8.3 Composition of Functions

In sections 8.1 and 8.2 we examined how to combine functions (constructing new functions) through the standard algebraic operations of addition, subtraction, multiplication and division. Here we will learn another method for combining functions, but **we won't be using a standard algebraic operation.**

The concept we define as **Composition of Functions** is **very useful for Calculus** (among other things) as some of you will see next semester.

The basic idea is that given two functions $f(x)$ and $g(x)$, we can define the composition of the two by

inserting one function into the other

The "algebraic" notation may seem a little weird, but don't make fun. Math has feelings too.

Definition 8.3.1

Given two functions $f(x)$ and $g(x)$ we write the composition of $f(x)$ and $g(x)$ as

$$(f \circ g)(x)$$

We can also write the composition of $g(x)$ and $f(x)$ as

$$(g \circ f)(x)$$

The "Algebraic Meaning" of Composition

$$(f \circ g)(x) = f(g(x))$$

outer

inner fn

$$(g \circ f)(x) = g(f(x))$$

outer

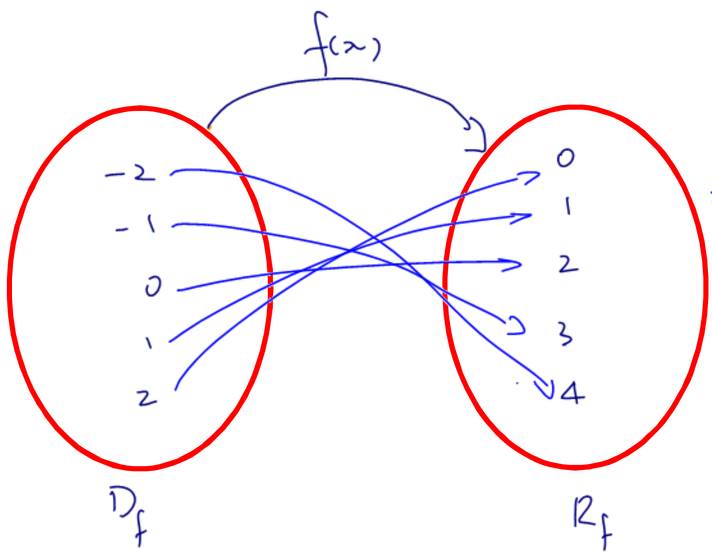
inner

Note: It is **very helpful** to keep in mind the distinction between the **inner** and **outer** functions

The Domain of a Composition of Functions

Recall the basic "**machinery**" of any function: **"Plug a (domain) number into the function, and get a (range) number out."**

Picture



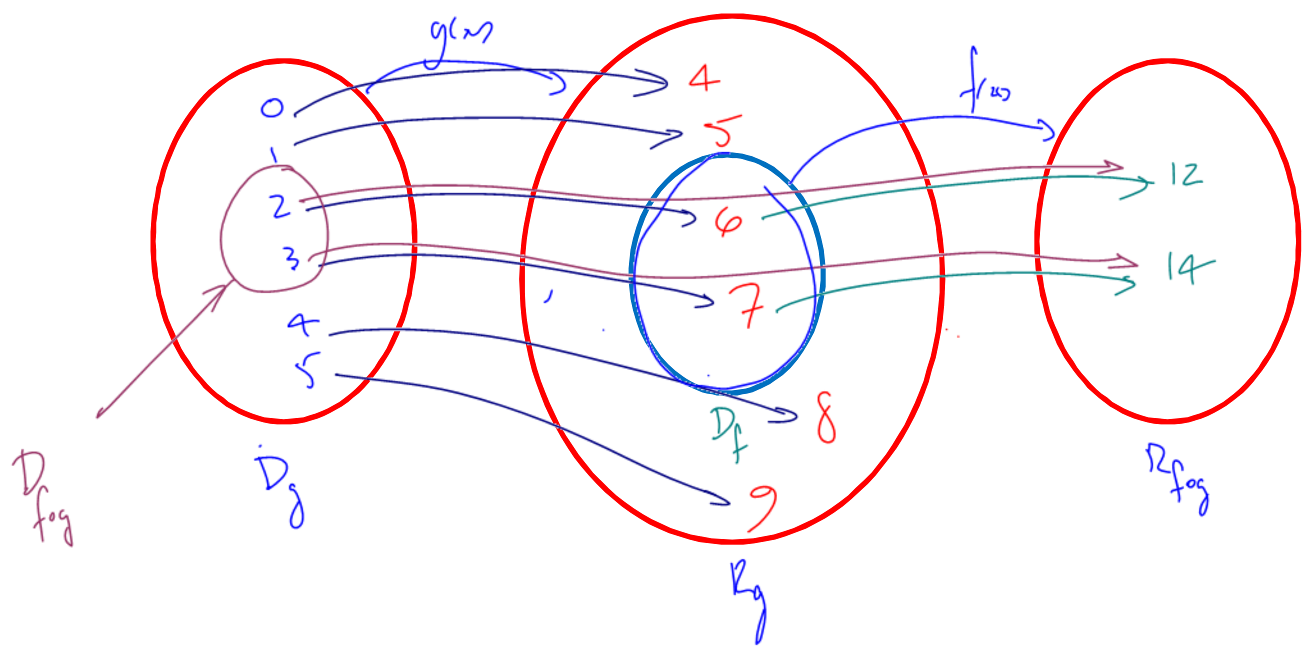
Graph of $f(x)$ is $= \{(x, f(x)) \mid (-2,4), (-1,3), (0,2), (1,1), (2,0)\}$

