Hwk Check

Pg 475

#6c) Evaluate $\log_6(6\sqrt{6})$

$= \log_6\left(6^{3/2}\right)$

$= 3/2$

$\log_a(a^m) = m$

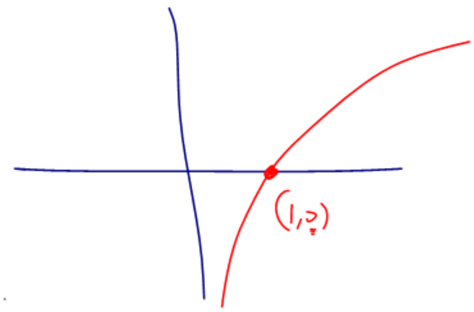
$6^1 \cdot 6^{\frac{1}{2}}$

$= 6^{3/2}$

#8 $\log_5(3) + \log_5\left(\frac{1}{3}\right) = 0$

$\Rightarrow \log_5(1)$

$= 0$



(6d) $\log_2\left(36^{\frac{1}{2}}\right) - \log_2\left(72^{\frac{1}{2}}\right)$

$= \log_2\left(\frac{36^{\frac{1}{2}}}{72^{\frac{1}{2}}}\right)$

$= \log_2\left(\left(\frac{36}{72}\right)^{\frac{1}{2}}\right)$

$= \frac{1}{2}\log_2\left(\frac{1}{2}\right)$

$= \frac{1}{2} \cdot \log_2\left(2^{-1}\right) = -\frac{1}{2}.$

7.5 Solving Exponential Equations

Recall that solving an equation means (in usual terms) "getting *x* by itself". That's generally pretty easy to do, for a student in grade 12.

e.g. Solve for *x*: $2x^3 - 8x = 0$

$$\Rightarrow 2x(x^2 - 4) = 0$$

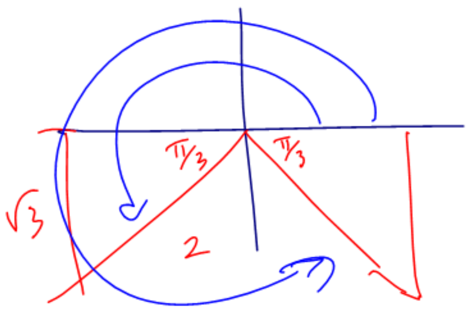
$$\Rightarrow 2x(x-2)(x+2) = 0$$

$$\therefore x = 0, 2, -2.$$

Easy as pi! However, "getting *x* by itself" **isn't always** a matter of standard **algebra**.

e.g. Solve for *x*: $\sin(x) = -\dfrac{\sqrt{3}}{2}$

$$x = \frac{4\pi}{3}, \frac{5\pi}{3}$$



$$x = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

We now turn our attention to **Solving Exponential Equations** ("algebraically").

Example 7.5.1

Solve $5^{x-2} = 1.7$

No matter what type of equation, "**getting *x* by itself**" requires using "**inverse operations**". So, for **Exponential Equations**, it's **Logarithms to the rescue**!

$$\Rightarrow \log(5^{x-2}) = \log(1.7)$$

Take log of both sides

$$(x-2)\cdot\log(5) = \log(1.7)$$

$$\Rightarrow x - 2 = \frac{\log(1.7)}{\log(5)}$$

$$7(x-2) = 5$$

$$x - 2 = \frac{5}{7} \Rightarrow x = \frac{5}{7} + 2$$

$$\therefore x = \frac{\log(1.7)}{\log(5)} + 2 = 2.33$$

Example 7.5.2

From your text: Pg. 485 #3a

Solve $x = \log_3(243)$

There are two methods for this problem, but one method is very rare…and, yes, I know this is a logarithmic equation in a lesson on exponential equations…but logs and exponentials are related!!!!!!!!!!

