



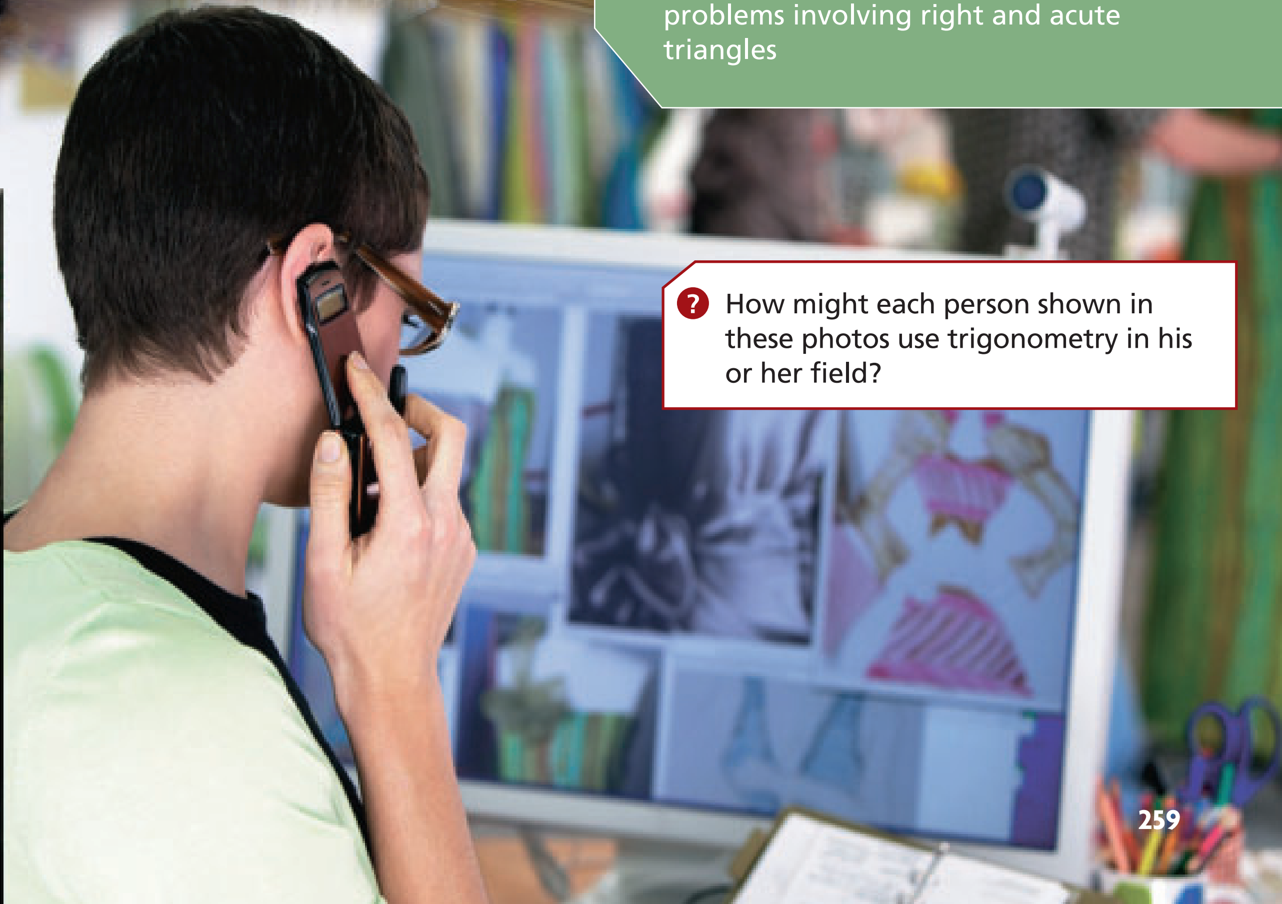
Chapter 5

Trigonometry and Acute Triangles

GOALS

You will be able to

- Use trigonometric ratios to determine angles and sides in right triangles
- Use technology to investigate and verify the sine and cosine laws
- Apply trigonometry to solve real-life problems involving right and acute triangles



? How might each person shown in these photos use trigonometry in his or her field?

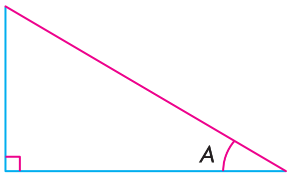
5 Getting Started

WORDS You Need to Know

1. Match the term with the picture or example that best illustrates its definition.

a) acute angle
b) right angle
c) Pythagorean theorem
d) **hypotenuse**
e) **opposite side**
f) **adjacent side**

i)



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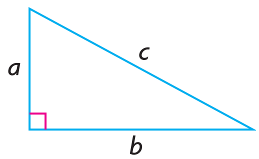
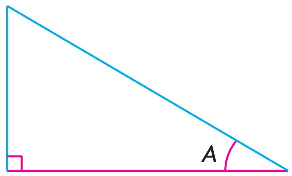


$a^2 + b^2 = c^2$

iii)



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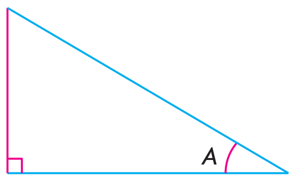

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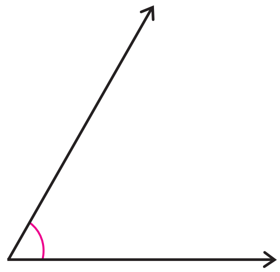
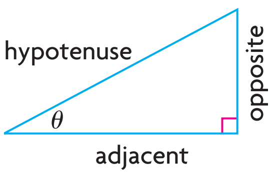


vi)

SKILLS AND CONCEPTS You Need



Determining and Using the Primary Trigonometric Ratios in Right Triangles

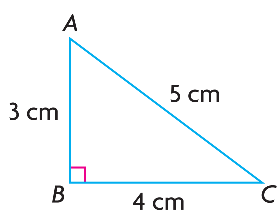
For a right angle triangle, the three primary trigonometric ratios are:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

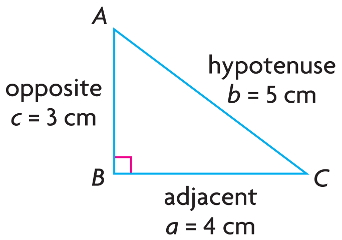
The opposite side, adjacent side, and hypotenuse are determined by their positions relative to a particular angle in the triangle.

EXAMPLE

State the primary trigonometric ratios for $\angle C$.



Solution



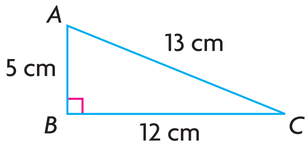
$$\sin C = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5}$$

$$\cos C = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}$$

$$\tan C = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{4}$$

2. State the primary trigonometric ratios for $\angle A$.

a)



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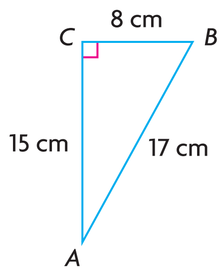
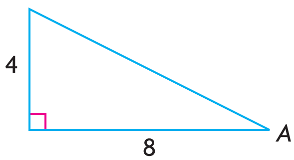


3. Calculate the measure of the indicated side or angle to the nearest unit.

a)



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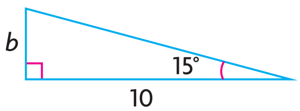
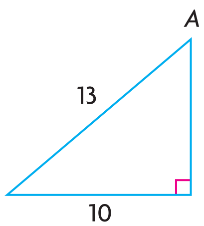
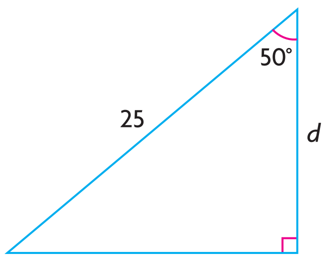


c)

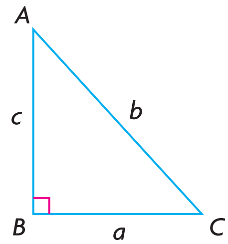


d)



Communication *Tip*

It is common practice to label the vertices of a triangle with upper case letters. The side opposite each angle is labelled with the corresponding lower case letter.



Study *Aid*

For help, see Essential Skills Appendix, A-15.

PRACTICE

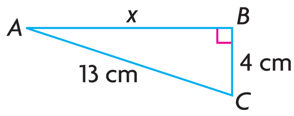
Study *Aid*

For help, see Essential Skills Appendix.

Question	Appendix
4	A-4
5, 6, 7, 10, 11	A-15

4. Use the Pythagorean theorem to determine the value of x to the nearest unit.

a)



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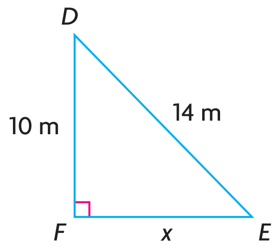


5. Using the triangles in question 4, determine the primary trigonometric ratios for each given angle. Then determine the angle measure to the nearest degree.

a) $\angle A$ **b)** $\angle D$ **c)** $\angle C$

Tech *Support*

For help setting a graphing calculator to degree mode and determining angles using the inverse trigonometric keys, see Technical Appendix, B-12.

6. Use a calculator to evaluate to four decimal places.

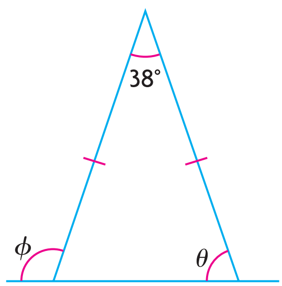
a) $\sin 50°$ **b)** $\cos 11°$ **c)** $\tan 72°$

7. Use a calculator to determine θ to the nearest degree.

a) $\cos \theta = 0.6820$ **b)** $\tan \theta = 0.1944$ **c)** $\sin \theta = 0.9848$

8. Determine each unknown angle to the nearest degree.

a)



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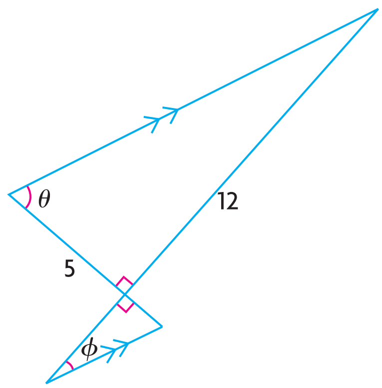
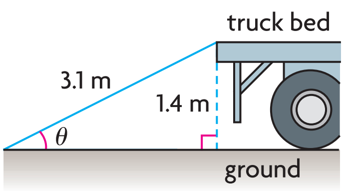




9. Rudy is unloading a piano from his truck. The truck bed is 1.4 m above ground, and he extended the ramp to a length of 3.1 m.

a) At what angle does the ramp meet the ground? Round your answer to the nearest degree.

b) Rudy needs the angle to be 15° or less so that he can control the piano safely. How much more should he extend the ramp? Round your answer to the nearest tenth of a metre.

