# 7.2 Evaluating Logarithms (Part 1)

We will learn to "evaluate" through many examples, keeping in mind **the fact that logarithms and exponentials are inverses of each other**.

**Example 7.2.1**

Evaluate

a) $\log_2(32)$

Let $x = \log_2(32)$

reminder $a^m = a^n$, $m = n$

$2^x = 32$

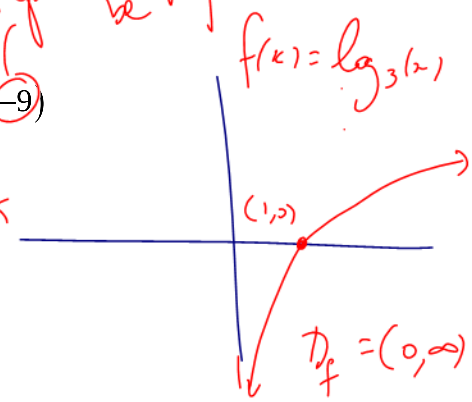
$\Rightarrow 2^x = 2^5$

$\therefore x = 5$

b) $\log_3(-9)$

argument cannot be negative

No sol'n




c) $\log_4\left(\frac{1}{64}\right)$

Let $x = \log_4\left(\frac{1}{64}\right)$

$4^x = \frac{1}{64}$

$4^x = 64^{-1}$

$4^x = (4^3)^{-1}$

$4^x = 4^{-3}$

$\therefore x = -3$

d) $\log_{254}(1)$

Let $x = \log_{254}(1)$

$\Rightarrow 254^x = 1$

$\Rightarrow 254^x = (254)^0$

$\therefore x = 0$.

e) $\log_4(32)$

Let $x = \log_4(32)$

$\Rightarrow 4^x = 32$

$\Rightarrow (2^2)^x = 2^5$

$\Rightarrow 2^{2x} = 2^5$

$\therefore 2x = 5$

$x = \frac{5}{2}$

32 cannot be written base 4, it can be written base 2 (and so can 4)

f) $\log_3\left(\frac{1}{81}\right)$

$\Rightarrow$ Let $x = \log_3\left(\frac{1}{81}\right)$

$\Rightarrow 3^x = 81^{-1}$

$\Rightarrow 3^x = 3^{-4}$

$\Rightarrow x = -4$

g) $\log_5\left(\sqrt[4]{125}\right)$

Let $x = \log_5\left(\sqrt[4]{125}\right)$

$5^x = (125)^{\frac{1}{4}}$

$\Rightarrow 5^x = (5^3)^{\frac{1}{4}}$

$\Rightarrow 5^x = 5^{\frac{3}{4}}$

$\therefore x = \frac{3}{4}$

h) $\log_9\left(\sqrt{27}\right)$

Let $x = \log_9\left(27^{\frac{1}{2}}\right)$

$(3^2)^x \Rightarrow 9^x = 27^{\frac{1}{2}}$

$\Rightarrow 3^{2x} = 3^{3/2}$

both base 3

$\Rightarrow 2x = 3/2$

$x = 3/4$

Practice Problem: Pg. 466 #3

# 7.3 Evaluating Logarithms (Part 2)

Here we take this idea of evaluation up a notch. We will continue learning "by example".

**Example 7.3.1**

Evaluate

a) $\log_3\left(\sqrt[5]{27}\right)$

Let $x = \log_3\left(\sqrt[5]{27}\right)$

$\Rightarrow 3^x = 27^{\frac{1}{5}}$

$\Rightarrow 3^x = 3^{3/5}$

$\therefore x = 3/5$

b) $\log_5(32)$

Let $x = \log_5(32)$

$\Rightarrow 5^x = 32$

CAN'T DO THIS W/ ALGEBRA

use GraphCalc.

Sketch $y_1 = 5^x$, $y_2 = 32$

find the POI

By GraphCalc $x = 2.15$

c) $\log_3(19)$

Let $x = \log_3(19)$

$\Rightarrow 3^x = 19$

By GraphCal.

$x = 2.68$

d) $\log(1000)$

No NUMBER!

$\Rightarrow$ BASE 10 !!

$\log_{10}(1000) = 3$

$(10^x = 10^3 \Rightarrow x = 3)$

e) $\log\left(10^6\right)$

$= 6$

f) $\log_3\left(3^5\right)$ same base

$= 5$ exponent is answer

g) $\log(27)$ punch into calculator.

$= 1.43.$

**Example 7.3.2**

From your text: Pg. 467 #12

**Half-life** is the time it takes for half of a sample of a radioactive element to decay. The function $M(t) = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$ can be used to calculate the mass remaining if the half-life is $h$ and the initial mass is $P$. The half-life of radium is 1620 years.

**a)** If a laboratory has 5 g of radium, how much will there be in 150 years?

**b)** How many years will it take until the laboratory has only 4 g of radium?

a) $M(150) = 5\left(\frac{1}{2}\right)^{\frac{150}{1620}}$

$= 4.69$ g.

b) Looking for t

$4 = 5\left(\frac{1}{2}\right)^{\frac{t}{1620}}$

$\Rightarrow \underbrace{5\left(\frac{1}{2}\right)^{t/1620} - 4}_{\text{graph: look for the zero}} = 0$

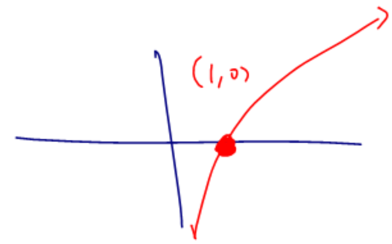
by graph calc

$t = 521.5$ years.

Example 7.3.4

From your text: Pg. 467 #13

The function $s(d) = 0.159 + 0.118 \log d$ relates the slope, $s$, of a beach to the average diameter, $d$, in millimetres, of the sand particles on the beach. Which beach has a steeper slope: beach $A$, which has very fine sand with $d = 0.0625$, or beach $B$, which has very coarse sand with $d = 1$? Justify your decision.




A: $s(0.0625) = 0.159 + 0.118 \underbrace{\log(0.0625)}_{\text{negative}}$

$= 0.159 - \text{"something"} < 0.159$

B: $s(1) = 0.159 + 0.118 \log(1)$ → 0

$= 0.159$

∴ Beach B has the steeper slope.

**Class/Homework for Section 7.3**

**Pg. 466 – 468 #4 – 7, 9, 10, 11, 14 – 17, 19**