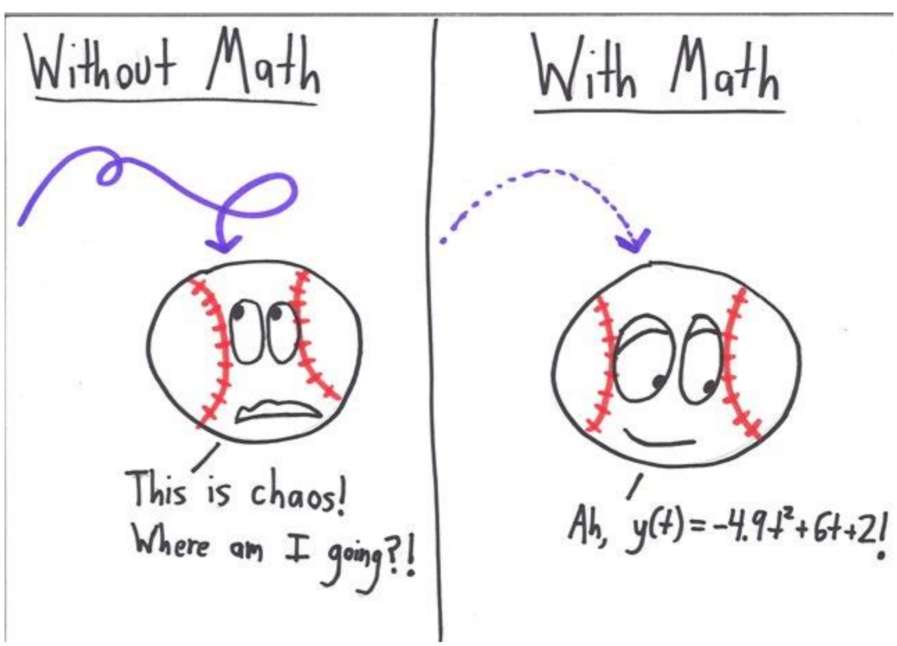
Functions & Applications 11

MCF3M

Course Notes

Chapter 1: Introduction to Quadratics



Homework

Contents with suggested problems from the Nelson Textbook. You are welcome to ask for help, from myself or your peers, with any of the following problems. They will be handed in on the day of the Unit Test as a homework check. Alternatively, you may be handing your homework in more regularly for me to correct.

Section 1.1 – Page 13
#1-5, 7, 11, 13

Section 1.3 – Page 33
#3, 4, 5, 6ab, 11a, 12

Section 1.6 – Page 56
For this homework, use the method shown in class. Ignore the textbook if the instructions are confusing.
#1 (just state the transformations)
#3, 5abc, 7abc

Section 1.7 – Page 64
#2-7 (your goal is finding the domain and range of a line)

1.1 Characteristics of a Function

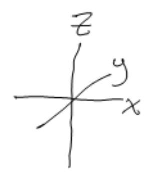
Learning Goal: We are learning how to identify the difference between a function and a relation.

In Grades 9 and 10, you learned about lines ($y = mx + b$) and parabolas ($y = ax^2 + bx + c$). Little did you know, these are called *functions*. Before we get into a formal definition of a function, let's first look at something more familiar, a *relation*.

A ***relation*** is an equation where there is an independent variable and a dependent variable connected in some way

Ex: Pay: \$11 per hour → $y = 11x$

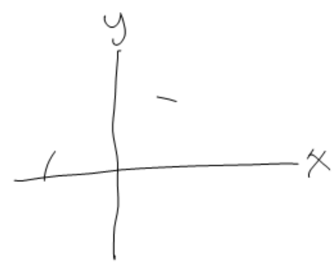
$x^2 + y^2 = 9$



Typically,

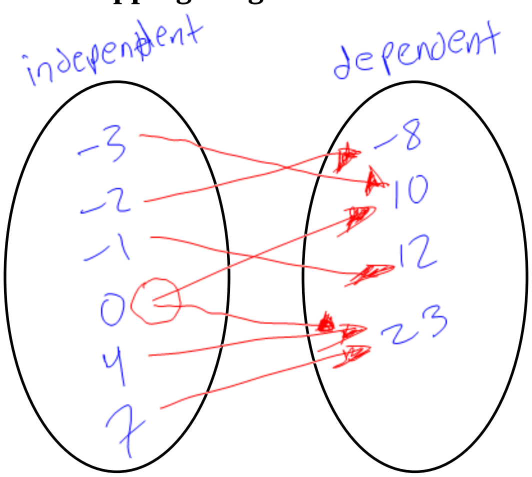
the independent variable is the "x"

the dependent variable is the "y"



