

8.2 Product and Quotient Combinations

Definition 8.2.1

Given two functions $f(x)$, and $g(x)$ with domains D_f and D_g respectively, then we can **construct** new functions:

$$F(x) = (f \cdot g)(x)$$

$$= f(x) \cdot g(x)$$

$$D_{f \cdot g} = D_f \cap D_g$$

$$G(x) = \left(\frac{f}{g} \right)(x)$$

$$= \frac{f(x)}{g(x)}$$

$$D_{\frac{f}{g}} = D_f \cap D_g, g(x) \neq 0$$

Example 8.2.1

Determine $(f \cdot g)(x)$ given $f(x) = \{(-2, 3), (-1, 5), (0, 3), (1, -3), (2, -5)\}$ and

$g(x) = \{(-1, 4), (0, -2), (1, 7), (2, -2), (3, 2)\}$

$$(f \cdot g)(x) = \{(-1, 20), (0, -6), (1, -21), (2, 10)\}$$

$$D_f = \{-2, -1, 0, 1, 2\}$$

$$D_g = \{-1, 0, 1, 2, 3\}$$

$$D_{f \cdot g} = \{-1, 0, 1, 2\}$$

Example 8.2.2

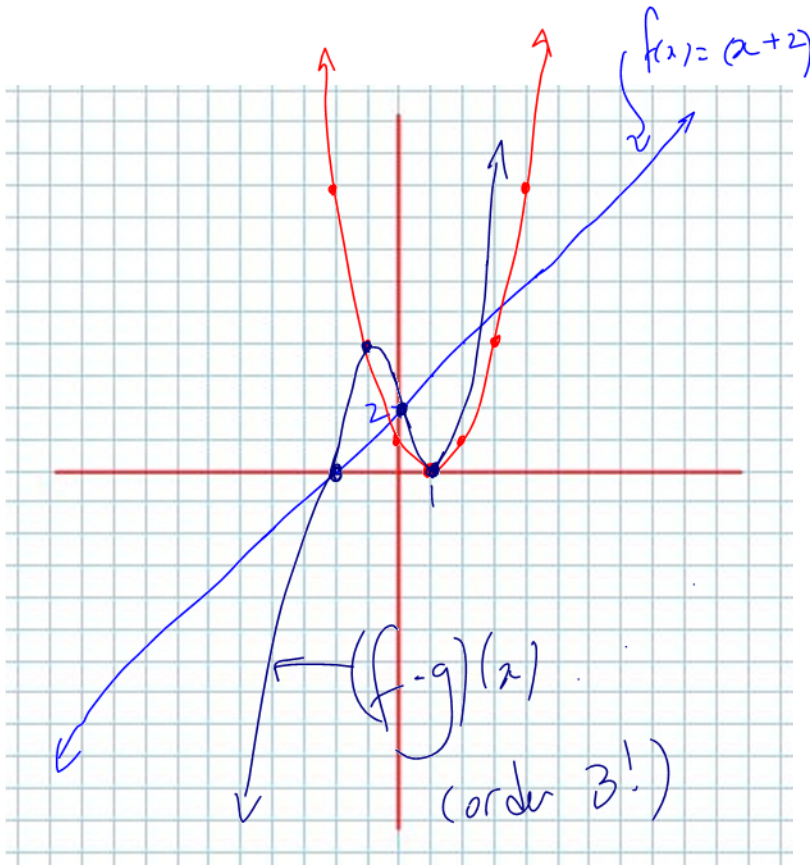
From your text: Pg. 537 #2 for #1e

Sketch the given pair of function on the same set of axes. State their domains. Use your sketch to draw $(f \cdot g)(x)$. State $(f \cdot g)(x)$ and $D_{f \cdot g}$.

$$f(x) = x + 2, \quad g(x) = x^2 - 2x + 1$$

$$D_f = (-\infty, \infty) = \{x \in \mathbb{R}\} \quad = (x-1)^2 \quad \text{vertex } (1, 0)$$

$$D_g = (-\infty, \infty)$$



$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$(f \cdot g)(-2) = f(-2) \cdot g(-2) = (0)(4) = 0$$

$$(-2, 0) = 0$$

$$(f \cdot g)(-1) = f(-1) \cdot g(-1) = (+1)(1) = 1$$

$$(f \cdot g)(0) = f(0) \cdot g(0) = (2)(1) = 2$$

$$(f \cdot g)(1) = f(1) \cdot g(1) = (3)(1) = 3$$

Example 8.2.3

Determine the domain of $D_{f \cdot g}$ and D_{f+g} given $f(x) = \sqrt{2x+3}$ and $g(x) = \sec(x)$

$$D_f = [-3/2, \infty)$$

$$\begin{aligned} 2x+3 &\geq 0 \\ 2x &\geq -3 \\ x &\geq -3/2 \end{aligned}$$

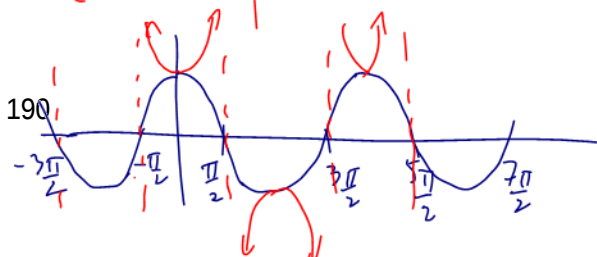
$$\sec(x) = \frac{1}{\cos(x)}$$

$$D_g = \left\{ x \in \mathbb{R} \mid x \neq \frac{(2n+1)\pi}{2}, n \in \mathbb{Z} \right\}$$

⇒ Domain has "issues" when $\cos(x) = 0$
all "odd $\frac{\pi}{2}$'s"

$$D_{f \cdot g} = \left\{ x \geq -3/2 \mid x \neq \frac{(2n+1)\pi}{2}, n \in \mathbb{Z} \right\}$$

$$D_{f+g} = \left\{ x \geq -3/2 \mid x \neq \frac{(2n+1)\pi}{2}, n \in \mathbb{Z} \right\}$$



Note $\sec(x) \neq 0$

Example 8.2.4

From your text: Pg. 542 #2 for 1d.

Sketch the given pair of function on the same set of axes. State their domains. Use your

sketch to draw $\left(\frac{f}{g}\right)(x)$. State $\left(\frac{f}{g}\right)(x)$ and $D_{\frac{f}{g}}$.

$$f(x) = x + 2 \text{ and } g(x) = \sqrt{x - 2}$$

you do this

$$D_{\frac{f}{g}} = (2, \infty)$$

$\leftarrow x=2$ makes $g(x) = 0$!

$$D_f = \{x \in \mathbb{R}\}$$

$$D_g = \{x \in \mathbb{R} \mid x \geq 2\} \\ \text{or } [2, \infty)$$

$$D_f \cap D_g = [2, \infty)$$

Class/Homework for Section 8.2 – READ ex 4 Pg. 536

Pg. 537 - 539 #1bd, 3, 8bd, 10, 15

Pg. 542 # 1aef, 2 (for #1aef)