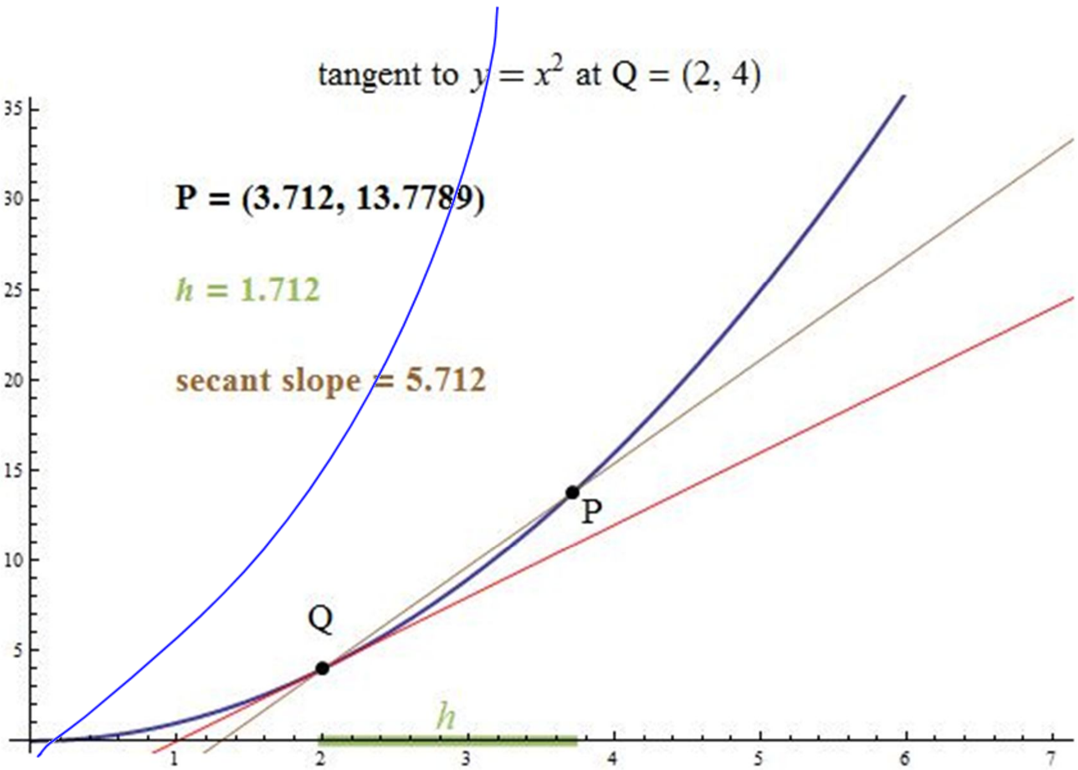
ADVANCED FUNCTIONS

Chapter 9 –Rates of Change and the Tangent Problem

(*Material adapted from Chapter 2 of your text*)



Chapter 9 – Rates of Change and the Tangent Problem

Contents with suggested problems from the Nelson Textbook (Chapter 2)

9.1 Average Rate of Change – The AROC

From Physics we learn that we can calculate the average velocity of some moving object through the formula

a measure of the change in displacement relative to change in time

$$v_{avg} = \frac{\Delta s}{\Delta t} = \;=\; \frac{s_2 - s_1}{t_2 - t_1}$$

$s_2 - s_1$ ← change in displacement

$t_2 - t_1$ ← change in time

In general we can calculate the **A**verage **R**ate **O**f **C**hange [AROC], for some given function $f(x)$, over an interval of time (the domain) $t \in [t_1, t_2]$ using the formula:

calculated

$$AROC = \frac{\Delta f}{\Delta t} = \frac{f(t_2) - f(t_1)}{t_2 - t_1}$$

Example 9.1.1

Consider the displacement function $s(t) = 100 - 4.9t^2$, which is being used to describe the displacement (*s* in *m*) of a falling body from the top of a 100*m* high cliff after *t* seconds.