10. Explain, using examples, how you could use trigonometry to calculate

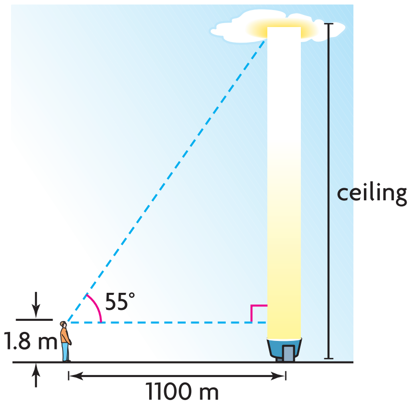
a) the measure of a side in a right triangle when you know one side and one angle

b) the measure of an angle when you know two sides

APPLYING What You Know

Landing at an Airport

For an airplane to land safely, the base of the clouds, or *ceiling*, above the airport must be at least 600 m. Grimsby Airport has a spotlight that shines perpendicular to the ground, onto the cloud above. At 1100 m from the spotlight, Cory measures the angle between the spotlight's vertical beam of light, himself, and the illuminated spot on the base of the cloud to be 55°. Cory's eyes are 1.8 m above ground.



? With this cloud ceiling, is it safe for a plane to land at Grimsby Airport?

A. Sketch the right triangle that models this problem. Represent the given information on your sketch and any other information you know.

B. Label the hypotenuse and the sides that are opposite and adjacent to the 55° angle.

C. Which trigonometric ratio (sine, cosine, or tangent) would you use to solve the problem? Justify your choice.

D. Use the trigonometric ratio that you chose in part C to write an equation to calculate the height of the cloud ceiling. Solve the equation and round your answer to the nearest metre.

E. Add 1.8 m to account for Cory's eyes being above ground. Determine whether it is safe for the plane to land.

5.1 Applying the Primary Trigonometric Ratios

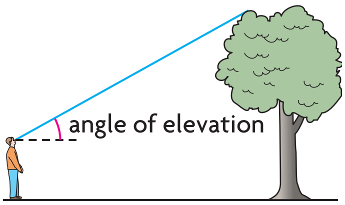
GOAL

Use primary trigonometric ratios to solve real-life problems.

LEARN ABOUT the Math

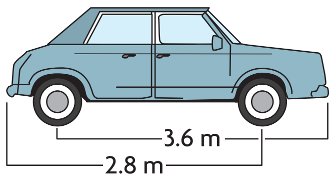
angle of elevation

the angle between the horizontal and the line of sight when looking up at an object



Eric's car alarm will sound if his car is disturbed, but it is designed to shut off if the car is being towed at an angle of elevation of more than 15°. Mike's tow truck can lift a bumper no more than 0.88 m higher than the bumper's original height above ground. Eric's car has these measurements:

- The front bumper is 3.6 m from the rear axle.
- The rear bumper is 2.8 m from the front axle.

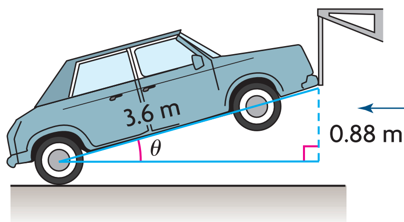


? Will Mike be able to tow Eric's car without the alarm sounding?

EXAMPLE 1 Selecting a strategy to solve a problem involving a right triangle

Determine whether the car alarm will sound.

Jason's Solution: Calculating the Angle of Elevation



If Eric's car is towed from the front, the car forms a right triangle with a hypotenuse of 3.6 m. The side opposite the angle of elevation, θ, is 0.88 m.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

In a right triangle, the sine ratio relates an angle to the opposite side and the hypotenuse.

$$\sin \theta = \frac{0.88}{3.6}$$

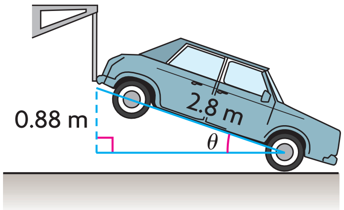
$$\theta = \sin^{-1}\left(\frac{0.88}{3.6}\right)$$

I used the inverse sine ratio to calculate the angle.

$$\theta \doteq 14°$$

I rounded to the nearest degree.

Since the angle is less than 15°, the car has not been lifted enough to shut off the alarm.



If the car is towed from the rear, the car still forms a right triangle. The opposite side is still 0.88, but the hypotenuse is now 2.8 m.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

To determine the angle of elevation, θ, I used the sine ratio, since I knew the lengths of the opposite side and the hypotenuse.

$$\sin \theta = \frac{0.88}{2.8}$$

$$\theta = \sin^{-1}\left(\frac{0.88}{2.8}\right)$$

I used the inverse sine ratio to calculate the angle.

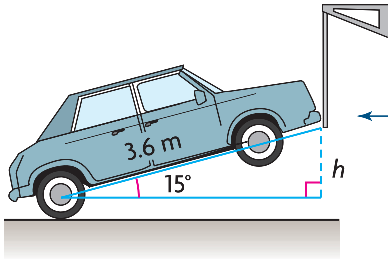
$$\theta \doteq 18°$$

I rounded to the nearest degree.

Since the angle is greater than 15°, the car alarm will shut off.

For the car alarm to shut off, Mike should tow Eric's car from the back.

Monica's Solution: Calculating the Minimum Height



For an angle of elevation of 15°, I wanted to know the height the car must be lifted to shut off the alarm. I started with the front of the car being lifted. I drew a right triangle and labelled the height as *h*. I knew an angle and the hypotenuse.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

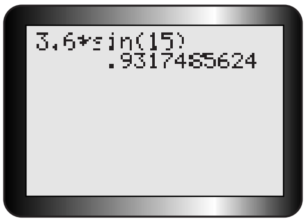
Side *h* is opposite the 15° angle, so I used the sine ratio to calculate *h*.

$$\sin 15° = \frac{h}{3.6}$$

$$3.6 \times \sin 15° = \cancel{3.6}^{1} \times \frac{h}{\cancel{3.6}_{1}}$$

I solved for *h* by multiplying both sides of the equation by 3.6.

$$h = 3.6 \times \sin 15°$$



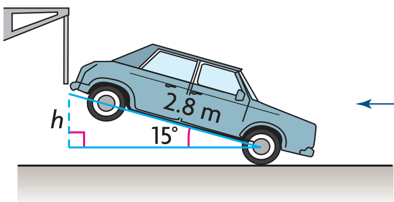
I used a calculator to evaluate and rounded to the nearest hundredth.

$$h \doteq 0.93 \text{ m}$$

When the front of the car is lifted 15°, the front bumper is about 0.93 m above its original height.

Mike can't raise the car high enough from the front to shut off the alarm.

This won't work because Mike can lift the bumper only 0.88 m.



I used the same method, but with the rear of the car being lifted.

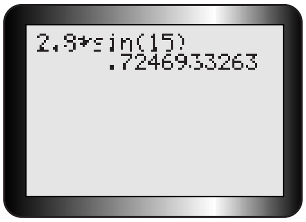
$$\sin 15° = \frac{h}{2.8}$$

I used the sine ratio because I knew the opposite side and the hypotenuse.

$$2.8 \times \sin 15° = \overset{1}{\cancel{2.8}} \times \frac{h}{\underset{1}{\cancel{2.8}}}$$

I solved for h by multiplying both sides of the equation by 2.8.

$$h = 2.8 \times \sin 15°$$



I used a calculator to evaluate and rounded to the nearest hundredth.

$$h \doteq 0.72 \text{ m}$$

When the rear of the car is lifted 15°, the rear bumper is about 0.72 m above its original height.

For the alarm to shut off, Mike should tow Eric's car from the rear.

This will work, since Mike can lift a car more than that.

Reflecting

A. Compare the two solutions. How are they the same and how are they different?

B. Which solution do you prefer? Why?

C. Could the cosine or tangent ratios be used instead of the sine ratio to solve this problem? Explain.

APPLY the Math

EXAMPLE 2 Selecting the appropriate trigonometric ratios to determine unknown sides

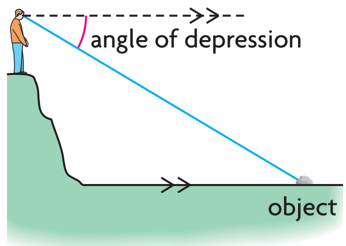
A hot-air balloon on the end of a taut 95 m rope rises from its platform. Sam, who is in the basket, estimates that the angle of depression to the rope is about 55°.

a) How far, to the nearest metre, did the balloon drift horizontally?

b) How high, to the nearest metre, is the balloon above ground?

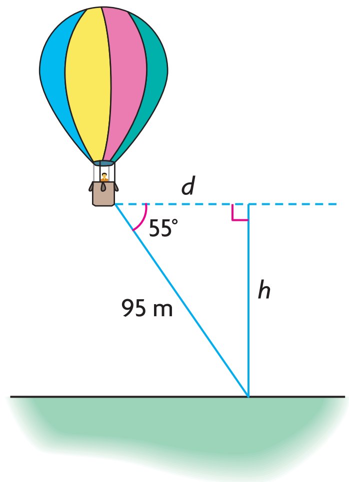
c) Viewed from the platform, what is the angle of elevation, to nearest degree, to the balloon?

angle of depression
the angle between the horizontal and the line of sight when looking down at an object



Kumar's Solution

a)



I drew a sketch and labelled the angle of depression as 55°. I labelled the horizontal distance the balloon drifted as *d* and the height of the balloon as *h*. Relative to the 55° angle, *d* is the adjacent side and *h* is the opposite side.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

I used the cosine ratio to calculate *d* because in a right triangle, cosine relates an angle to the adjacent side and the hypotenuse.

$$\cos 55° = \frac{d}{95}$$

$$95 \times \cos 55° = \cancel{95}^{1} \times \frac{d}{\cancel{95}_{1}}$$

I solved for *d* by multiplying both sides of the equation by 95.

$$d = 95 \times \cos 55°$$

I used a calculator to evaluate.

$$d \doteq 54 \text{ m}$$

The balloon drifted about 54 m horizontally.

b)

$$\sin 55° = \frac{h}{95}$$

I used the sine ratio to calculate *h* because in a right triangle, sine relates an angle to the opposite side and the hypotenuse.

$$95 \times \sin 55° = \cancel{95}^{1} \times \frac{h}{\cancel{95}_{1}}$$

I solved for *h* by multiplying both sides of the equation by 95.