Change of Base

$$x = \frac{\log(243)}{\log(3)} = 5$$

Invert

$$3^x = 243$$

$$\Rightarrow 3^x = 3^5$$

$$x = 5$$

Example 7.5.3

Solve $32^{3x-5} = \left(4^{\frac{5}{2}}\right)^x$

$$\Rightarrow 32^{3x-5} = 4^{\frac{5x}{2}}$$

Notice 32 & 4 can both be written as base 2

$$\left(2^5\right)^{3x-5} = \left(2^2\right)^{5x/2}$$

$$\Rightarrow 2^{15x-25} = 2^{5x}$$

$$\therefore 15x - 25 = 5x$$

$$10x = 25$$

$$x = 2.5$$

"x-stuff" = "non-x-stuff"

$$(3x-5)\cdot\log(32) = \frac{5x}{2}\cdot\log(4)$$

$$\Rightarrow 3x\log(32) - 5\log(32) = \frac{5x}{2}\log(4)$$

$$\Rightarrow 3x\log(32) - \frac{5x}{2}\log(4) = 5\log(32)$$

$$x\left(3\log(32) - \frac{5}{2}\log(4)\right) = 5\log(32)$$

$$x = \frac{(5\log(32))}{\left(3\log(32) - \frac{5}{2}\log(4)\right)} = 2.5$$

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Example 7.5.4

Solve $3^{x-4} = 17^{2x+1}$

'log' both sides

$$\Rightarrow (x-4)\cdot\log(3) = (2x+1)\cdot\log(17)$$

$$\Rightarrow x\cdot\log(3) - 4\log(3) = 2x\log(17) + \log(17)$$

$$\Rightarrow x\log(3) - 2x\log(17) = \log(17) + 4\log(3)$$

$5x = 12$

$$x\left(\log(3) - 2\log(17)\right) = \log(17) + 4\log(3)$$

$$\Rightarrow x = \frac{(\log(17) + 4\log(3))}{(\log(3) - 2\log(17))} = -1.58$$

Example 7.5.5

A naught

From your text: Pg. 485 #6b

A \$1000 investment is made in a trust fund that pays 12%/a, **compounded monthly**. How long will it take the investment to grow to \$5000?

$$A(t) = A_0(1+i)^n$$

A_0 = initial investment

i = interest rate / compounding period

$$= \frac{0.12}{12} = 0.01$$

$$5000 = \underline{1000}(1.01)^n$$

1000 in the way.

$\div 1000$

$n = 12t$

$$\Rightarrow 5 = 1.01^n$$

$$\Rightarrow \log(5) = n\log(1.01)$$

$$n = \frac{\log(5)}{\log(1.01)} = 161.75 \text{ months}$$

$\log(a^n) = n\log(a)$

$\therefore$ years is $\frac{161.75}{12} = 13.5$ years.

Example 7.5.6

From your text: Pg. 485 #7

A bacteria culture doubles every 15 minutes. How long will it take for a culture of 20 bacteria to grow to a population of 163 840 bacteria?

Doubling Formula:

$$P(t) = P_0(2)^{\frac{t}{D}}$$

Half-life Formula:

$$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

D: h must have the same time units in their problem

$P(t) = P_0(2)^{t/D}$

GIVEN

$P_0 = 20$

$D = 15$ min.

$P(t) = 163\,840$

WANT

t

$163840 = 20(2)^{t/15}$

$\div 20$ $\quad 8192 = 2^{t/15}$

$\log(8192) = \frac{t}{15}\log(2) \Rightarrow t = \dfrac{15\cdot\log(8192)}{\log(2)}$

$= 195$ minutes

$= 3.25$ hours

Class/Homework for Section 7.5

Pg. 485 – 486 #2, 3cdf, 4, 5, 6acd, 8adf (see Ex 2, Pg. 483), 10

Solve: A) $4^{2x+3} = 9^x$ $\quad x = -7.23$

B) $5^{x-2} = 3^{x+1}$ $\quad x = 8.45$

C) $(0.312)^{2x+1} = 4^{3x-1}$ $\quad x = 0.0341$