A relation can be represented in a few ways:

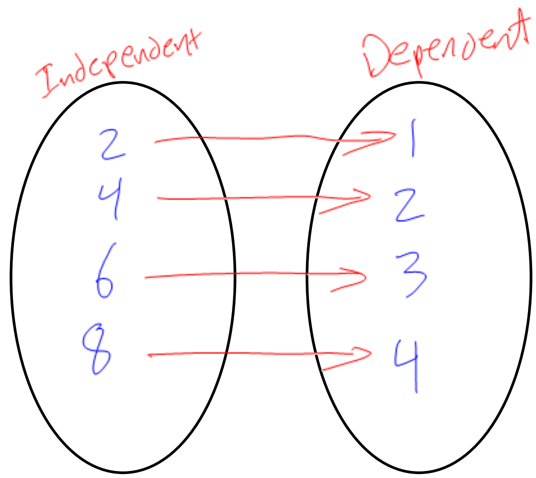
1. **Mapping Diagram**



Domain: $\{-3, -2, -1, 0, 4, 7\}$

Range: $\{-8, 10, 12, 23\}$

Not a fn, because 0 produces 10 & 23.



D: $\{2, 4, 6, 8\}$

R: $\{1, 2, 3, 4\}$

Is a function

2. **Equation**

$y=3x+4$

fn ✓

$y=x^2-8$

fn ✓

$y^2=4x^{10}+12x$

$(x=1)$

$y^2=4(1)^{10}+12(1)$

$y^2=4+12$

$\sqrt{y^2}=\sqrt{16}$ $y=\pm 4$

Not a function

3. **Table**

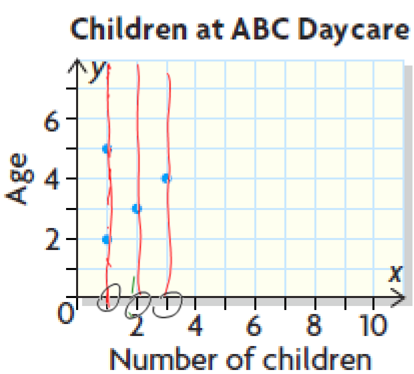
Independent km Driven	Dependent Cost of Rental
10	50
50	80
70	95
100	110

D: {10, 50, 70, 100}

R: {50, 80, 95, 110}

Is a fn

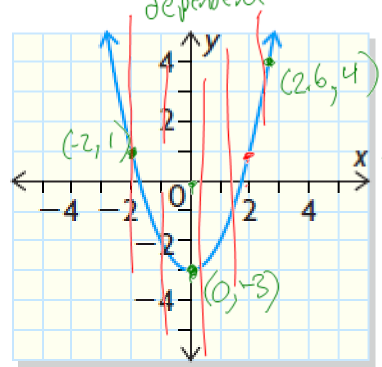
4. **Graph**



Not a fn fails V.L.T.

D: {1, 2, 3}

R: {2, 3, 4, 5}



Is a fn. Passes V.L.T.

5. **Set of Ordered Pairs**

{(-1,-3), (0,1), (1,1), (2,9)}

(x, y) x, y x, y x, y

{ } mean "a set, or a collection"

D: {-1, 0, 1, 2} (the x values)

R: {-3, 1, 9} (the y values)

fn ✓

{(1,4), (3,2), (0,5), (5,6), (3,0)}

D: {1, 3, 0, 5}

R: {4, 2, 5, 6, 0}

Not a fn. 3 gives two answers 2 + 0.

Before we define what a function is, we first need to define a few other things:

Domain: - is the set of all possible "x" values. (Independent)
- all of the x values that produce a result.

$y=\sqrt{x}$, $x=9$? $y=\sqrt{9}$ $=3$ works! 9 is a domain value. $x=-5$ $y=\sqrt{-5}$ = ERROR! -5 is not a domain value.

Range: - the set of all possible "y values". (Dependent)
- all of the possible answers/results.

Set Notation:
a fancy math notation to represent the domain + range values.

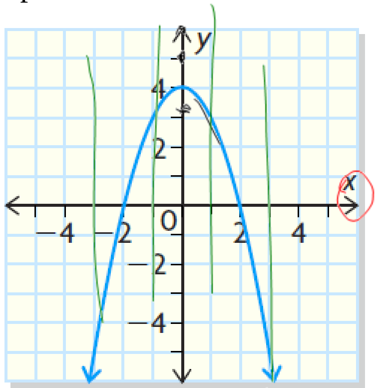
Function:
A function (fn) is a special type of relation, where the independent variable has a 1 to 1 relationship with the dependent variable. Each "x" produces only 1 "y".

