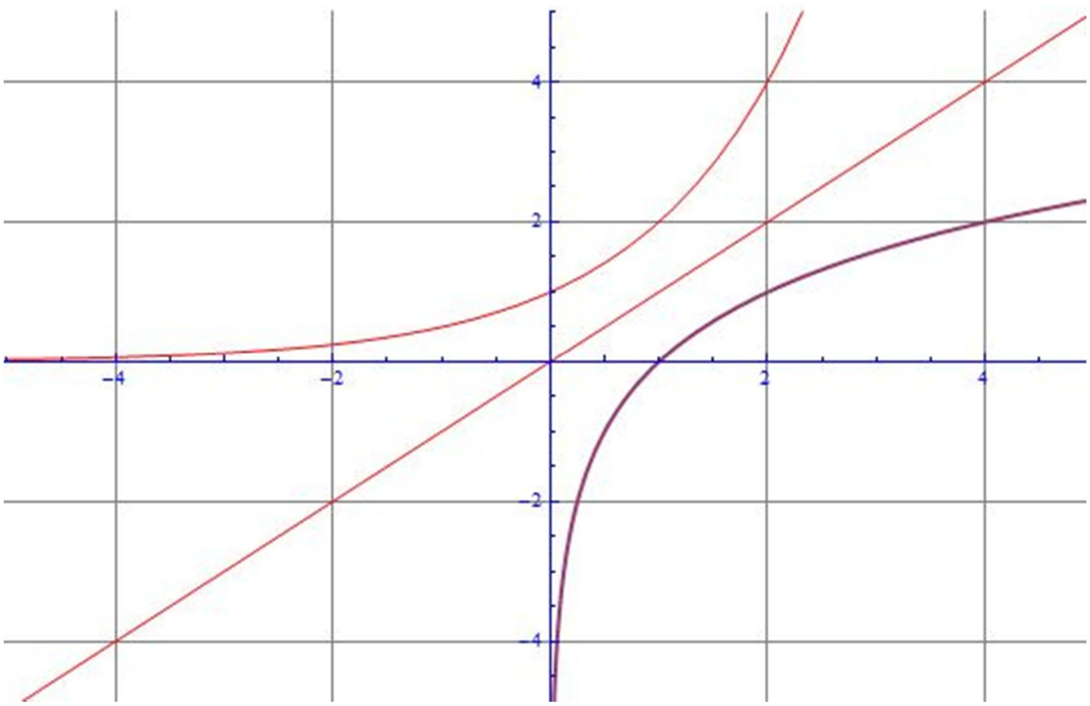
ADVANCED FUNCTIONS

Chapter 7 –Exponential and Logarithmic Functions

(*Material adapted from Chapter 8 of your text*)



Chapter 7 – Exponential and Logarithmic Functions

Contents with suggested problems from the Nelson Textbook (Chapter 8)

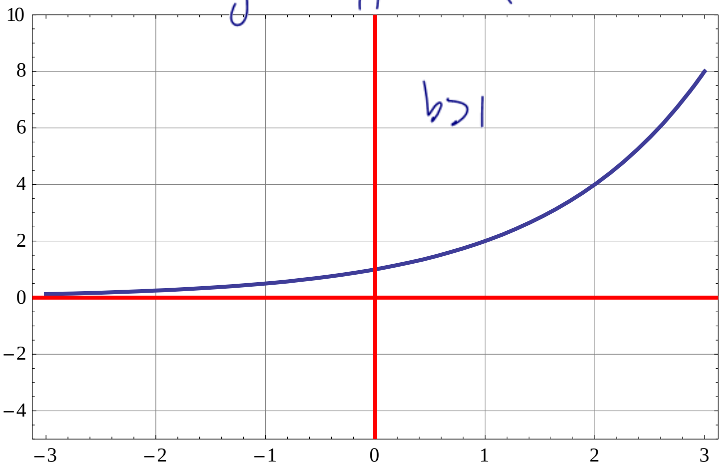
7.1 Exploring the Logarithmic Function

In grade 11 Functions, you spent a bunch of time considering Exponential Functions, and it seems like a good idea to spend a little time reviewing that type of function.