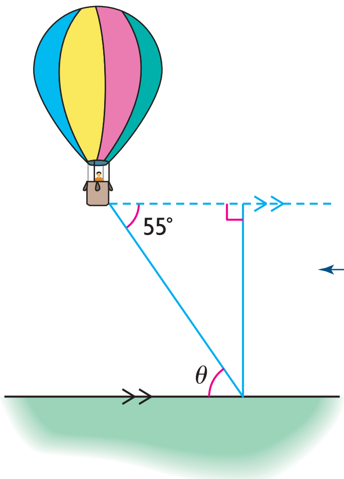
$$h = 95 \times \sin 55°$$

I used a calculator to evaluate.

$h \doteq 78$ m

The balloon is about 78 m above ground.

c)



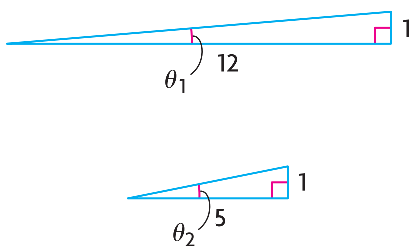
I labelled the angle of elevation to the balloon as θ. The angle of depression $(55°)$ and θ are alternate angles between parallel lines. So both angles are equal.

The angle of elevation to the balloon from the platform is $55°$.

EXAMPLE 3 Solving a problem by using a trigonometric ratio

A wheelchair ramp is safe to use if it has a minimum slope of $\frac{1}{12}$ and a maximum slope of $\frac{1}{5}$. What are the minimum and maximum angles of elevation to the top of such a ramp? Round your answers to the nearest degree.

Nazir's Solution



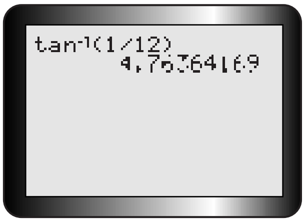
I drew a sketch of each ramp and labelled the angles of elevation as θ_1 and θ_2. Slope is rise divided by run, so, in both triangles, the rise is the vertical side and the run is the horizontal side. Relative to θ_1 and θ_2, these sides are opposite and adjacent.

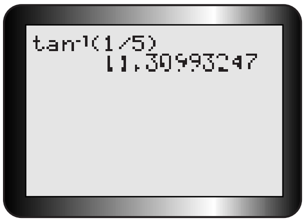
$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

The tangent ratio relates an angle in a right triangle to the opposite side and the adjacent side.

$$\tan \theta_1 = \frac{1}{12} \qquad \tan \theta_2 = \frac{1}{5}$$

$$\theta_1 = \tan^{-1}\left(\frac{1}{12}\right) \qquad \theta_2 = \tan^{-1}\left(\frac{1}{5}\right)$$





I used the inverse tangent function on my calculator to calculate each angle.

$$\theta_1 \doteq 5^\circ \qquad \theta_2 \doteq 11^\circ$$

The angle of elevation to the top of the wheelchair ramp must be between 5° and 11° to be safe.

In Summary

Key Idea

- The primary trigonometric ratios can be used to determine side lengths and angles in right triangles. Which ratio you use depends on what you want to calculate and what you know about the triangle.

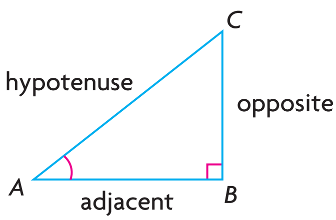
Need to Know

- For any right $\triangle ABC$, the primary trigonometric ratios for $\angle A$ are:

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$



- Opposite and adjacent sides are named relative to their positions to a particular angle in a triangle.
- The hypotenuse is the longest side in a right triangle and is always opposite the 90° angle.

CHECK Your Understanding

1. Use a calculator to evaluate to four decimal places.

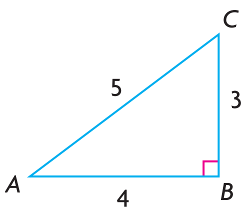
a) $\sin 15°$ **b)** $\cos 55°$

2. Use a calculator to determine the angle to the nearest degree.

a) $\tan^{-1}\left(\frac{2}{5}\right)$ **b)** $\sin^{-1}(0.7071)$

3. State all the primary trigonometric ratios for $\angle A$ and $\angle D$. Then determine $\angle A$ and $\angle D$ to the nearest degree.

a)



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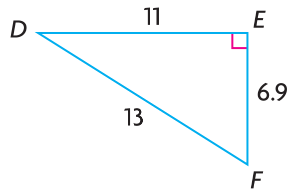
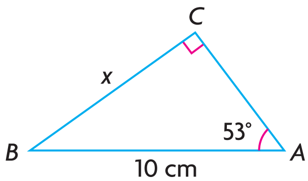


4. For each triangle, calculate x to the nearest centimetre.

a)



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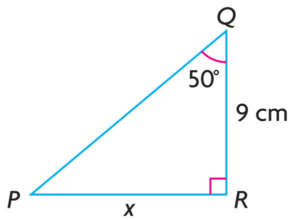
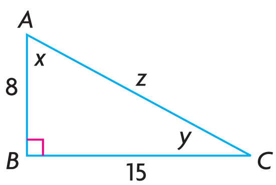


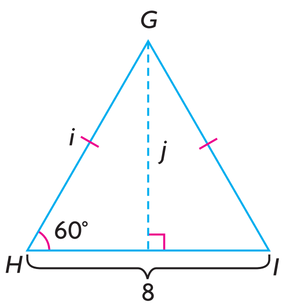
PRACTISING

5. Determine all unknown sides to the nearest unit and all unknown interior angles to the nearest degree.
K

a)



b)



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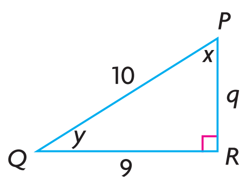

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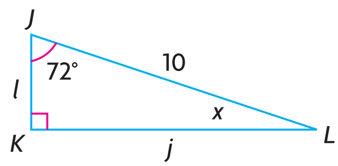
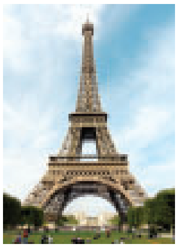
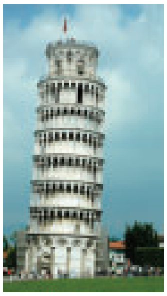
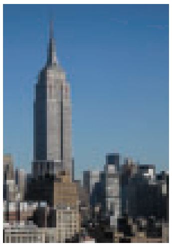




Eiffel Tower



Leaning Tower of Pisa



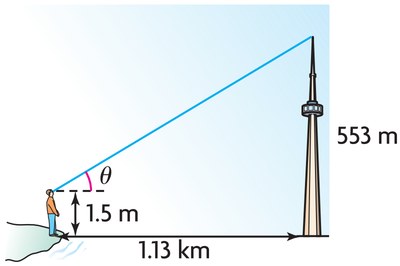
Empire State Building

Big Ben's clock tower

6. Predict the order, from tallest to shortest, of these famous landmarks. Then use the given information to determine the actual heights to the nearest metre.
   a) Eiffel Tower, Paris, France
      68 m from the base, the angle of elevation to the top is 78°.
   b) Empire State Building, New York, New York
      267 m from the base, the angle of elevation to the top is 55°.
   c) Leaning Tower of Pisa, Pisa, Italy
      The distance from a point on the ground to the tallest tip of the tower is 81 m with an angle of elevation of 44°.
   d) Big Ben's clock tower, London, England
      81 m from the base, the angle of elevation is 50°.

7. Manpreet is standing 8.1 m from a flagpole. His eyes are 1.7 m above ground. The top of the flagpole has an angle of elevation of 35°. How tall, to the nearest tenth of a metre, is the flagpole?

8. **A** The CN Tower is 553 m tall. From a position on one of the Toronto Islands 1.13 km away from the base of the tower, determine the angle of elevation, to the nearest degree, to the top of the tower. Assume that your eyes are 1.5 m above ground.



9. Devin wants to estimate the slope of the road near his apartment building. He uses a level that is 1.2 m long and holds it horizontally with one end touching the ground and the other end 39 cm above ground.
   a) What is the slope of the road, to the nearest tenth, at this point?
   b) What angle, to the nearest degree, would represent the slant of the road?

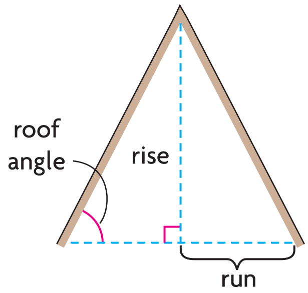
10. A 200 m cable attached to the top of an antenna makes an angle of 37° with the ground. How tall is the antenna to the nearest metre?

11. An underground parking lot is being constructed 8.00 m below ground level.
    a) If the exit ramp is to rise at an angle of 15°, how long will the ramp be? Round your answer to the nearest hundredth of a metre.
    b) What horizontal distance, to the nearest hundredth of a metre, is needed for the ramp?

12. The pitch of a roof is the rise divided by the run. If the pitch is greater than 1.0 but less than 1.6, roofers use planks fastened to the roof to stand on when shingling it. If the pitch is greater than 1.6, scaffolding is needed. For each roof angle, what equipment (scaffolding or planks), if any, would roofers need?

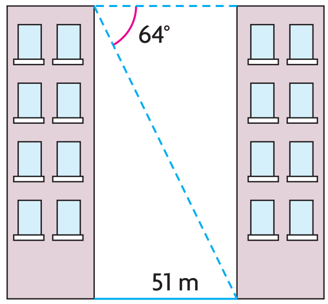
T

a) $\theta = 34°$ **b)** $\theta = 60°$ **c)** $\theta = 51°$



13. To estimate the width of a river near their school, Charlotte and Mavis have a device to measure angles and a pole of known length. Describe how they might calculate the width of the river with this equipment.

14. Ainsley's and Caleb's apartment buildings are exactly the same height. Ainsley measures the distance between the buildings as 51 m and observes that the angle of depression from the roof of her building to the bottom of Caleb's is about 64°. How tall, to the nearest metre, is each building?



15. To use an extension ladder safely, the base must be 1 m out from the wall for every 2 m of vertical height.

a) What is the maximum angle of elevation, to the nearest degree, to the top of the ladder?

b) If the ladder is extended to 4.72 m in length, how high can it safely reach? Round your answer to the nearest hundredth of a metre.

c) How far out from the wall does a 5.9 m ladder need to be? Round your answer to the nearest tenth of a metre.

16. Martin is installing an array of solar panels 3.8 m high in his backyard. The array needs to be tilted 60° from the ground. Municipal bylaws restrict residents from having any secondary structures taller than 3.0 m. Will Martin be able to build his array? Include calculations in your explanation.

