The time has arrived.
Advanced Fns Final Lesson.

9.3 Instantaneous Rate of Change – The IROC (part 2)

The Difference Quotient

Suppose we wish to calculate the Instantaneous Rate of Change (IROC) of some function, $f(x)$, at $x = 2$. Last day we saw three things:

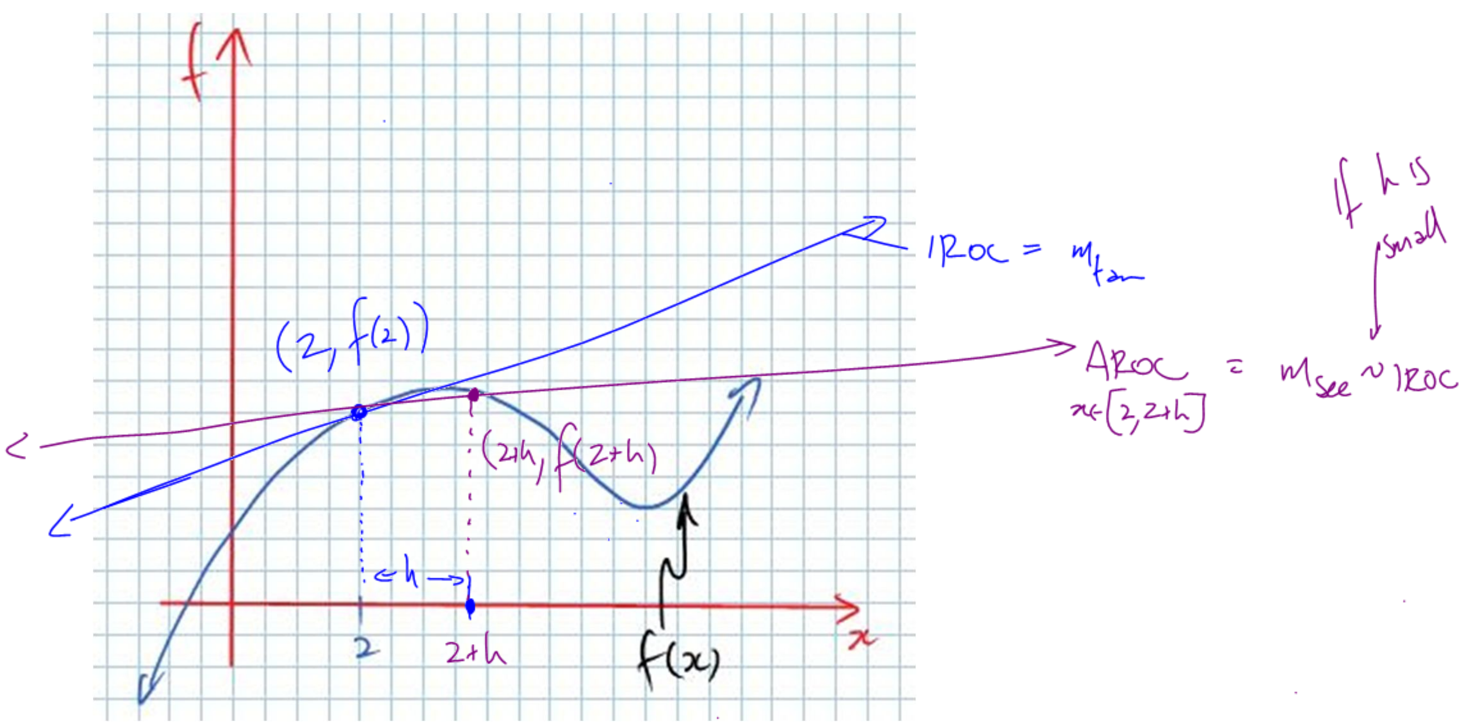
1) IROC (number) = the slope of a tangent to $f(x)$ at a given domain value!

2) We cannot calculate the slope of a tangent!

3) We can estimate the value of the IROC with the AROC (secant slope) if the interval for the AROC is small

Rather than using a "centered interval" approach, we now consider the so-called **Difference Quotient** (which can be much more useful than the centered interval approach).

Consider the sketch:



$$m_{sec} = \frac{f(2+h) - f(2)}{(2+h) - 2} = \frac{f(2+h) - f(2)}{h}$$

Now, we understand that

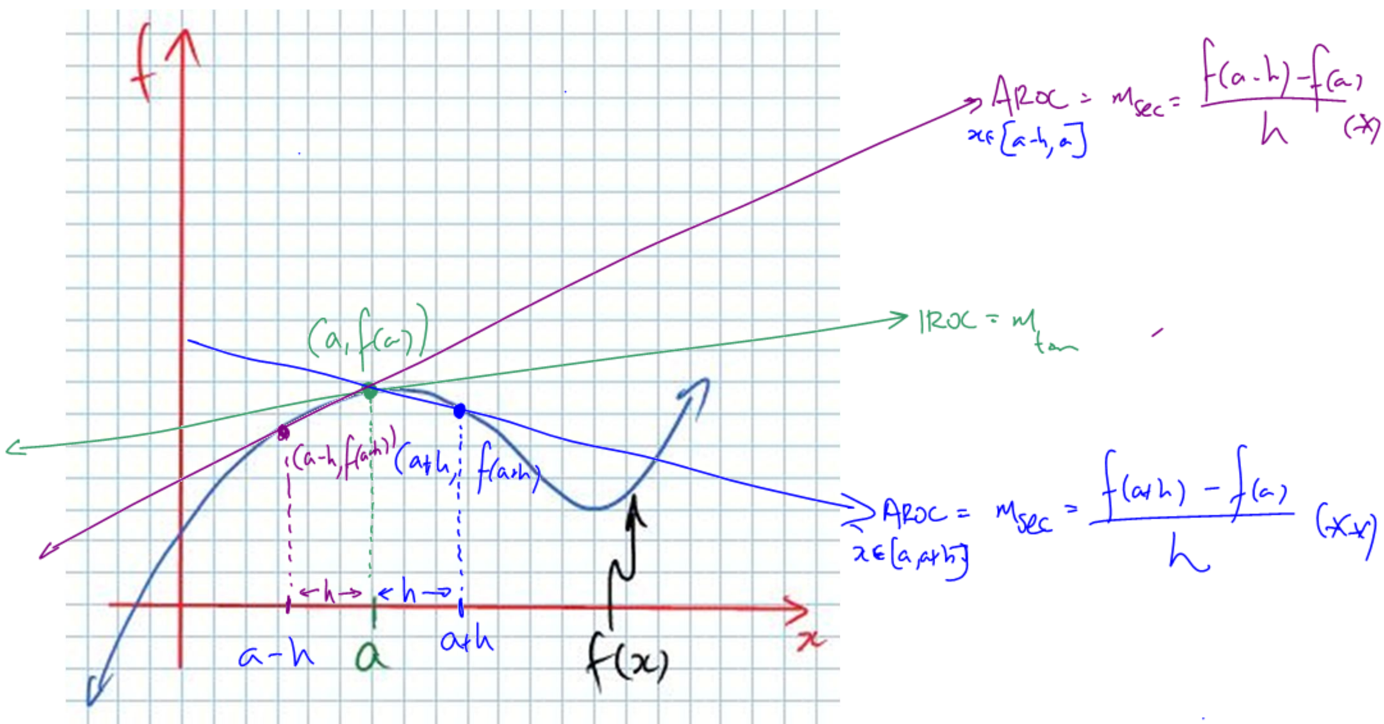
$$IROC \sim AROC = m_{\sec} = \frac{f(2+h)-f(2)}{h}, \text{ for small } h, \text{ at } x=2.$$

"Def'n" of the difference quotient

* In general, if we wish to approximate the IROC of $f(x)$ at some (general) domain value $x = a$, then

$$IROC \sim AROC = \frac{f(a+h)-f(a)}{h}, \text{ for small } h \text{ at } x = a$$

Note: "h" can be either positive or negative. Consider the sketch:



The text suggests averaging (*) & (**)

No need if h is small enough..

Example 9.3.1

Given $s(t) = 2t^2 - 3t - 5,$ determine a difference quotient which will estimate the IROC of $s(t)$ at $t = a$. Use that difference quotient to estimate the IROC at $t = 3$ using $h = 0.0001$.

$$\text{IROC} \sim \frac{s(a+h) - s(a)}{h}$$

for $a = 3,\ h = 0.0001$

$$\text{IROC} \sim \frac{s(3.0001) - s(3)}{0.0001}$$

$$= \frac{\left[\left(2(3.0001)^2 - 3(3.0001) - 5\right) - \left(2(3)^2 - 3(3) - 5\right)\right]}{0.0001}$$

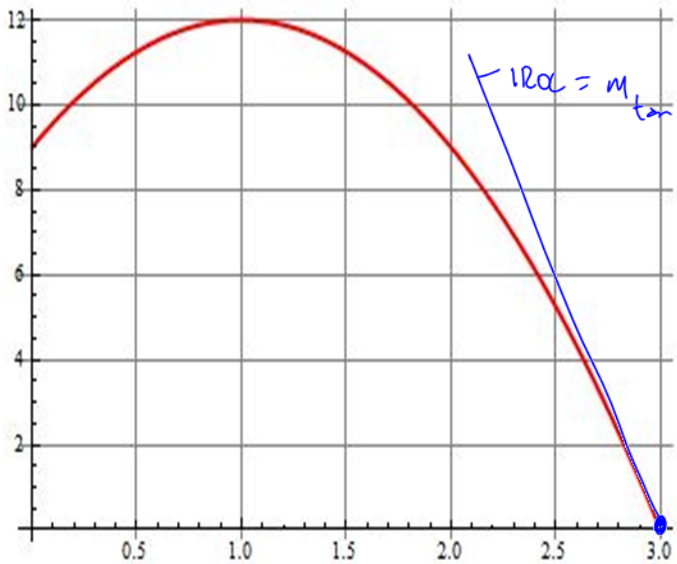
$$\doteq 9.0002$$

state WHERE you are estimating the IROC!

$\therefore$ The IROC $\sim$ 9 units at $t = 3$

Example 9.3.2

Consider the water-balloon problem from Example 9.2.1. The water-balloon "flies" according to the function $s(t) = -3(t-1)^2 + 12$. Estimate the instantaneous velocity (the IROC) of the balloon when it hits the ground (at $t = 3$ sec).



Note: we cannot use a centered interval approach (there is no domain past $t=3$)

Note further: we must use a negative value for h

Let $h = -0.0001$

$$IROC \approx \frac{s(3-0.0001) - s(3)}{-0.0001}$$

$$\doteq -11.9997$$

$\therefore$ The balloon hits the ground at approximately -12 m/sec at $t = 3$ sec

Class/Homework

Determine an estimate for the IROC of the given function at the indicated domain value using a difference quotient. Use $h = 0.001$ for your estimation.

a) $f(x) = x^2 - 3x + 1$ at $x = 2$ *IROC* ~ 1

b) $h(t) = 2^t - 3$ at $t = 0$ *IROC* ~ 0.693

c) $g(x) = \sin(x)$ at $x = \pi$ *IROC* ~ -1

d) $s(t) = \dfrac{t+1}{t-2}$ at $t = 3$ *IROC* ~ -3

e) $g(x) = x^3 + 2$ at $x = 3$ *IROC* ~ 27