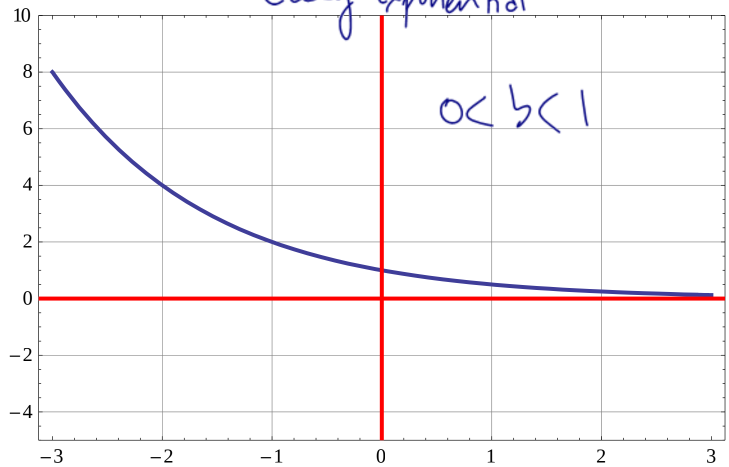
"basic" f= $f(x) = b^x$

Consider the sketches of the graphs of the functions $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^x$:

growth exponential



decay exponential



Features of $f(x) = 2^x$:

Domain: $x \in \mathbb{R}$

Range: $f(x) > 0$

Axis Intercepts:

x int none

y int (0, 1)

Asymptotes:

H.A. $y=0$

V.A. none

Intervals of inc/decrease:

inc : $(-\infty, \infty)$

dec: never

Features of $g(x) = \left(\frac{1}{2}\right)^x$:

Domain: $x \in \mathbb{R}$

Range: $g(x) > 0$

Axis Intercepts:

x int : NONE

y int (0,1)

Asymptotes:

V.A. none

H.A $y=0$

Intervals of inc/decrease:

inc : never

dec: $(-\infty, \infty)$

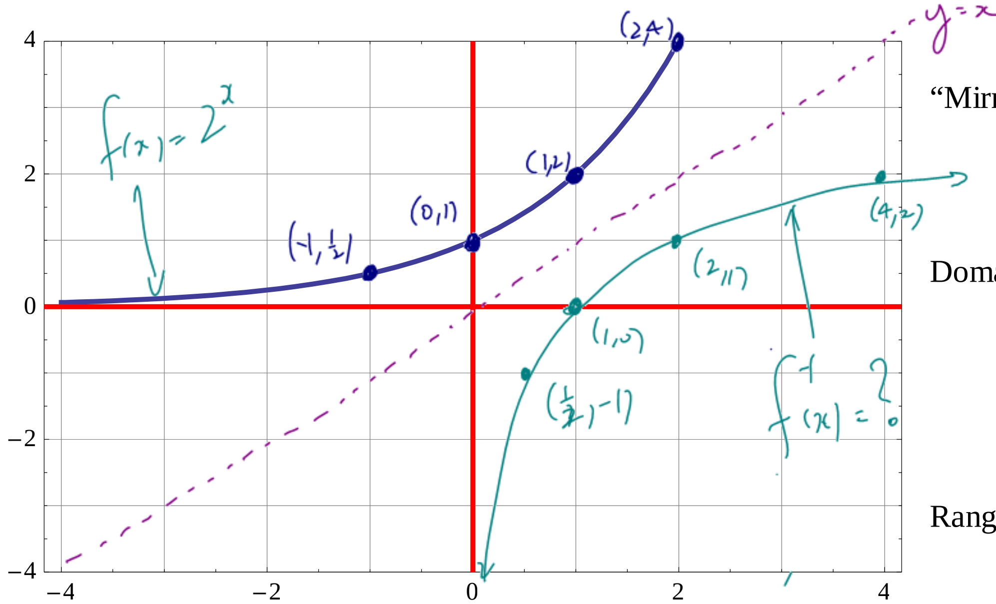
One important aspect of functions is the concept of **inverse**. We have in fact spent a good chunk of time talking about the inverses of functions. In case you've forgotten, let's take a look at how to find the inverse of a function.