C

Extending

17. When poured into a pile, gravel will naturally form a cylindrical cone with a slope of approximately 34°. If a construction foreman has room only for a pile that is 85 m in diameter, how tall, to the nearest metre, will the pile be?

18. Nalini and Jodi are looking at the top of the same flagpole. They are standing in a line on the same side of the flagpole, 50.0 m apart. The angle of elevation to the top of the pole is 11° from Jodi's position and 7° from Nalini's position. The girls' eyes are 1.7 m above ground. For each question, round your answer to the nearest tenth of a metre.

a) How tall is the flagpole?

b) How far is each person from the base of the flagpole?

c) If Nalini and Jodi were standing in a line on opposite sides of the pole, how tall would the flagpole be? How far would each person be from its base?

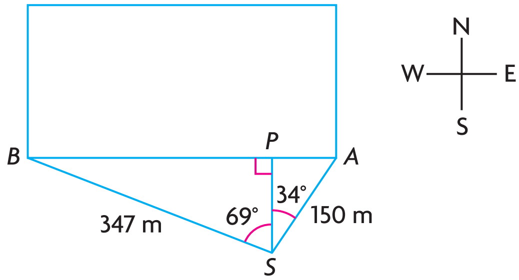
5.2 Solving Problems by Using Right-Triangle Models

GOAL

Solve real-life problems by using combinations of primary trigonometric ratios.

LEARN about the Math

A surveyor stands at point S and uses a laser transit and marking pole to sight two corners, A and B, on one side of a rectangular lot. The surveyor marks a reference point P on the line AB so that the line from P to S is perpendicular to AB. Corner A is 150 m away from S and 34° east of P. Corner B is 347 m away from S and 69° west of P.



Communication *Tip*

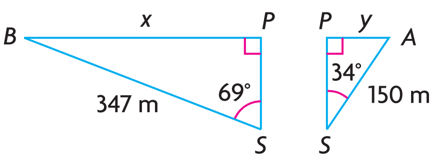
For the highest degree of accuracy, save intermediate answers by using the memory keys of your calculator. Round only after you do the very last calculation.

? How long, to the nearest metre, is the lot?

EXAMPLE 1 Selecting a strategy to determine a distance

Calculate the length of the building lot to the nearest metre.

Leila's Solution: Using the Sine Ratio



The length of the lot is $BP + PA$. I sketched the two right triangles that these sides formed. I labelled BP as x and PA as y.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

In each triangle, I knew an angle and the hypotenuse. I needed to calculate the sides (x and y) opposite each given angle. So I used the sine ratio.

$$\sin 69° = \frac{x}{347} \qquad \sin 34° = \frac{y}{150}$$

$$347 \times \sin 69° = \overset{1}{\cancel{347}} \times \frac{x}{\underset{1}{\cancel{347}}} \qquad 150 \times \sin 34° = \overset{1}{\cancel{150}} \times \frac{y}{\underset{1}{\cancel{150}}}$$

To solve for x, I multiplied both sides of the first equation by 347.
To solve for y, I multiplied both sides of the second equation by 150.

$$x = 347 \times \sin 69° \qquad y = 150 \times \sin 34°$$

$$x \doteq 323.952\,408 \qquad y \doteq 83.878\,935\,52$$

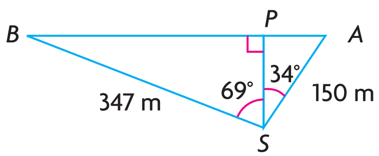
I used a calculator to evaluate.

$$AB = BP + PA$$
$$= x + y$$
$$= 323.952\,408 + 83.878\,935\,52$$
$$\doteq 408 \text{ m}$$

I added x and y to determine the length of the lot.

The lot is about 408 m long.

Tony's Solution: Using the Cosine Ratio and the Pythagorean Theorem



The length of the lot is $BP + PA$. PS is a shared side of $\triangle BPS$ and $\triangle APS$. If I knew PS, then I could use the Pythagorean theorem to determine BP and PA.

In $\triangle APS$:

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos 34° = \frac{PS}{150}$$

AS is the hypotenuse and PS is adjacent to the 34° angle. So, to determine PS, I used the cosine ratio.

$$150 \times \cos 34° = \overset{1}{\cancel{150}} \times \frac{PS}{\underset{1}{\cancel{150}}}$$
$$PS = 150 \times \cos 34°$$
$$PS \doteq 124.356 \text{ m}$$

To solve for PS, I multiplied both sides of the equation by 150 and used a calculator to evaluate.

In $\triangle APS$:

$$AS^2 = PS^2 + PA^2$$
$$150^2 = 124.356^2 + PA^2$$
$$PA^2 = 150^2 - 124.356^2$$
$$PA^2 = 7036$$
$$PA = \sqrt{7036}$$
$$PA \doteq 83.880\,867\,9 \text{ m}$$

In $\triangle BPS$:

$$BS^2 = PS^2 + BP^2$$
$$347^2 = 124.356^2 + BP^2$$
$$BP^2 = 347^2 - 124.356^2$$
$$BP^2 = 104\,945$$
$$BP = \sqrt{104\,945}$$
$$BP \doteq 323.952\,157 \text{ m}$$

Next, I used the Pythagorean theorem to determine PA and BP .

$$AB = BP + PA$$
$$= 83.880\,867\,9 + 323.952\,157$$
$$\doteq 408 \text{ m}$$

I added BP and PA to determine the length of the lot.

The lot is about 408 m long.

Reflecting

A. Explain why the length of the lot cannot be determined directly using either the Pythagorean theorem or the primary trigonometric ratios.

B. How are Leila's and Tony's solutions the same? How are they different?

C. Why did the surveyor choose point P so that PS would be perpendicular to AB?

APPLY the Math

EXAMPLE 2 Using trigonometric ratios to calculate angles

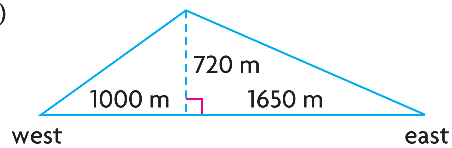
Karen is a photographer taking pictures of the Burlington Skyway bridge. She is in a helicopter hovering 720 m above the bridge, exactly 1 km horizontally from the west end of the bridge. The Skyway spans a distance of 2650 m from east to west.

a) From Karen's position, what are the angles of depression, to the nearest degree, of the east and west ends of the bridge?

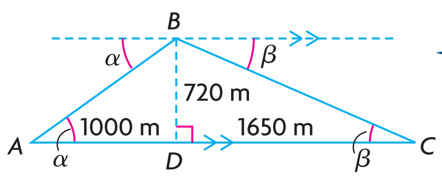
b) If Karen's camera has a wide-angle lens that can capture 150°, can she get the whole bridge in one shot?

Ali's Solution

a)



I sketched the position of the helicopter relative to the bridge. The horizontal distance from the helicopter to the west end of the bridge is 1000 m. So I subtracted 1000 from 2650 to determine the horizontal distance from the helicopter to the east end.



I drew a dashed line parallel to the base of the triangle and labelled the angles of depression as α and β. In $\triangle ABD$, I knew that $\angle BAD = \alpha$ because $\angle BAD$ and α are alternate angles between parallel lines. By the same reasoning, $\angle BCD = \beta$.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

I knew the opposite and adjacent sides in each right triangle. So, to determine α and β, I used the tangent ratio.

$$\tan \alpha = \frac{720}{1000} \qquad \tan \beta = \frac{720}{1650}$$

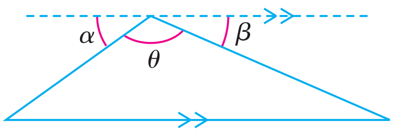
$$\alpha = \tan^{-1}\left(\frac{720}{1000}\right) \qquad \beta = \tan^{-1}\left(\frac{720}{1650}\right)$$

$$\alpha \doteq 35.8° \qquad \beta \doteq 23.6°$$

I used the inverse tangent function on my calculator to evaluate each angle.

The angles of depression are about 36° (west end) and about 24° (east end).

b)



To determine whether Karen can get the whole bridge in one shot, I needed to know θ, the angle between the east and west ends of the bridge from Karen's perspective.

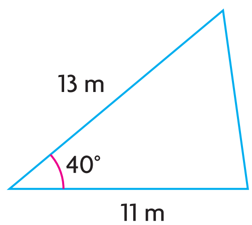
$$\begin{aligned}\theta &= 180° - \alpha - \beta \\ &= 180° - 35.8° - 23.6° \\ &= 120.6°\end{aligned}$$

I subtracted α and β from 180°.

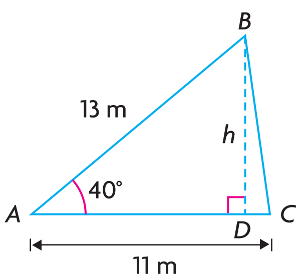
Since the angle from Karen's perspective is less than 150°, she will be able to get the whole bridge in one shot.

EXAMPLE 3 Using trigonometry to determine the area of a triangle

Elise's parents have a house with a triangular front lawn as shown. They want to cover the lawn with sod rather than plant grass seed. How much would it cost to put sod if it costs $13.75 per square metre?



Tina's Solution



I drew a dashed line for the height of the triangle and labelled it as h. The formula for the area of a triangle is $A = \frac{1}{2} b \times h$.

I knew the base, but I needed to calculate h before calculating the area.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

In right $\triangle ABD$, I knew the hypotenuse and the side adjacent to the 40° angle. The height is the side opposite the 40° angle, so I used the sine ratio.

$$\sin 40° = \frac{h}{13}$$

$$13 \times \sin 40° = \cancel{13}^{1} \times \frac{h}{\cancel{13}_{1}}$$

To solve for h, I multiplied both sides of the equation by 13 and used a calculator to evaluate.

$$h = 13 \times \sin 40°$$

$$h \doteq 8.356 \text{ m}$$

$$A = \frac{1}{2} b \times h$$

I substituted the base and height into the formula for the area of a triangle.

$$= \frac{1}{2}(11 \times 8.35)$$

$$\doteq 45.958 \text{ m}^2$$

$$\text{cost} = 45.958 \times 13.75$$

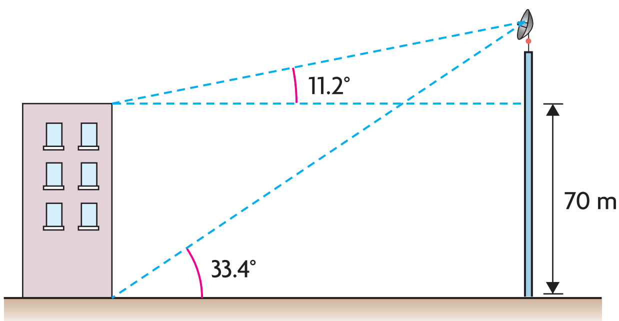
$$\doteq \$631.92$$

I multiplied the area by the cost per square metre to determine the cost to sod the lawn. I rounded to the nearest cent.

