ADVANCED FUNCTIONS

in

Chapter 8 –Combing Functions

(*Material adapted from Chapter 9 of your text*)

Chapter 8 – Combining Functions

Contents with suggested problems from the Nelson Textbook (Chapter 9)

8.1 Sums and Differences of Functions – *Pg 185 - 188*
Pg. 451 #1c, 4, Finish 5 – 7, 9, 11

8.2 Product and Quotient Combinations – *Pg 189 – 191*
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Pg. 542 # 1aef, 2 (for #1aef)

8.3 Composition of Functions – *Pg 192 – 195*
READ Ex. 4 Pg550 making sure you understand the concept of a function composed with its inverse.
Pg. 552 – 553 #1abf, 2bdf, 5bcdf (use tech to graph), 6bcd, 7bcdef, 13

8.1 Sums and Differences of Functions

Definition 8.1.1

Given two functions $f(x)$, and $g(x)$ with domains D_f and D_g respectively, then we can **construct** new functions:

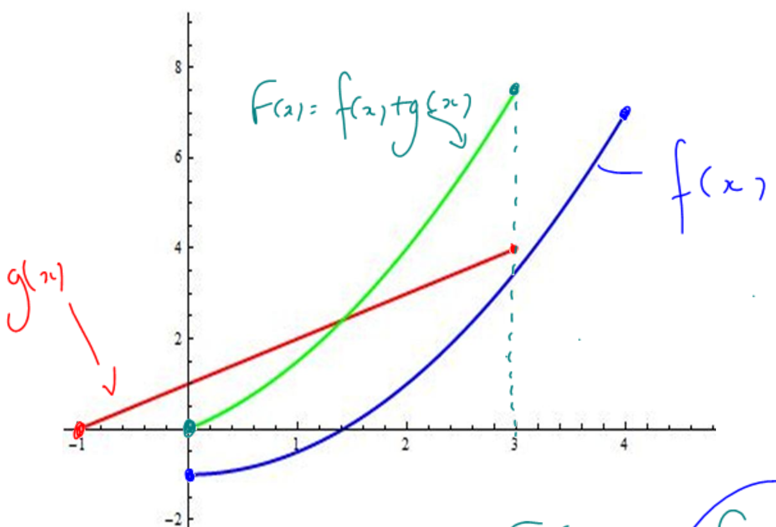
$$F(x) = (f+g)(x) \qquad\qquad G(x) = (f-g)(x)$$

where the **meaning** of the notation for the "sum" and "difference" functions is as follows:

$F(x) = f(x) + g(x)$ $\qquad\qquad$ $G(x) = f(x) - g(x)$

Example 8.1.1

Consider the sketch (so that we can get at the domain of sum and difference functions):



$D_f = [0, 4]$

$D_g = [-1, 3]$

$D_{f+g} = [0, 3]$

$F(-1) = f(-1) + g(-1)$

undefined

$\therefore$ $F(-1)$ is undefined

we can only use domain values which are common to both $f(x)$ & $g(x)$!

Note: the domain of a sum/difference fn is the INTERSECTION of the domains of the known fns

In general, given functions $f(x)$ and $g(x)$ with domains D_f and D_g, then the combined function

$$F(x) = (f \pm g)(x)$$

intersection (values in common)

$$D_F = D_f \cap D_g$$

Example 8.1.2

Determine the domain of $F(x) = (f - g)(x)$ for $f(x) = \sqrt{x}$, and $g(x) = \log(-(x-2))$.

$-(x-2) > 0$

$\Rightarrow x - 2 < 0$

$x < 2$

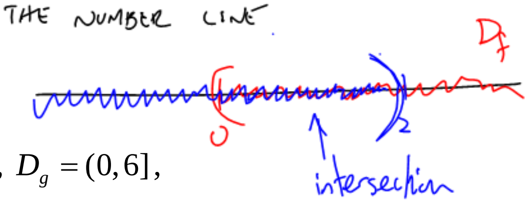
$D_F = D_f \cap D_g$

$= [0, 2)$

$D_f = [0, \infty)$

$D_g = (-\infty, 2)$

CONSIDER THE NUMBER LINE



Example 8.1.3

Given $f(x) = x^3 - 4x + 1$, D_f =[-4,5] and $g(x) = 2x^2 - 1$, $D_g = (0,6]$,

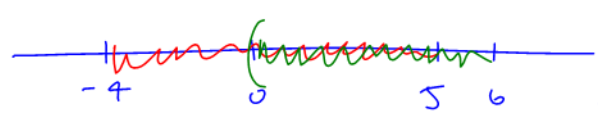
Determine: a) the order of $F(x) = (g - f)(x)$ $= g(x) - f(x)$