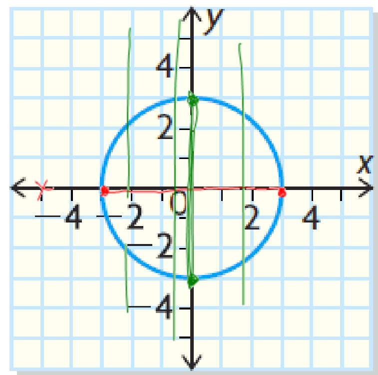
Vertical Line Test:
- used to determine if a graph is a function.
1) Draw vertical lines on the graph
2) If <u>any</u> vertical line touches the graph <u>more than once</u>, it is NOT a function.

Let's go back and determine the domain and range and whether or not each relation is a function.

Domain and Range can be represented in just words ("x can be any number"), but Math is all about representing things in numbers and symbols. This is what makes math universal, because people in Korea may not understand "x can be any number", but they would understand the symbols used to represent that.

fn ✓





Not a fn
fails V.L.T.

the identified variable

Real Numbers

Domain: $\{x \in \mathbb{R}\}$

"belongs to"

x belongs to the set of all real numbers.

x can be any number

Range: $\{y \in \mathbb{R} \mid y \leq 4\}$

"such that / but"

y can be any number, but y must be less than or equal to 4.

Domain: $\{x \in \mathbb{R} \mid -3 \leq x \leq 3\}$

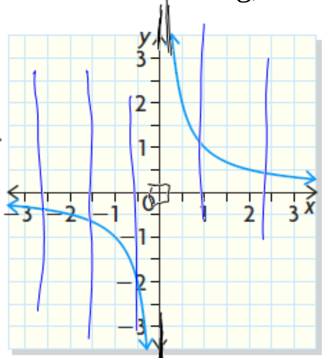
x can be any number, but x must be between −3 and 3.

Range: $\{y \in \mathbb{R} \mid -3 \leq y \leq 3\}$

For the following, determine the domain and range using set notation, and then state if it is a function.

a)

$y = \frac{1}{x}$

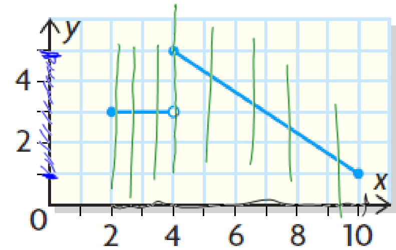


D: $\{x \in \mathbb{R} \mid x \neq 0\}$

R: $\{y \in \mathbb{R} \mid y \neq 0\}$

Is a fn

b)



$\leq, \geq$ • means the point is included

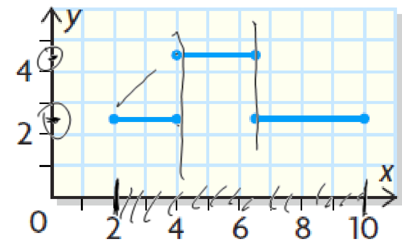
$<, >$ ○ means the point is not included

D: $\{x \in \mathbb{R} \mid 2 \leq x \leq 10\}$

R: $\{y \in \mathbb{R} \mid 1 \leq y \leq 5\}$

Fn ✓

c)



D: $\{x \in \mathbb{R} \mid 2 \leq x \leq 10\}$

R: $\{2.5, 4.5\}$

Not a Fn

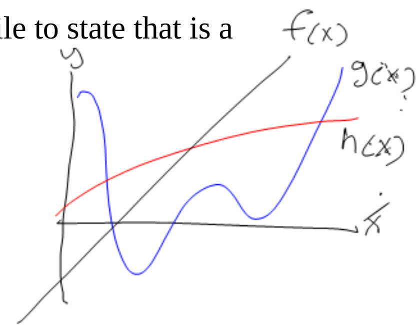
Success Criteria:

- I can tell the difference between a function and a relation
  - I can use the vertical line test as a tool to tell if a relation is a function
- I can find the domain and range of a function or relation
- I can use set notation to state domain and range
- I can represent a function or relation using a table of values, set of ordered pairs, graph, mapping diagram, or equation

1.3 Working with Function Notation

Learning Goal: We are learning how to use Function Notation

When we determine that a relation is a function, such as $y = 3x + 4$, it is worthwhile to state that is a function by giving it a name and indicating what the independent variable is.



$$y = 3x + 4 \rightarrow f(x) = 3x + 4$$

"f of x" or "f at x"

x is the independent variable, which is used to determine the functional value (formerly known as *y*).