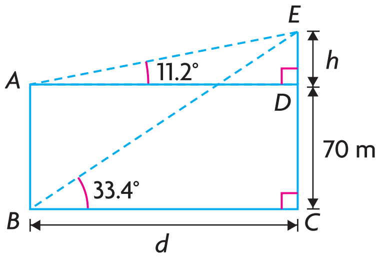
Elise's parents would have to spend \$631.92 to sod their lawn.

EXAMPLE 4 Solving a problem by using trigonometric ratios

A communications tower is some distance from the base of a 70 m high building. From the roof of the building, the angle of elevation to the top of the tower is 11.2°. From the base of the building, the angle of elevation to the top of the tower is 33.4°. Determine the height of the tower and how far it is from the base of the building. Round your answers to the nearest metre.



Pedro's Solution



I drew a sketch and labelled the distance between the tower and the building as *d*. The tower is more than 70 m tall, so I labelled the extra height above 70 m as *h*.

In $\triangle BEC$: In $\triangle AED$:

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 33.4^\circ = \frac{h + 70}{d} \qquad \tan 11.2^\circ = \frac{h}{d}$$

In $\triangle BEC$, *d* is adjacent to the 33.4° angle and the opposite side is $70 + h$. In $\triangle AED$, *d* is adjacent to the 11.2° angle and the opposite side is *h*. Since I knew the adjacent and opposite sides in each right triangle, I used the tangent ratio.

$$d \times \tan 33.4^\circ = \overset{1}{\cancel{d}} \times \frac{h + 70}{\underset{1}{\cancel{d}}} \qquad d \times \tan 11.2^\circ = \overset{1}{\cancel{d}} \times \frac{h}{\underset{1}{\cancel{d}}}$$

$$d \times \tan 33.4^\circ = h + 70 \qquad d \times \tan 11.2^\circ = h$$

$$d = \frac{h + 70}{\tan 33.4^\circ}$$

To solve for *d* in the first equation, I multiplied both sides by *d* and then divided both sides by tan 33.4°. To solve for *h* in the second equation, I multiplied both sides by *d*.

$$d = \frac{d \times \tan 11.2^\circ + 70}{\tan 33.4^\circ}$$

$$\tan 33.4^\circ \times d = d \times \tan 11.2^\circ + 70$$

$$d(\tan 33.4^\circ - \tan 11.2^\circ) = 70$$

$$0.461\ 373\ 177\ 9d \doteq 70$$

I substituted the second equation into the first and solved for *d*. I multiplied both sides of the equation by tan 33.4°. Then I simplified.

$$d = \frac{70}{0.461\ 373\ 177\ 9}$$

$$\doteq 151.721\ 000\ 2 \text{ m}$$

$$h = 151.721\ 000\ 2 \times \tan 11.2^\circ$$

$$\doteq 30 \text{ m}$$

To determine *h*, I substituted the value of *d* into the second equation.

$$70 + 30 = 100 \text{ m}$$

To determine the height of the tower, I added 70 to *h*.

The tower is about 100 m tall and about 152 m from the base of the building.

In Summary

Key Idea

- If a situation involves calculating a length or an angle, try to represent the problem with a right-triangle model. If you can, solve the problem by using the primary trigonometric ratios.

Need to Know

- To calculate the area of a triangle, you can use the sine ratio to determine the height. For example, if you know a, b, and $\angle C$, then

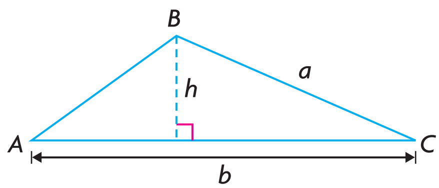
$$\sin C = \frac{h}{a}$$

$$h = a \sin C$$

The area of a triangle is

$$A = \frac{1}{2} b \times h$$

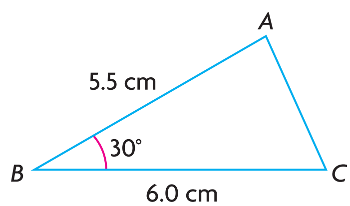
$$A = \frac{1}{2} b(a \sin C)$$



CHECK Your Understanding

1. Calculate the area of each triangle to the nearest tenth of a square centimetre.

a)



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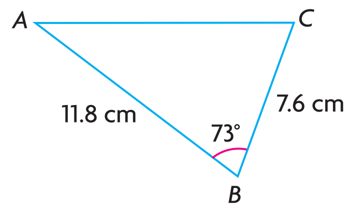
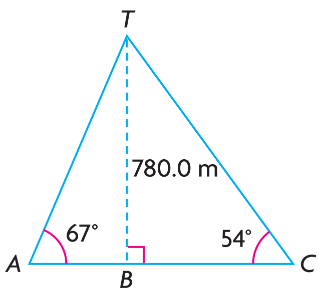
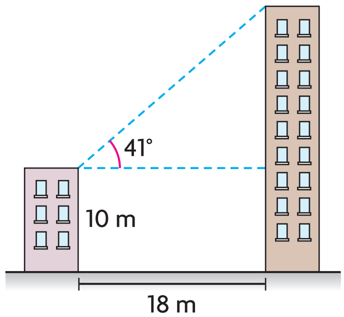




2. A mountain is 780.0 m high. From points A and C, the angles of elevation to the top of the mountain are 67° and 54° as shown at the left. Explain how to calculate the length of a tunnel from A to C.

3. Karen and Anna are standing 23 m away from the base of a 23 m high house. Karen's eyes are 1.5 m above ground and Anna's eyes are 1.8 m above ground. Both girls observe the top of the house and measure its angle of elevation. Which girl will measure the greater angle of elevation? Justify your answer.

PRACTISING

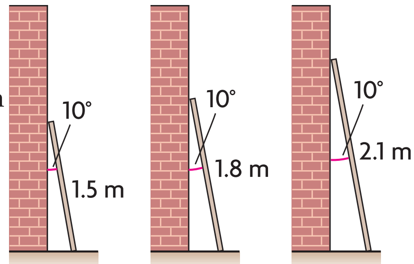


4. The angle of elevation from the roof of a 10 m high building to the top of another building is $41°$. The two buildings are 18 m apart at the base.
K

a) Which trigonometric ratio would you use to solve for the height of the taller building? Why?

b) How tall, to the nearest metre, is the taller building?

5. If the angle of elevation to the top of the pyramid of Cheops in Giza, Egypt, is $17.5°$, measured 348 m from its base, can you calculate the height of the pyramid accurately? Explain your reasoning.



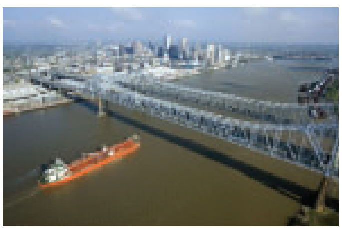
6. Darrin wants to lean planks of wood that are 1.5 m, 1.8 m, and 2.1 m long against the wall inside his garage. If the top of a plank forms an angle of less than $10°$ with the wall, the plank might fall over. If the bottom of a plank sticks out more than 30 cm from the wall, Darrin won't have room to park his car. Will Darrin be able to store all three planks in the garage? Justify your answer with calculations.

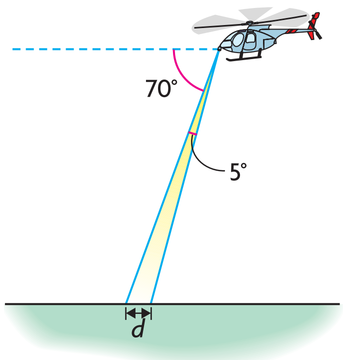
7. Jordan is standing on a bridge over the Welland Canal. His eyes are 5.1 m above the surface of the water. He sees a cargo ship heading straight toward him. From his position, the bow appears at an angle of depression of $5°$ and the stern appears at an angle of depression of $1°$. For each question, round your answer to the nearest tenth of a metre.

a) What is the straight-line distance from the bow to Jordan?

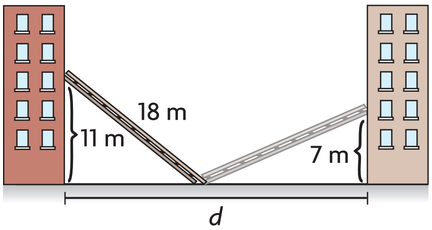
b) What is the straight-line distance from the stern to Jordan?

c) What is the length of the ship from bow to stern?



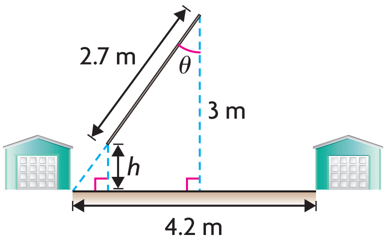


8. A searchlight is mounted at the front of a helicopter flying 125 m above ground. The angle of depression of the light beam is $70°$. An observer on the ground notices that the beam of light measures $5°$. How wide, to the nearest metre, is d, the spot on the ground?
T



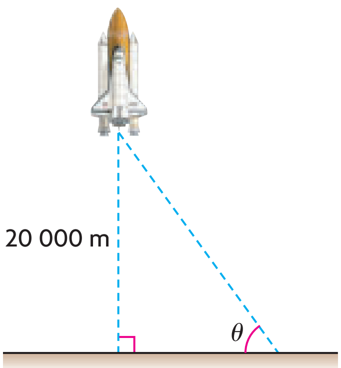
9. An 18 m long ladder is leaning against the wall of a building. The top of the ladder reaches a window 11 m above ground. If the ladder is tilted in the opposite direction, without moving its base, the top of the ladder can reach a window in another building that is 7 m above ground. How far apart, to the nearest metre, are the two buildings?

10. Kyle and Anand are standing on level ground on opposite sides of a tree. Kyle measures the angle of elevation to the treetop as 35°. Anand measures an angle of elevation of 30°. Kyle and Anand are 65 m apart. Kyle's eyes and Anand's eyes are 1.6 m above ground. How tall, to the nearest tenth of a metre, is the tree?

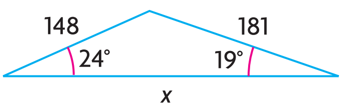


11. A tree branch 3 m above ground runs parallel to Noel's garage and his neighbour's. Noel wants to attach a rope swing on this branch. The garages are 4.2 m apart and the rope is 2.7 m long.
    a) What is the maximum angle, measured from the perpendicular, through which the rope can swing? Round your answer to the nearest degree.
    b) What is the maximum height above ground, to the nearest tenth of a metre, of the end of the rope?