Over the given time intervals determine the average rate of change (the AROC) of displacement for a stone dropped from the edge of the cliff:

a) $t = \overset{t_1}{0}$ to $t = \overset{t_2}{1}$ seconds. b) $t \in [1,2]$ (seconds). c) $t \in [0,3]$.

a) $AROC = \frac{\Delta s}{\Delta t} = \frac{(s(t_2) - s(t_1))}{(t_2 - t_1)}$

$= \frac{s(1) - s(0)}{1 - 0}$

$= \frac{95.1 - 100}{1}$

$= -4.9 \text{ m/sec}$

b) $AROC = \frac{(s(2) - s(1))}{(2-1)}$

$= -14.7 \text{ m/sec}$

note that the stone is "speeding up"

c) $AROC = \frac{s(3) - s(0)}{3 - 0}$

$= -14.7 \text{ m/sec}$

Q. what is the AROC?

$s(0) = 100$

$s(4.5) = 0$

Sketch $s(t) = -4.9t^2 + 100$

'time' is restricted

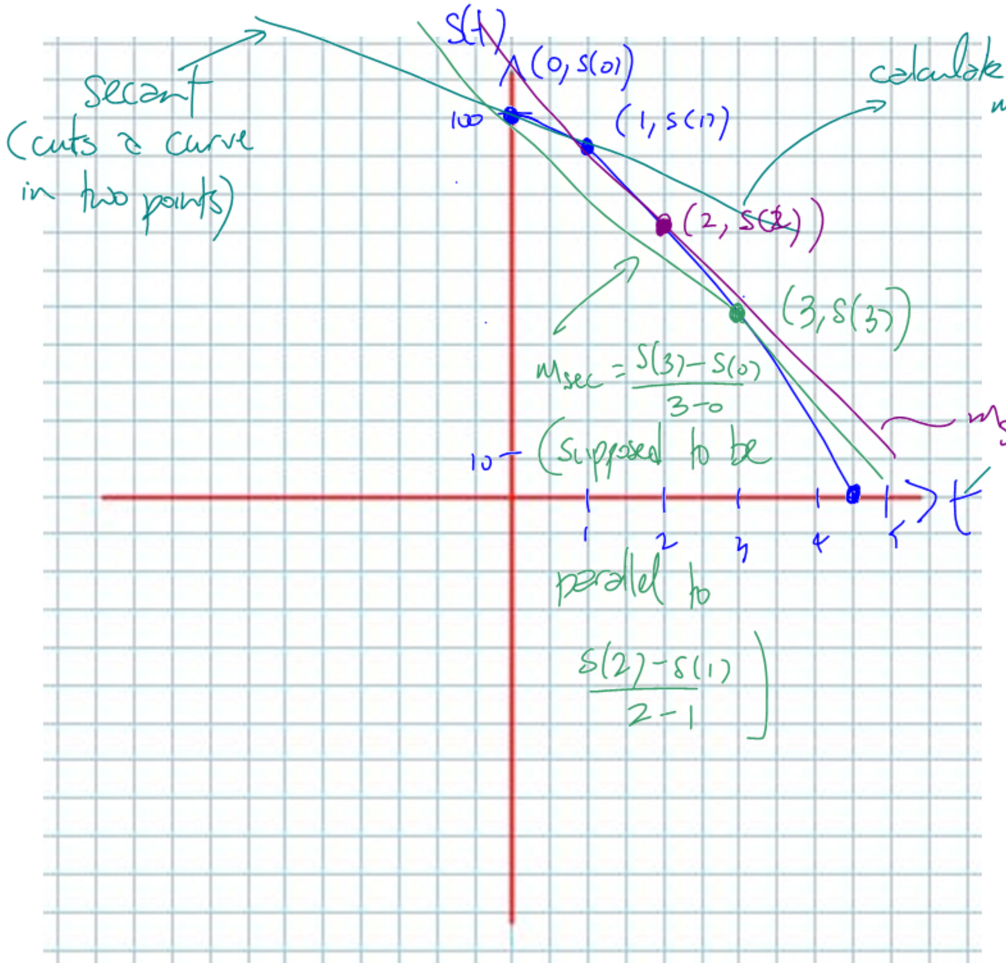
A picture of the situation in example 9.1.1:

Note: when $s(t) = 0$ we have the "end" of the domain.

$\Rightarrow -4.9t^2 + 100 = 0$

$\Rightarrow t \doteq 4.5$

$t \in [0, 4.5]$



calculate

$m_{sec} = \frac{\text{rise}}{\text{run}}$

$= \frac{s(1) - s(0)}{1 - 0}$

$=$ AROC on $t \in [0, 1]$!

$m_{sec} = \frac{s(2) - s(1)}{2 - 1} =$ AROC on $t \in [1, 2]$

The Slope of a Secant IS

the AROC of a fn $f(t)$ over the domain interval $t \in [t_1, t_2]$ (or $t_1 \leq t \leq t_2$)

Class/Homework for Section 9.1

Pg. 76 – 77 #1 (important question), 2, 4, 9, 10