Let's look at how this works: Given $f(x) = 3x + 4$, evaluate $f(2)$. This means that *x*= 2 . Just put 2 into both *x's*, then evaluate.

$f(2) = 3(2) + 4$

$f(2) = 10$, therefore we have the point $(2, 10)$

Given $f(x) = 2x^2 + 3x - 1$, evaluate

a) $f(3)$ b) $f\left(\frac{1}{2}\right)$ c) $f(5-3)$ d) $f(5) - f(4)$

a) $f(3) = 2(3)^2 + 3(3) - 1$

$= 2(9) + 9 - 1$

$= 18 + 9 - 1$

$\boxed{= 26}$

$(3, 26)$

b) $f(\frac{1}{2}) = 2(\frac{1}{2})^2 + \frac{3}{1}(\frac{1}{2}) - 1$

$= \frac{2}{1}(\frac{1}{4}) + \frac{3}{2} - \frac{1}{1}$

$= \frac{1}{2} + \frac{3}{2} - \frac{2}{2}$

$= \frac{2}{2} = 1$

So, $f(\frac{1}{2}) = 1$

c) So, $f(2)$?

$f(2) = 2(2)^2 + 3(2) - 1$

$= 2(4) + 6 - 1$

$= 8 + 6 - 1$

$f(2) = 13$

So, $\boxed{f(2) = 13 \text{ OR } (2, 13)}$

d) $f(x) = y$

Find $f(5)$

$f(5) = 2(5)^2 + 3(5) - 1$

$= 2(25) + 15 - 1$

$= 50 + 15 - 1$

$f(5) = 64$

$f(4) = 2(4)^2 + 3(4) - 1$

$= 32 + 12 - 1$

$f(4) = 43$

So, $f(5) - f(4) = 64 - 43$

$\boxed{= 21}$

Given $g(x) = 5x - 8$, determine the x so that $g(x) = 18$. $\boxed{y = 18}$

$\underbrace{g(x)}_{\;} = \underbrace{y}_{\;}$

Gave you "y", now find x.

$18 = 5x - 8$
$+8 \qquad +8$

$\frac{26}{5} = \frac{5x}{5}$

$\therefore x = 5.2$

So, $g(5.2) = 18$

From your textbook, pg 32 #2.

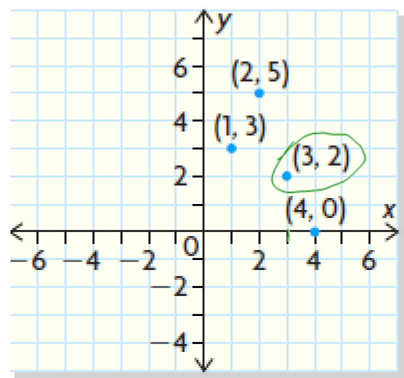
Evaluate $f(3)$ for each of the following.

a) $\{(1, 2), (2, 0), \boxed{(3, 1)}, (4, 2)\}$

b)

x	1	2	3	4
y	2	3	4	5

c)



a) $f(\underset{x}{3}) = \underset{y}{1}$

b) $f(\underset{x}{3}) = \underset{y}{4}$

c) $f(\underset{x}{3}) = \underset{y}{2}$

Lastly, something a little strange: given $h(x) = 2x^2 - 3x + 4$, evaluate $h(a)$ and $h(x-2)$.

$h(a) = 2a^2 - 3a + 4$

b) $h(x-2) = 2(x-2)^2 - 3(x-2) + 4$

$= 2(x-2)(x-2) - 3(x-2) + 4$

$= 2(x^2 - 2x - 2x + 4) - 3x + 6 + 4$

$= 2(x^2 - 4x + 4) - 3x + 10$

$= 2x^2 - 8x + 8 - 3x + 10$

$\boxed{h(x-2) = 2x^2 - 11x + 18}$

Success Criteria:

- I can recognize that "*x*" represents the domain value in the function notation "*f(x)*"
- I can represent that *f(x)* is the range value (y-value) that corresponds to the x input
  - *y = f(x)*

1.6 Graphing Quadratics with Transformations

Learning Goal: We are learning to use transformations to sketch the graphs of quadratic functions

Graphing a Quadratic function (and other functions) requires an understanding of *transformations*. Transformations are values which change the shape, direction, and position of the function. In a quadratic function,

$$\underbrace{f(x)=x^2}_{\text{parent}} \rightarrow \underbrace{f(x)=\underline{a}(x-\underline{h})^2+\underline{k}}_{\text{transformed}}$$

Vert - a is the vertical stretch which stretches out the y-values from the base function. Strech means multiply

$a=2$, $\times 2$
$a=0.4$, $\times 0.4$

Hor - h is the horizontal shift which "shifts" the x-values from the base function. shift means add.