12. A regular hexagon has a perimeter of 50 cm.
    a) Calculate the area of the hexagon to the nearest square centimetre.
    b) The hexagon is the base of a prism of height 100 cm. Calculate the volume, to the nearest cubic centimetre, and the surface area, to the nearest square centimetre, of the prism. Recall that volume = area of base × height. The surface area is the sum of the areas of all faces of the prism.



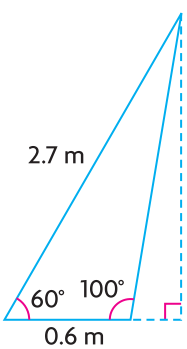
13. **A** Lucien wants to photograph the separation of the solid rocket boosters from the space shuttle. He is standing 14 500 m from the launch pad, and the solid rocket boosters separate at 20 000 m above ground.
    a) Assume that the path of the shuttle launch is perfectly vertical up until the boosters separate. At what angle, to the nearest degree, should Lucien aim his camera?
    b) How far away is the space shuttle from Lucien at the moment the boosters separate? Round your answer to the nearest metre.



14. **C** Sven wants to determine the unknown side of the triangle shown at the left. Even though it is not a right triangle, describe how Sven could use primary trigonometric ratios to determine *x*.

15. A pendulum that is 50 cm long is moved 40° from the vertical. What is the change in height from its initial position? Round your answer to the nearest centimetre.

Extending



16. Calculate the area of the triangle shown at the left to the nearest tenth of a square metre.

5.3 Investigating and Applying the Sine Law in Acute Triangles

GOAL

Verify the sine law and use it to solve real-life problems.

YOU WILL NEED

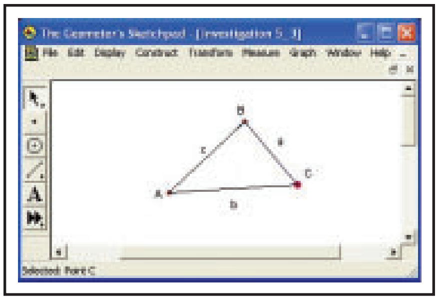
- dynamic geometry software

INVESTIGATE the Math

The trigonometric ratios sine, cosine, and tangent are defined only for right triangles. In an oblique triangle, these ratios no longer apply.

oblique triangle
a triangle (acute or obtuse) that does not contain a right angle

? In an oblique triangle, what is the relationship between a side and the sine of the angle opposite that side?



A. Use dynamic geometry software to construct any acute triangle.

B. Label the vertices A, B, and C. Then name the sides a, b, and c as shown.

C. Measure all three interior angles and all three sides.

D. Using one side length and the angle opposite that side, choose **Calculate ...** from the **Measure** menu to evaluate the ratio $\frac{\text{side length}}{\sin(\text{opposite angle})}$.

E. Repeat part D for the other sides and angles. What do you notice?

F. Drag any vertex of your triangle. What happens to the sides, angles, and ratios? Now drag the other two vertices and explain.

G. Express your findings
- in words
- with a mathematical relationship

Tech *Support*

For help using dynamic geometry software, see Technical Appendix, B-21 and B-22.

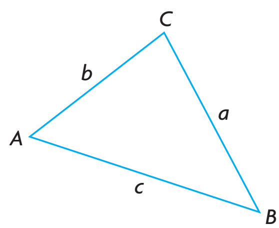
Reflecting

H. Are the reciprocals of the ratios you found equal? Explain how you know.

I. The relationship that you verified is the sine law. Why is this name appropriate?

J. Does the sine law apply to all types of triangles (obtuse, acute, and right)? Explain how you know.

sine law
in any acute triangle, the ratios of each side to the sine of its opposite angle are equal



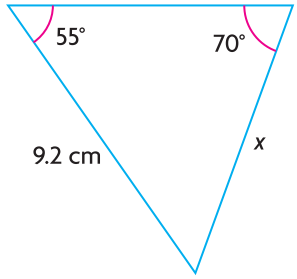
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \text{ or}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

APPLY the Math

EXAMPLE 1 Using the sine law to calculate an unknown length

Determine x to the nearest tenth of a centimetre.



Barbara's Solution

$$\frac{x}{\sin 55°} = \frac{9.2}{\sin 70°}$$

The triangle doesn't have a 90° angle. So it isn't convenient to use the primary trigonometric ratios.

I chose the sine law because I knew two angles and the side opposite one of those angles.

I wrote the ratios with the sides in the numerators to make the calculations easier.

$$\overset{1}{\cancel{\sin 55°}} \times \frac{x}{\underset{1}{\cancel{\sin 55°}}} = \sin 55° \times \frac{9.2}{\sin 70°}$$

To solve for x, I multiplied both sides of the equation by sin 55°.

$$x = \sin 55° \times \frac{9.2}{\sin 70°}$$

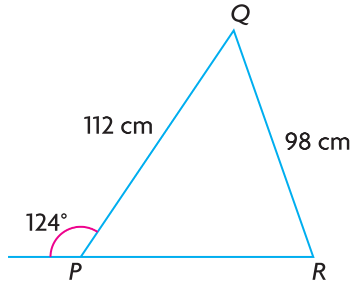
I used a calculator to evaluate.

$$x \doteq 8.0 \text{ cm}$$

The unknown side of the triangle is about 8.0 cm long.

EXAMPLE 2 Selecting the sine law as a strategy to calculate unknown angles

Determine all the interior angles in $\triangle PQR$.
Round your answers to the nearest degree.



Tom's Solution

$$\angle QPR = 180° - 124°$$
$$= 56°$$

$\angle QPR$ and 124° add up to 180°. So, to determine $\angle QPR$, I subtracted 124° from 180°.

$$\frac{\sin R}{112} = \frac{\sin 56°}{98}$$

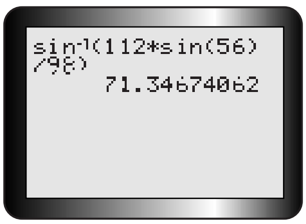
$\triangle PQR$ doesn't have a 90° angle. So I chose the sine law, since I knew two sides and an angle opposite one of those sides.

I wrote the ratios with the angles in the numerators to make it easier to solve for $\angle R$.

$$\overset{1}{\cancel{112}} \times \frac{\sin R}{\underset{1}{\cancel{112}}} = 112 \times \frac{\sin 56°}{98}$$

To solve for $\angle R$, I multipled both sides of the equation by 112.

$$\sin R = 112 \times \frac{\sin 56°}{98}$$



I used the inverse sine function on a calculator to evaluate.

$$\angle R \doteq 71°$$
$$\angle Q = 180° - \angle QPR - \angle R$$
$$= 180° - 56° - 71°$$
$$= 53°$$

All three interior angles add up to 180°. So I subtracted $\angle R$ and $\angle QPR$ from 180° to determine $\angle Q$.

The interior angles in $\triangle PQR$ are 56°, 53°, and 71°.

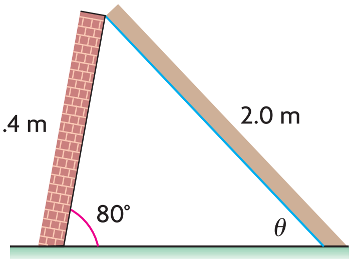
EXAMPLE 3 Solving a problem by using the sine law

A wall that is 1.4 m long has started to lean and now makes an angle of 80° with the ground. A 2.0 m board is jammed between the top of the wall and the ground to prop the wall up. Assume that the ground is level.

a) What angle, to the nearest degree, does the board make with the ground?

b) What angle, to the nearest degree, does the board make with the wall?

c) How far, to the nearest tenth of a metre, is the board from the base of the wall?



Isabelle's Solution

a) $$\frac{\sin \theta}{1.4} = \frac{\sin 80°}{2.0}$$

This triangle doesn't have a 90° angle. So I chose the sine law, since I knew two sides and an angle opposite one of those sides.

$$\overset{1}{\cancel{1.4}} \times \frac{\sin \theta}{\underset{1}{\cancel{1.4}}} = 1.4 \times \frac{\sin 80°}{2.0}$$

To solve for θ, I multiplied both sides of the equation by 1.4.

$$\sin \theta = 1.4 \times \frac{\sin 80°}{2.0}$$

$$\theta = \sin^{-1}\left(1.4 \times \frac{\sin 80°}{2.0}\right)$$

I used the inverse sine ratio on a calculator to determine the angle.

$$\theta \doteq 44°$$

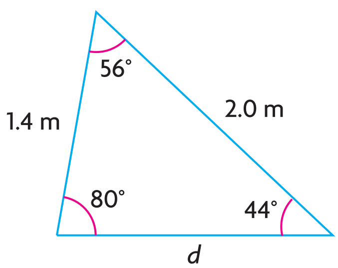
The board makes an angle of about 44° with the ground.

b) $180° - 80° - 44° = 56°$

The interior angles add up to 180°. So I subtracted the two known angles from 180°.

The board makes an angle of about 56° with the wall.

c)



I labelled the distance along the ground between the wall and the board as *d*.

$$\frac{d}{\sin 56°} = \frac{2.0}{\sin 80°}$$

Then I used the sine law.

$$\overset{1}{\cancel{\sin 56°}} \times \frac{d}{\underset{1}{\cancel{\sin 56°}}} = \sin 56° \times \frac{2.0}{\sin 80°}$$

To solve for d, I multiplied both sides of the equation by sin 56° and used a calculator to evaluate.

$$d = \sin 56° \times \frac{2.0}{\sin 80°}$$

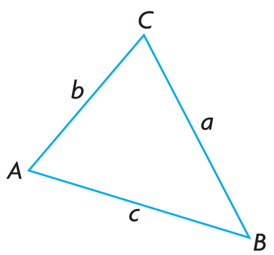
$$d \doteq 1.7 \text{ m}$$

The board is about 1.7 m from the base of the wall.

In Summary

Key Idea

- The sine law states that, in any acute $\triangle ABC$, the ratios of each side to the sine of its opposite angle are equal.