b) D_F

c) an algebraic representation for $F(x)$

a) Order 3

b) $D_F = D_f \cap D_g = (0, 5]$



c) $F(x) = (g - f)(x) = g(x) - f(x)$

$= (2x^2 - 1) - (x^3 - 4x + 1)$

$= -x^3 + 2x^2 + 4x - 2$

Example 8.1.4

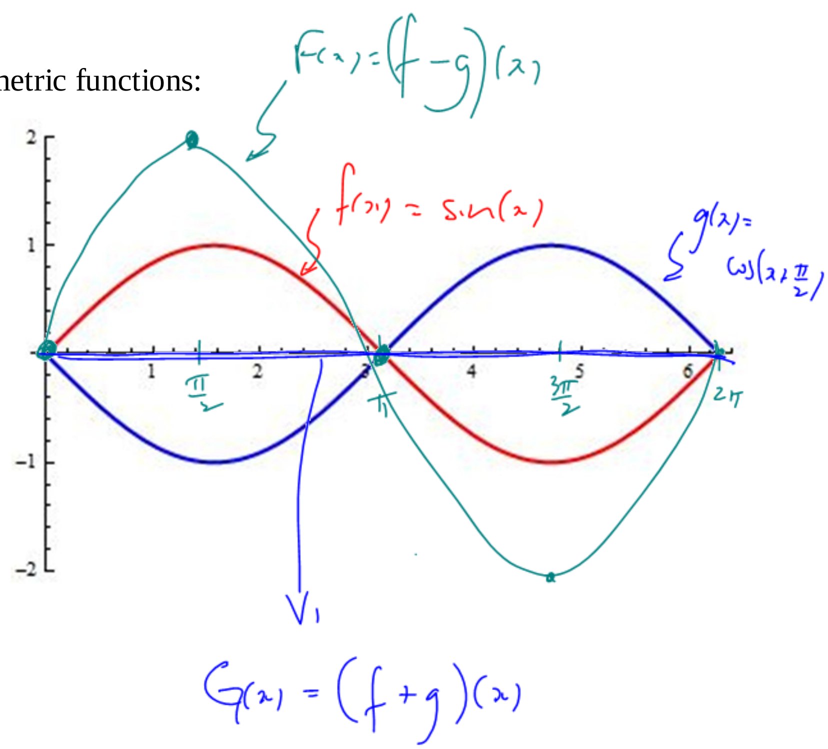
Consider the sketches of the trigonometric functions:

$$f(x)=\sin(x),\ D_f=[0,2\pi]$$

$$g(x)=\cos\left(x+\frac{\pi}{2}\right),\ D_g=[0,2\pi]$$

Sketch a) $F(x)=(f-g)(x)$

b) $G(x)=(f+g)(x)$



a) $F(x) = f(x) - g(x)$

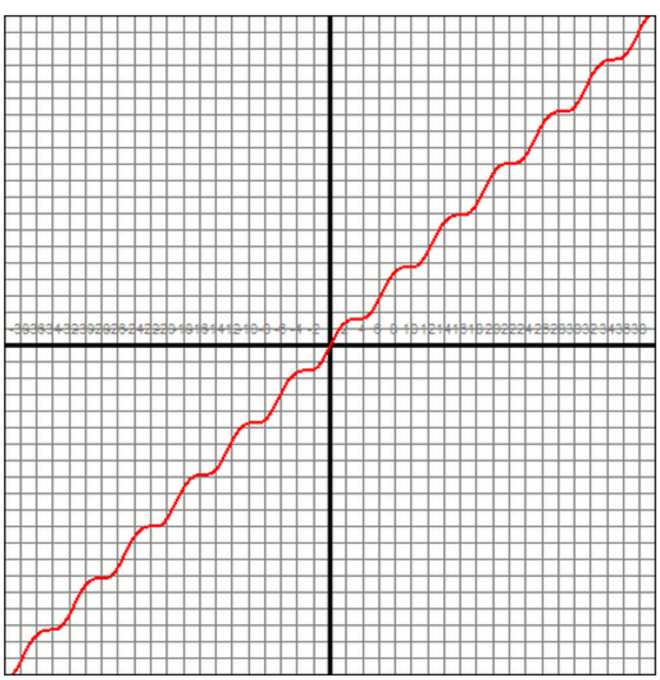
eg $F\left(\frac{3\pi}{2}\right) = f\left(\frac{3\pi}{2}\right) - g\left(\frac{3\pi}{2}\right)$

$= -1 - 1 = -2$

Example 8.1.5

Consider the functions $f(x)=x,\ g(x)=\sin(x),\ x\in\mathbb{R}$.

What does $F(x)=(f+g)(x)$ look like? (see Ex. 3 Pg. 526)



Note that $F(x)$ contains the behaviours of BOTH fns used in its construction

Example 8.1.6

From your text: Pg. 528 #1ae

Let $f = \{(-4, 4), (-2, 4), (1, 3), (3, 5), (4, 6)\}$ and $g = \{(-4, 2), (-2, 1), (0, 2), (1, 2), (2, 2), (4, 4)\}$.

$D_f = \{-4, -2, 1, 3, 4\}$

$D_g = \{-4, -2, 0, 1, 2, 4\}$

Determine: a) $(f+g)(x)$ e) $(f+f)(x)$

$(f+g)(-4) = f(-4) + g(-4) = 4+2 = 6$

$D_{f+g} = \{-4, -2, 1, 4\}$

a) $(f+g)(x) = \{(-4, 6), (-2, 5), (1, 5), (4, 10)\}$

e) $(f+f)(x) = \{(-4, 8), (-2, 8), (1, 6), (3, 10), (4, 12)\}$

Example 8.1.7

From your text: Pg. 529 #7ab

Given $f(x) = \dfrac{1}{3x-4}$ and $g(x) = \dfrac{1}{x-2}$, determine $(f+g)(x)$ and $D_{(f+g)}$

$(f+g)(x) = f(x) + g(x)$

$= \dfrac{1}{3x-4} + \dfrac{1}{x-2}$

$x - 2 + 3x - 4$

$= \dfrac{4x-6}{(3x-4)(x-2)}$

$D_{f+g} = \{x \in \mathbb{R} \mid x \neq \frac{4}{3}, x \neq 2\}$

or

$(-\infty, \frac{4}{3}) \cup (\frac{4}{3}, 2) \cup (2, \infty)$

Class/Homework for Section 8.1

Pg. 528 - 530 #1ce, 2, 3, 6ad, 10 (recall even/odd fns), 11 (see ex3 pg. 526), 12, 9ac (not symmetry)