There are **two methods**. One is a **geometric approach**, and the other (more useful) method is an **algebraic approach**.

Inverse of a function geometrically

Consider the sketch of the graph of the function $f(x) = 2^x$:

Switch x & y



"Mirror" : $y = x$

Domain of $f(x)$ is the range of $f^{-1}(x)$

Range of $f(x)$ is the domain of $f^{-1}(x)$

Why isn't this technique very useful for the mathematician? If we want to calculate, we need an eqn.

Inverse of a function algebraically

Recall that the basic technique for finding the algebraic inverse of a function is to switch x and $f(x)$ and then solve for $f^{-1}(x)$.

$\pm\left(\frac{x-2}{2}\right)^{\frac{1}{2}}$

e.g. Determine the inverse of $f(x) = 2(x-1)^2 + 2$

$$\Rightarrow x = 2\left(f^{-1}(x) - 1\right)^2 + 2$$

$$\Rightarrow x - 2 = 2\left(f^{-1}(x) - 1\right)^2$$

$$\Rightarrow \left(f^{-1}(x) - 1\right)^2 = \frac{x-2}{2}$$

$$f^{-1}(x) - 1 = \pm\sqrt{\frac{x-2}{2}}$$

$$\boxed{f^{-1}(x) = \pm\sqrt{\frac{x-2}{2}} + 1}$$

O.K., now let's try this same technique for an exponential function.

Example 7.1.1

Determine the inverse of $f(x) = 2^x$ (First we switch x and $f(x)$)

$\boxed{x = 2^{f^{-1}(x)}}$ (this is called the exponential form of the inverse)

O.K. now we just need to isolate for $f^{-1}(x)$ … ie, we need to find the inverse of operation of exponentiation

Definition 7.1.1

Given $y = a^x$

$x = \log_a(y)$

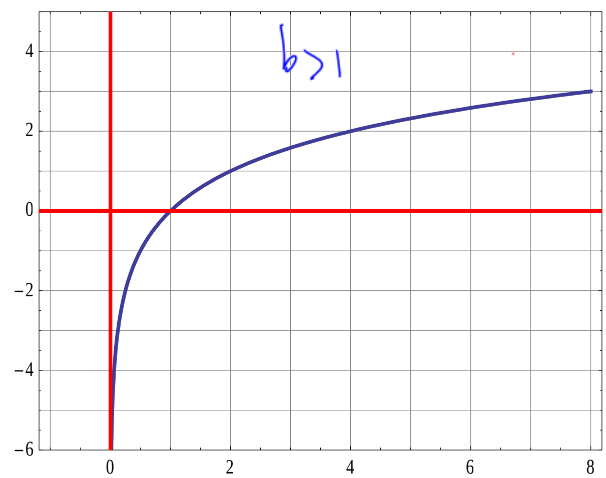
Note: A **Logarithm** is an

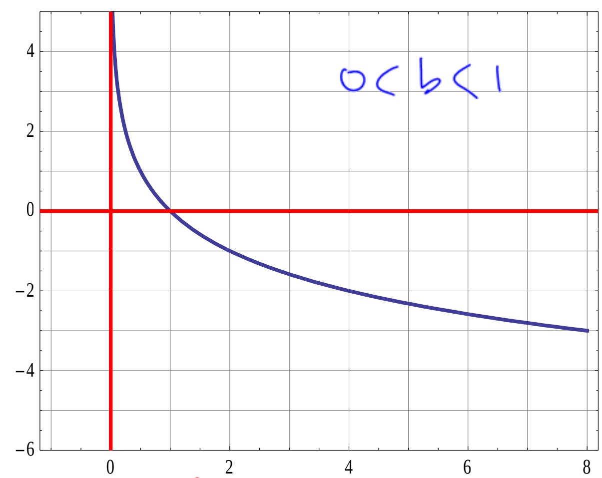
EXPONENT

We will use the following form for Logarithmic Functions:

general log fn $f(x) = \log_b(x)$

Basic Sketches of the General Logarithmic Function





$f(x) = \log_2(x)$

Domain: $x > 0$

Range: $f(x) \in \mathbb{R}$

Axis Int's:
x_{int} $(1,0)$
y_{int} NONE

Asymptotes:
V.A. $x = 0$
H.A. NONE

Intervals of inc/dec
inc: $(0, \infty)$
dec: never

$g(x) = \log_{\frac{1}{2}}(x)$

Domain: $x > 0$

Range: $g(x) \in \mathbb{R}$

Axis Int's:
x_{int} $(1,0)$
y_{int} NONE

Asymptotes
V.A. $x = 0$
H.A. NONE.

Intervals of inc/dec
inc: never
dec: $(0, \infty)$

Example 7.1.2

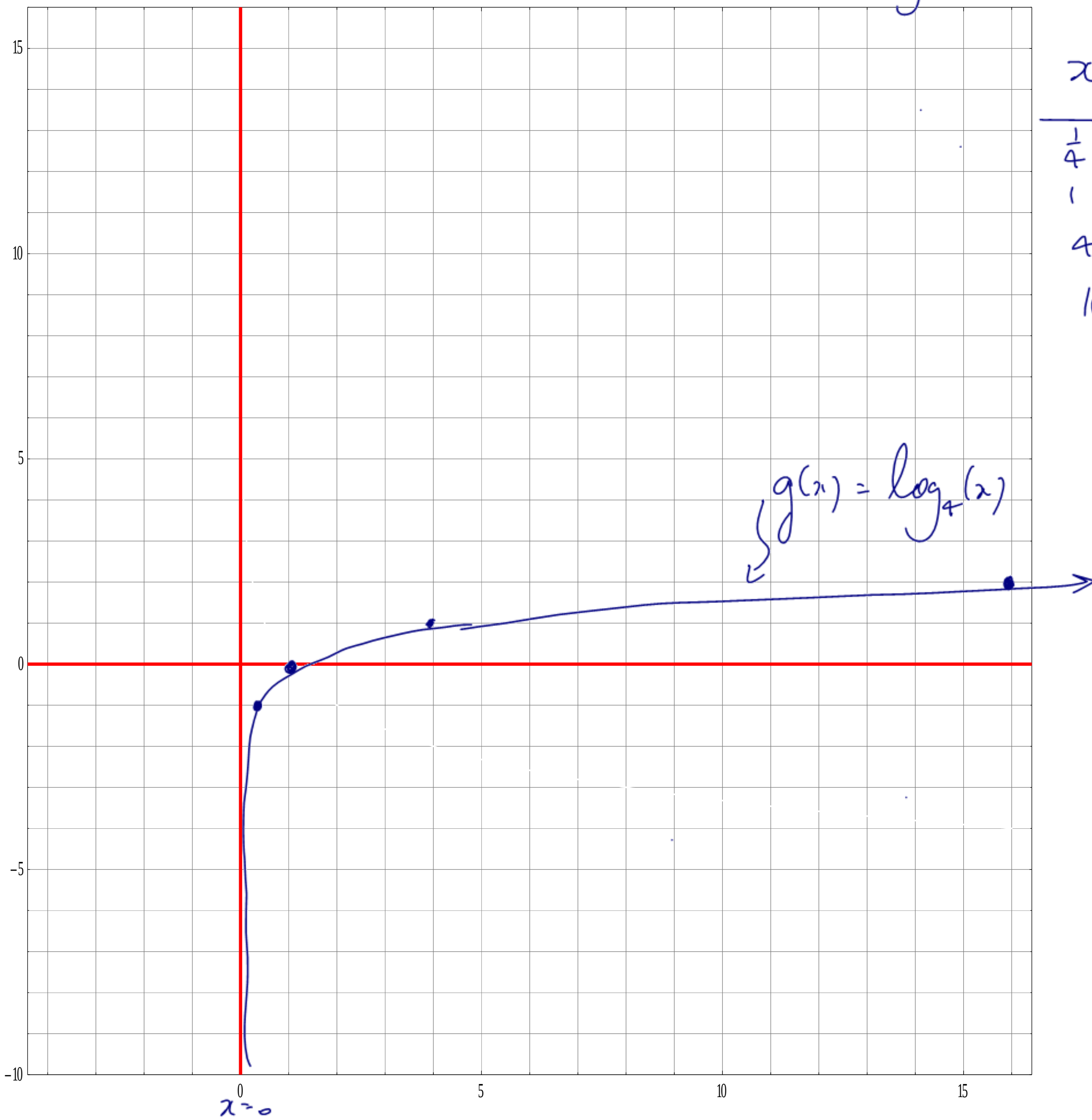
From your text: Pg. 451 #1a)

