Chapter 7 – Exponential and Logarithmic Equations
Practice Test

1. Evaluate $\log_5(20000) - \log_5(32)$

$\log_5\left(\frac{20000}{32}\right) = \log_5(625) = \log_5(5^4)$

$= 4$

2. Determine $\log_6\left(\sqrt[5]{36}\right)$

Let $x = \log_6(36^{\frac{1}{5}})$

$\Rightarrow 6^x = 36^{\frac{1}{5}}$

$= (6^2)^{\frac{1}{5}}$

$6^x = 6^{2/5}$

$\therefore x = 2/5$

3. Using a change of base (to base 10) calculate, correct to three decimal places:
$\log_9(319)$

$= \frac{\log(319)}{\log(9)} = 2.624$

4. Evaluate $\log_3\left(\frac{1}{243}\right)$

$\Rightarrow$ let $x = \log_3(243^{-1})$

$\Rightarrow 3^x = 243^{-1}$

$3^x = (3^5)^{-1}$

$3^x = 3^{-5}$

$\therefore x = -5$

5. Solve, correct to two decimal places if necessary:

a) $3\cdot\left(7^x\right)=50421$

$\div 3 \Rightarrow 7^x = 16807$

$x \cdot \log(7) = \log(16807)$

$\Rightarrow x = \dfrac{\log(16807)}{\log(7)}$

$= 5$

b) $1.24^x = 0.39$

$x \cdot \log(1.24) = \log(0.39)$

$\Rightarrow x = \dfrac{\log(0.39)}{\log(1.24)}$

$= -4.38$

c) $3^{2x-1} = 7^{x+1}$

$\Rightarrow (2x-1)\log(3) = (x+1)\log(7)$

$\Rightarrow 2x\log(3) - \log(3) = x\log(7) + \log(7)$

$\Rightarrow 2x\log(3) - x\log(7) = \log(7) + \log(3)$

$\therefore x = \dfrac{\log(7)+\log(3)}{2\log(3)-\log(7)}$

$= 12.11$

6. Solve, correct to one decimal place: $\left(5^{x+2}\right)+2\left(5^x\right)=54$

$\Rightarrow 5^x \cdot 5^2 + 2\cdot 5^x = 54$

$5^x(25+2) = 54$

$5^x(27) = 54$

$\div 27$

$5^x = 2$

$x \cdot \log(5) = \log(2)$

$x = \dfrac{\log(2)}{\log(5)} = 0.4$

7. Solve:

a) $\log_2(x-2)=\log_2(2x-7)$

Adm. Dom

$x > 2$

$x > \frac{7}{2}$

$\Rightarrow x > \frac{7}{2}$

$x-2 = 2x-7$

$-x = -5$

$x = 5$.

b) $\log_3(2x+5)=\log_3(12)-\log_3(6)$

Adm. Dom

$x > -5/2$

$\Rightarrow \log_3(2x+5) = \log_3(12/6)$

$\Rightarrow \log_3(2x+5) = \log_3(2)$

$\Rightarrow 2x+5 = 2$

$2x = -3$

$\Rightarrow x = -3/2$

8. Solve $\log_4(3x-1)=2$

Adm. Dom
$x > \frac{1}{3}$.

$3x-1 = 4^2$

$3x-1 = 16$

$3x = 17$

$x = \frac{17}{3}$

9. Solve $\log_5(x)+\log_5(2x-9)=1$

Adm. Dom
$x > 0$
$x > 9/2$
$\Rightarrow x > 9/2$

$\Rightarrow \log_5(x(2x-9)) = 1$

$x(2x-9) = 5^1$

$\Rightarrow 2x^2 - 9x - 5 = 0$

$(2x+1)(x-5) = 0$

$\therefore x = -\frac{1}{2}$ or $x = 5$

$x = -\frac{1}{2}$ inadmissible

10. If $\log_3(x)=1.2$, evaluate $\log_3\left(\frac{\sqrt[3]{x}}{27}\right)$.

$\log_3\left(\frac{x^{\frac{1}{3}}}{27}\right) = \log_3(x^{\frac{1}{3}}) - \log_3(27)$

$= \frac{1}{3}\log_3(x) - \log_3(3^3)$

$= \frac{1}{3}(1.2) - 3$

$= 0.4 - 3 = -2.6$.

11. Solve $2\log(x+1)-\log(3-x)=\log(2)$

Adm Dm

$x > -1$

$x < 3$

$\Rightarrow -1 < x < 3$

$\Rightarrow \log((x+1)^2) - \log(3-x) = \log(2)$

$\log\left(\frac{(x+1)^2}{3-x}\right) = \log(2)$

$\Rightarrow \frac{(x+1)^2}{3-x} = 2 \Rightarrow (x+1)^2 = 6-2x$

$\Rightarrow x^2+2x+1 = 6-2x$

$x^2+4x-5=0$

$(x+5)(x-1)=0$

$\therefore x=-5$ or $x=1$

inadmissible.

12. A certain bacterium has a doubling time of 12 minutes. How long will it take an initial population of 60 bacteria to grow to 245,760 bacteria? Note: the doubling formula is given by $P(t)=P_0(2)^{\frac{t}{D}}$

GIVEN

$D=12$

$P_0=60$

$P(t)=245760$

$245760 = 60(2^{t/12})$

$\div 60 \quad 4096 = 2^{t/12}$

$\log(4096) = \frac{t}{12}\cdot\log(2)$

$\Rightarrow t = \frac{12\cdot\log(4096)}{\log(2)} = 144$ min.