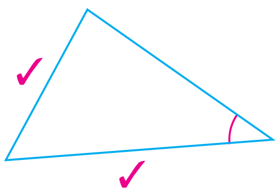
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or

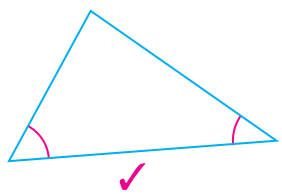
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Need to Know

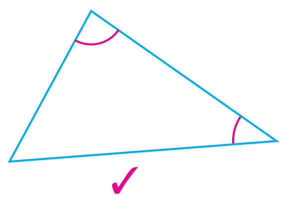
- The sine law can be used only when you know
  - two sides and the angle opposite a known side or



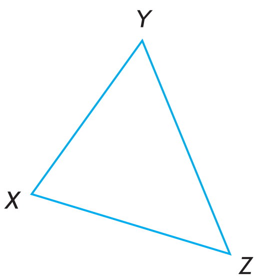
  - two angles and any side



or



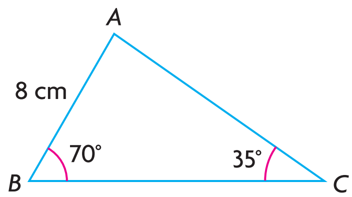
CHECK Your Understanding



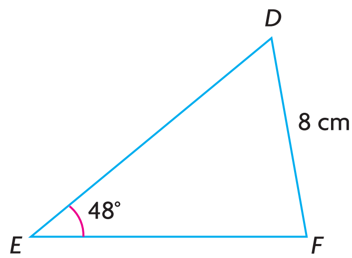
1. a) Given $\triangle XYZ$ at the left, label the sides with lower case letters.
   b) State the sine law for $\triangle XYZ$.

2. Solve each equation. Round x to the nearest tenth of a unit and θ to the nearest degree.
   a) $\dfrac{x}{\sin 22°} = \dfrac{11.6}{\sin 71°}$
   b) $\dfrac{13.1}{\sin \theta} = \dfrac{29.2}{\sin 65°}$

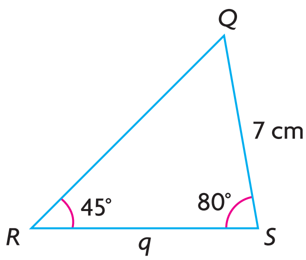
3. Use the sine law to calculate b to the nearest centimetre and $\angle D$ to the nearest degree.
   a)



   b)

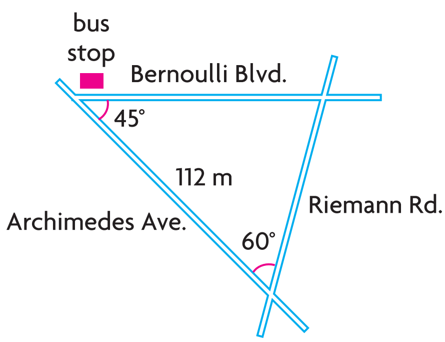


PRACTISING



4. Given $\triangle RQS$ at the left, determine q to the nearest centimetre.
   K

5. Archimedes Avenue and Bernoulli Boulevard meet at an angle of 45° near a bus stop. Riemann Road intersects Archimedes Avenue at an angle of 60°. That intersection is 112 m from the bus stop.
   a) At what angle do Riemann Road and Bernoulli Boulevard meet? Round your answer to the nearest degree.
   b) How far, to the nearest metre, is the intersection of Riemann Road and Bernoulli Boulevard from the bus stop?



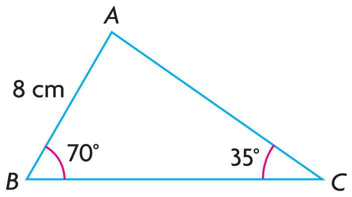
6. In $\triangle ABC$, two sides and an angle are given. Determine the value of $\angle C$ to the nearest degree and the length of b to the nearest tenth of a centimetre.

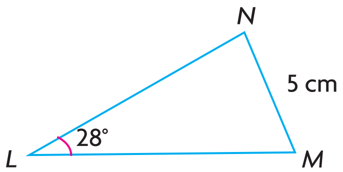
a) $a = 2.4$ cm, $c = 3.2$ cm, $\angle A = 28°$
b) $a = 9.9$ cm, $c = 11.2$ cm, $\angle A = 58°$
c) $a = 8.6$ cm, $c = 9.4$ cm, $\angle A = 47°$
d) $a = 5.5$ cm, $c = 10.4$ cm, $\angle A = 30°$

7. An isosceles triangle has two 5.5 cm sides and two 32° angles.

T **a)** Calculate the perimeter of the triangle to the nearest tenth of a centimetre.
b) Calculate the area of the triangle to the nearest tenth of a square centimetre.

8. Solve each triangle. Round each length to the nearest centimetre and each angle to the nearest degree.

a)


b)


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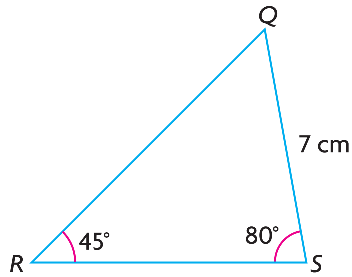
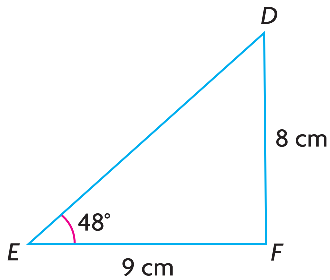


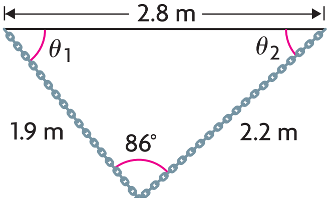
d)


Communication Tip

To solve a triangle, determine the measures of all unknown sides and angles.

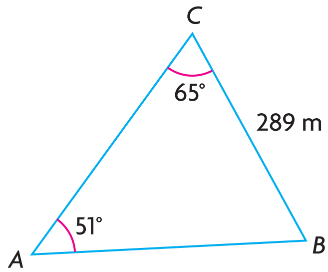
9. Solve each triangle. Round each length to the nearest tenth of a unit and each angle to the nearest degree.

a) $\triangle ABC$: $a = 10.3$, $c = 14.4$, $\angle C = 68°$
b) $\triangle DEF$: $\angle E = 38°$, $\angle F = 48°$, $f = 15.8$
c) $\triangle GHJ$: $\angle G = 61°$, $g = 5.3$, $j = 3.1$
d) $\triangle KMN$: $k = 12.5$, $n = 9.6$, $\angle N = 42°$
e) $\triangle PQR$: $p = 1.2$, $r = 1.6$, $\angle R = 52°$
f) $\triangle XYZ$: $z = 6.8$, $\angle X = 42°$, $\angle Y = 77°$



10. Toby uses chains and a winch to lift engines at his father's garage. Two hooks in the ceiling are 2.8 m apart. Each hook has a chain hanging from it. The chains are of length 1.9 m and 2.2 m. When the ends of the chains are attached, they form an angle of 86°. In this configuration, what acute angle, to the nearest degree, does each chain make with the ceiling?

11. **A** Betsy installed cordless phones in the student centre at points *A*, *B*, and *C* as shown. Explain how you can use the given information to determine which two phones are farthest apart.

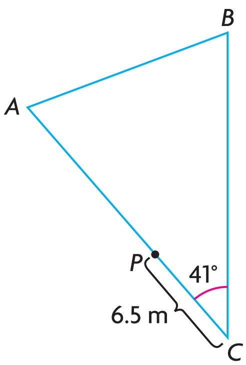


12. **C** Tom says that he doesn't need to use the sine law because he can always determine a solution by using primary trigonometric ratios. Is Tom correct? What would you say to Tom to convince him to use the sine law to solve a problem?

Extending

13. Two angles in a triangle measure 54° and 38°. The longest side of the triangle is 24 cm longer than the shortest side. Calculate the length, to the nearest centimetre, of all three sides.

14. Use the sine law to show why the longest side of a triangle must be opposite the largest angle.



15. A triangular garden is enclosed by a fence. A dog is on a 5 m leash tethered to the fence at point *P*, 6.5 m from point *C*, as shown at the left. If $\angle ACB = 41°$, calculate the total length, to the nearest tenth of a metre, of fence that the dog can reach.

16. Tara built a sculpture in the shape of a huge equilateral triangle of side length 4.0 m. Unfortunately, the ground underneath the sculpture was not stable, and one of the vertices of the triangle sank 55 cm into the ground. Assume that Tara's sculpture was built on level ground originally.

a) What is the length, to the nearest tenth of a metre, of the two exposed sides of Tara's triangle now?

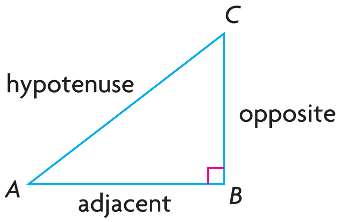
b) What percent of the triangle's surface area remains above ground? Round your answer to the nearest percent.

5 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: What are the primary trigonometric ratios, and how do you use them?

A: Given $\triangle ABC$, the primary trigonometric ratios for $\angle A$ are:



$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

To calculate an angle or side using a trigonometric ratio:

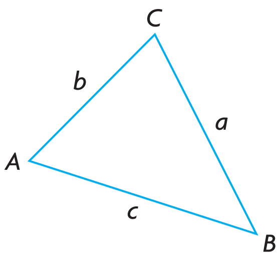
- Label the sides of the triangle relative to either a given angle or the one you want to determine.
- Write an equation that involves what you are trying to find by using the appropriate ratio.
- Solve your equation.

Study *Aid*

- See Lesson 5.1, Examples 1, 2, and 3 and Lesson 5.2, Examples 1 to 4.
- Try Mid-Chapter Review Questions 1 to 8.

Q: What is the sine law, and when can I use it?

A: In any acute triangle, the ratios of a side to the sine of its opposite angle are equal.

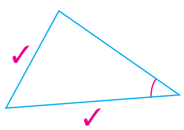


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

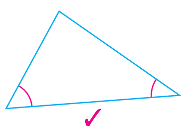
or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

- The sine law can be used only when you know
  - two sides and the angle opposite a known side or



  - two angles and any side



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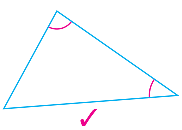


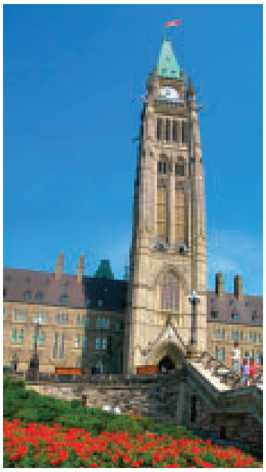
Study *Aid*

- See Lesson 5.3, Examples 1, 2, and 3.
- Try Mid-Chapter Review Questions 9, 10, and 11.

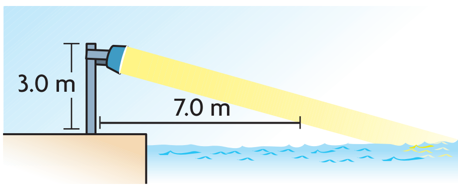
PRACTICE Questions

Lesson 5.1

1. From a spot 25 m from the base of the Peace Tower in Ottawa, the angle of elevation to the top of the flagpole is 76°. How tall, to the nearest metre, is the Peace Tower, including the flagpole?