Sketch the inverse of $f\left(x\right)=4^{x}$

$f^{-1}(x) = \log_4(x)$

x	4^x
-1	$\frac{1}{4}$
0	1
1	4
2	16

x	$f^{-1}(x)$
$\frac{1}{4}$	-1
1	0
4	1
16	2



Example 7.1.3

From your text: Pg. 451 #2

$f(x) = 4^x$

Write the equation of the above inverse function in i) exponential and ii) logarithmic form:

i) $x = 4^{f^{-1}(x)}$

ii) $g(x) = \log_4(x)$

Example 7.1.4

From your text":Pg. 451

a) #5, 6cd Write the inverse of the given exponential function in i) exponential and ii) logarithmic form:

$f(x)$ 1) $y=\left(\frac{1}{4}\right)^x$ 2) $y=m^x$

i) $x=\left(\frac{1}{4}\right)^y$ ii) $f^{-1}(x)$ $y=\log_{\frac{1}{4}}(x)$ i) $x=m^y$ ii) $y=\log_m(x)$

b) #7c) Write the equation of the given logarithmic function in exponential form:

$y=\log_3(x)$ $3^y=x$

c) #8c) is a question which can be very confusing if you don't carefully consider the context of the problem.

Write the equation of the INVERSE **FUNCTION** of given logarithmic function (in exponential form…well duh).

$f(x)=\log_3(x)$ $3^{f^{-1}(x)}=x$ ~~$f(x)=3^x$~~

$y=a^x$

$\Leftrightarrow x=\log_a(y)$

find the #

d) #9adf) Evaluate (remember that **evaluate** means to **calculate** a number)

1) $\log_2 4$ 2) $\log_5 1$ 3) $\log_3\sqrt{3}$

$=2$ $=0$ $5^{0}=1$ $=\log_3(3^{\frac{1}{2}})=\frac{1}{2}$

the # (exponent) raised on base 2 to get the answer 4

e) #11 For $y=\log_2 x$ determine the coordinates of the points on the graph with functional (y) values of $-2,-1,0,1,2$

$\Rightarrow -2=\log_2(x)$

$\Rightarrow 2^{-2}=x$

$\Rightarrow x=\frac{1}{4}$

$\Rightarrow \left(\frac{1}{4},-2\right)$

$\left(\frac{1}{2},-1\right)$, $(1,0)$

$(2,1)$ $(4,2)$

Class/Homework for Section 7.1

Pg. 451 #1c, 4, Finish 5 – 7, 9, 11