$(x-2)^2$
$(x-(+2))^2$ $h=2$

$(x+2)^2$
$(x-(-2))^2$ $h=-2$

Vert - k is the vertical shift which "shifts" the y-values of the base function. shift means add

The process to graphing is straight-forward.

1. Identify the transformations
2. Create starting points from the base "parent" function
3. Transform the starting points
4. Graph the transformed points

In general: $f(x)=\underline{a}(x-\underline{h})^2+\underline{k}$

① vertical Stretch of "a"
② Horizontal shift of "+h"
③ vertical shift of "+k"

$f(x)=x^2$

x	y
−2	4
−1	1
0	0
1	1
2	4

x+h	ay+k
−2 +h	4a + k
−1 +h	1a + k
0 +h	0a + k
1 +h	1a + k
2 +h	4a + k

Example: $f(x) = -2(x+4)^2 + 6$ $(x-(-4))^2$

① Vertical Stretch $\times -2$

② Hor Shift left 4 (-4)

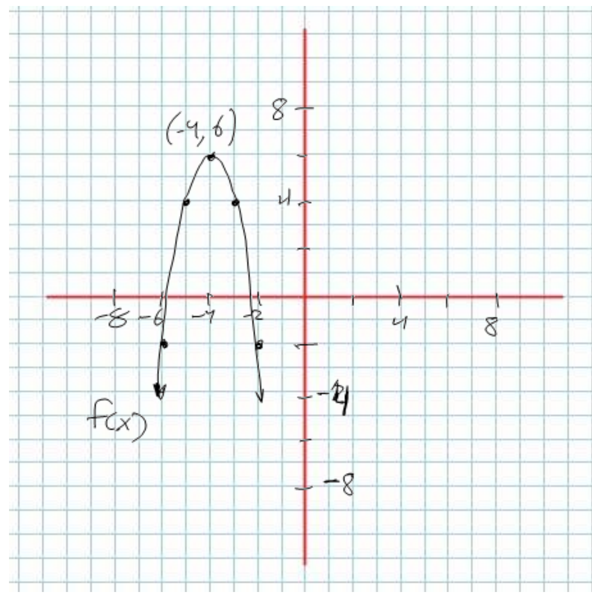
③ Vert Shift up 6 $(+6)$

x^2

x	y
-2	4
-1	1
0	0
1	1
2	4

x − 4	−2y + 6
−6	−2
−5	4
−4	6
−3	4
−2	−2

Transformed function



Example: $g(x) = \frac{1}{3}(x-2)^2 - 5$ $(+2)$

① Vert stretch of $\left(\frac{1}{3}\right)$

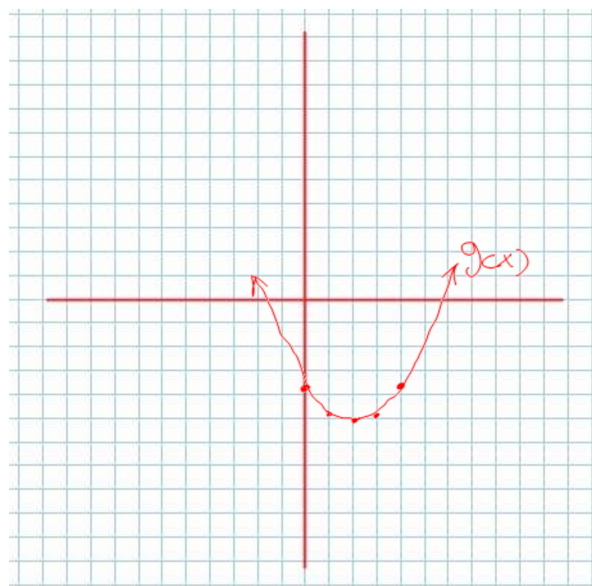
② Horizontal Shift Right 2 $(+2)$

③ Vert shift Down 5 (-5)

x^2

x	y
-2	4
-1	1
0	0
1	1
2	4

x + 2	$\frac{1}{3}(y) - 5$
0	−3.8
1	−4.7
2	−5
3	−4.7
4	−3.8



Success Criteria:

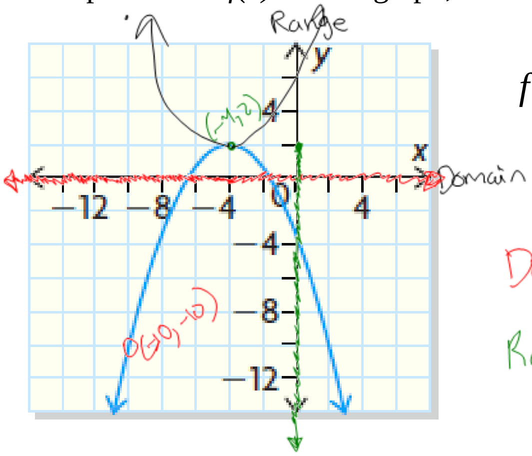
- I can identify the transformations *a, h, and k,* in the equation of a quadratic function
- I can develop a table of values for the base/parent function $f(x) = x^2$
- I can apply the transformations to the x and y values of the base/parent function in order to draw the transformed quadratic function

1.7 Domain and Range of Quadratic Functions

Learning Goal: We are learning to determine the domain and range of quadratic functions.

Now that we know about transformations, we can more easily derive the domain and range of Quadratic Functions.

Example: Given *f(x)* and its graph, state the domain and range.



$$f(x) = -\frac{1}{3}(x+4)^2 \boxed{+2}$$

Domain: $\{x \in \mathbb{R}\}$ ✓

Range: $\{f(x) \in \mathbb{R} \mid f(x) \le 2\}$

$f(x) = \frac{1}{3}(x+4)^2 + 2$

Range: $\{f(x) \in \mathbb{R} \mid x \ge 2\}$

In general, given $f(x) = a(x-h)^2 + k$, the domain is ***ALWAYS*** $\{x \in \mathbb{R}\}$

The range, however, depends on the vertical stretch, or "*a*":

(positive)
If $a > 0$, $\{f(x) \in \mathbb{R} \mid f(x) \ge k\}$

(negative)
If $a < 0$, $\{f(x) \in \mathbb{R} \mid f(x) \le k\}$

Determine the domain and range of each quadratic function:

$f(x) = \overset{a}{3}(x-4)^2 \overset{k}{-8}$

(4, -8)

Domain: $\{x \in \mathbb{R}\}$

Range: $\{f(x) \in \mathbb{R} \mid f(x) \ge -8\}$

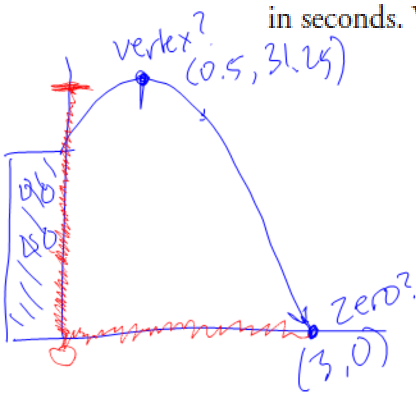
$g(x) = -23(x+365)^2 + 4303$

Domain: $\{x \in \mathbb{R}\}$

Range: $\{g(x) \in \mathbb{R} \mid g(x) \le 4303\}$

Sometimes the domain needs to be *restricted*. This means that instead of $\{x \in \mathbb{R}\}$, there will be some limitations to both the domain and the range.

Example: A baseball thrown from the top of a building falls to the ground below. The path of the ball is modelled by the function $h(t) = -5t^2 + 5t + 30$, where $h(t)$ is the height of the ball above ground, in metres, and t is the elapsed time in seconds. What are the domain and range of this function?



Domain: $\{t \in \mathbb{R} \mid 0 \le t \le 3\}$

Range: $\{h(t) \in \mathbb{R} \mid 0 \le h(t) \le 31.25\}$

Example: Find the domain and range on $f(x) = 3x^2 - 8x - 7$, where $x \ge 0$.

D: $\{x \in \mathbb{R} \mid x \ge 0\}$

R: $\{f(x) \in \mathbb{R} \mid f(x) \ge -12.333\}$

Did these by graphing on Desmos.com

Success Criteria:

- I can state the domain and range of a quadratic function using set notation
- I can apply restrictions to the domain and/or range to model real-life scenarios