Recall further that many functions cannot claim "all real numbers" as their (natural) domain.

e.g. Determine the domain of $f(x) = \sqrt{x+1}$ $\quad D_f = [-1, \infty)$

Now we consider the "machinery" for the composition function $(f \circ g)(x) = f(g(x))$

Picture:



Algebraic Definition of the Domain of a Composition of Two Functions

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Given two functions $f(x)$ and $g(x)$ with domains D_f and D_g respectively, then the domain of the composition of $f(x)$ and $g(x)$ is given by:

$$D_{(f \circ g)} = \{x \in \mathbb{R} \mid x \in D_g \quad \text{such that } g(x) \in D_f \}$$

Using words we might write that the **domain of a composition of functions** $(f \circ g)(x) = f(g(x))$ is **the set of all *x* values** which belong to the domain of the

inner function which have range values which are in the domain of the

outer function.

Example 8.3.1

Given $f(x) = 3x + 1$ and $g(x) = x^3 - 1$ determine:

a) $(f \circ g)(0)$

$= f(g(0))$

$= f(-1)$

$= -2$

$\Rightarrow$ pt $(0, -2)$

b) $g(f(0))$

$= g(1)$

$= (0)$

$\Rightarrow$ pt $(0, 0)$

c) $(f \circ f)(1)$

$= f(f(1))$

$= f(4)$

$= 13$ $\Rightarrow$ pt $(1, 13)$

Example 8.3.2

From your text: Pg. 552 #2 ac"g"

Given $f(x) = \{(0,1),(1,2),(2,5),(3,10)\}$ and $g(x) = \{(2,0),(3,1),(4,2),(5,3),(6,4)\}$

determine:

a) $(g \circ f)(2)$

$\Rightarrow g(f(2))$

$= g(5)$

$= 3$

c) $f(g(5))$

$f(g(5))$

$= f(3)$

$= 10$

g) $g(f(3))$

$g(10)$

not existing! (dne)

$\Rightarrow 3 \notin D_{g \circ f}$

Something Silly but Entirely Serious

Given $f(x) = 2x^2 - 1$ determine:

a) $f(2)$ b) $f(A)$ c) $f(\square)$ d) $f(\square + \triangle)$

$= 2(2)^2 - 1$
$= 7$

$= 2A^2 - 1$

$= 2(\square)^2 - 1$

$2(\square + \triangle)^2 - 1$

Example 8.3.3

From your text: Pg 552 #6ae

Given the functions $f(x)$ and $g(x)$ determine functional equations for $f(g(x))$ and $g(f(x))$ and determine their domains.

a) $f(x) = 3x$ and $g(x) = \sqrt{x-4}$ e) $f(x) = 10^x$ and $g(x) = \log(x)$

$f(x)$ & $g(x)$ are inverses of each other

$f(g(x)) = f(\sqrt{x-4})$
$= 3\sqrt{x-4}$

$g(f(x))$
$= g(3x)$
$= \sqrt{3x-4}$

$f(g(x))$
$= f(\log(x))$
$= 10^{\log(x)}$
$= x$

$g(f(x))$
$= g(10^x)$
$= \log_{10}(10^x)$
$= x$

Note: In General

$f(g(x)) \neq g(f(x))$ unless $f(x)$ & $g(x)$ are inverses of each other

& $f(f^{-1}(x)) = x$
& $f^{-1}(f(x)) = x$.

Example 8.3.4

From your text: Pg. 553 #7a

Given $h(x)$ find two functions $f(x)$ and $g(x)$ such that $h(x) = f(g(x))$

a) $h(x) = \sqrt{x^2 + 6}$

$g(x) = x^2 + 6$
$f(x) = \sqrt{x}$

or

$g(x) = x^2$
$f(x) = \sqrt{x+6}$

or . . .

Class/Homework for Section 8.3

READ Ex. 4 Pg550 making sure you understand the concept of a function composed with its inverse.
Pg. 552 – 553 #1abf, 2bdf, 5bcdf (use tech to graph), 6bcd, 7bcdef, 13