2. A spotlight on a 3.0 m stand shines on the surface of a swimming pool. The beam of light hits the water at a point 7.0 m away from the base of the stand.

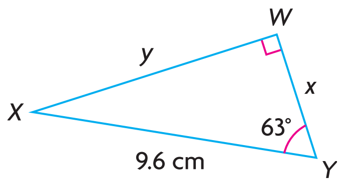


a) Calculate the angle the beam makes with the pool surface. Round your answer to the nearest degree.

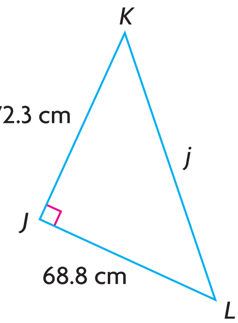
b) The beam reflects off the pool surface and strikes a wall 4.5 m away from the reflection point. The angle the beam makes with the pool is exactly the same on either side of the reflection point. Calculate how far up the wall, to the nearest tenth of a metre, the spotlight will appear.

3. Solve each triangle. Round each length to the nearest tenth of a unit and each angle to the nearest degree.

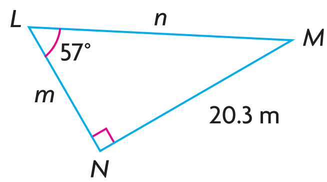
a)



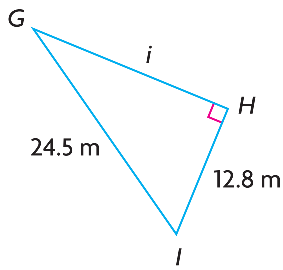
b)



c)



d)

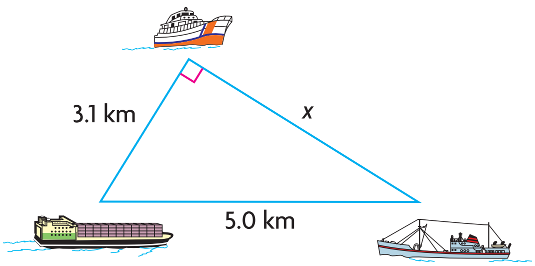


4. The sides of an isosceles triangle are 12 cm, 12 cm, and 16 cm. Use primary trigonometric ratios to determine the largest interior angle to the nearest degree. (*Hint:* Divide the triangle into two congruent right triangles.)

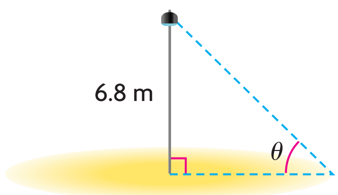
Lesson 5.2

5. The mainsail of the small sailboat shown below measures 2.4 m across the boom and 4.4 m up the mainmast. The boom and mast meet at an angle of 74°. What is the area of the mainsail to the nearest tenth of a square metre?

6. A coast guard boat is tracking two ships using radar. At noon, the ships are 5.0 km apart and the angle between them is 90°. The closest ship is 3.1 km from the coast guard boat. How far, to the nearest tenth of a kilometre, is the other ship from the coast guard boat?



7. An overhead streetlight can illuminate a circular area of diameter 14 m. The light bulb is 6.8 m directly above a bike path. Determine the angle of elevation, to the nearest degree, from the edge of the illuminated area to the light bulb.



8. Martin's building is 105 m high. From the roof, he spots his car in the parking lot. He estimates that it is about 70 m from the base of the building.

a) What is the angle of depression, to the nearest degree, from Martin's eyes to the car?

b) What is the straight-line distance, to the nearest metre, from Martin to his car?

Lesson 5.3

9. Solve each triangle. Round each length to the nearest unit and each angle to the nearest degree.

a) $\triangle DEF$: $\angle D = 67°$, $\angle F = 42°$, $e = 25$

b) $\triangle PQR$: $\angle R = 80°$, $\angle Q = 49°$, $r = 8$

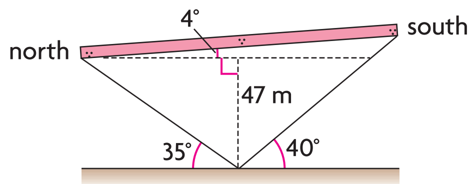
c) $\triangle ABC$: $\angle A = 52°$, $\angle B = 70°$, $a = 20$

d) $\triangle XYZ$: $\angle Z = 23°$, $\angle Y = 54°$, $x = 16$

10. From the bottom of a canyon, Rita stands 47 m directly below an overhead bridge. She estimates that the angle of elevation of the bridge is about 35° at the north end and about 40° at the south end. For each question, round your answer to the nearest metre.

a) If the bridge were level, how long would it be?

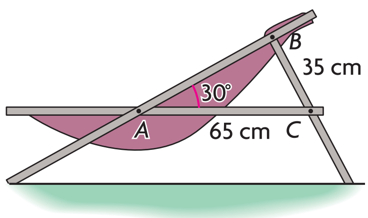
b) If the bridge were inclined 4° from north to south, how much longer would it be?



11. The manufacturer of a reclining lawn chair is planning to cut notches on the back of the chair so that you can recline at an angle of 30° as shown.

a) What is the measure of $\angle B$, to the nearest degree, for the chair to be reclined at the proper angle?

b) Determine the distance from A to B to the nearest centimetre.



5.4 Investigating and Applying the Cosine Law in Acute Triangles

YOU WILL NEED

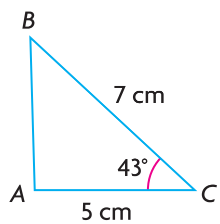
- dynamic geometry software

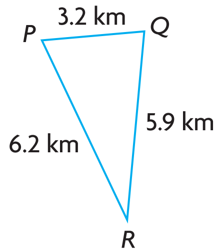
GOAL

Verify the cosine law and use it to solve real-life problems.

INVESTIGATE the Math

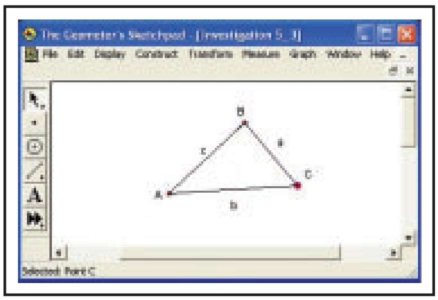
The sine law is useful only when you know two sides and an angle opposite one of those sides or when you know two angles and a side. But in the triangles shown, you don't know this information, so the sine law cannot be applied.





? How can the Pythagorean theorem be modified to relate the sides and angles in these types of triangles?

A. Use dynamic geometry software to construct any acute triangle.



B. Label the vertices A, B, and C. Then name the sides a, b, and c as shown.

C. Measure all three interior angles and all three sides.

D. Drag vertex C until $\angle C = 90°$.

E. What is the Pythagorean relationship for this triangle? Using the measures of a and b, choose **Calculate ...** from the **Measure** menu to determine $a^2 + b^2$. Use the measure of c and repeat to determine c^2.

Tech | *Support*

For help using dynamic geometry software, see Technical Appendix, B-21 and B-22.

F. Does $a^2 + b^2 = c^2$? If not, are they close? Why might they be off by a little bit?

G. Move vertex C farther away from AB to create an acute triangle. How does the value of $a^2 + b^2$ compare with that of c^2?

H. Using the measures of $a^2 + b^2$ and c^2, choose **Calculate ...** from the **Measure** menu to determine $a^2 + b^2 - c^2$. How far off is this value from the Pythagorean theorem? Copy the table shown and record your observations.

Triangle	a	b	c	$\angle C$	$a^2 + b^2$	c^2	$a^2 + b^2 - c^2$
1							
2							
3							
4							
5							

I. Move vertex C to four other locations and record your observations. Make sure that one of the triangles has $\angle C = 90°$, while the rest are acute.

J. Use a calculator to determine $2ab \cos C$ for each of your triangles. Add this column to your table and record these values. How do they compare with the values of $a^2 + b^2 - c^2$?

K. Based on your observations, modify the Pythagorean theorem to relate c^2 to $a^2 + b^2$.

Reflecting

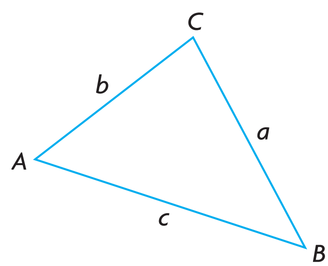
L. How is the Pythagorean theorem a special case of the relationship you found? Explain.

M. Based on your observations, how did the value of $\angle C$ affect the value of $a^2 + b^2 - c^2$?

N. Explain how you would use the cosine law to relate each pair of values in any acute triangle.
- a^2 to $b^2 + c^2$
- b^2 to $a^2 + c^2$

cosine law

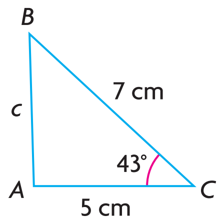
in any acute $\triangle ABC$,
$c^2 = a^2 + b^2 - 2ab \cos C$



APPLY the Math

EXAMPLE 1 Selecting the cosine law as a strategy to calculate an unknown length

Determine the length of c to the nearest centimetre.



Anita's Solution

$$a = 7 \text{ cm}$$
$$b = 5 \text{ cm}$$
$$\angle C = 43°$$

I can't be certain that the triangle has a 90° angle, and I can't use the sine law with the given information.

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c^2 = 7^2 + 5^2 - 2(7)(5)\cos 43°$$

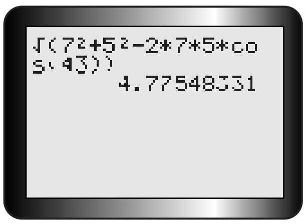
I chose the cosine law since I knew two sides and the **contained angle**, $\angle C$.

contained angle

the acute angle between two known sides

$$c = \sqrt{7^2 + 5^2 - 2(7)(5)\cos 43°}$$

To solve for c, I took the square root of both sides of the equation.



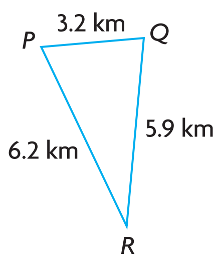
I used a calculator to evaluate.

$$c \doteq 5 \text{ cm}$$

Side c is about 5 cm long.

EXAMPLE 2 Selecting the cosine law as a strategy to calculate an unknown angle

Determine the measure of $\angle R$ to the nearest degree.



Kew's Solution

$$p = 5.9 \text{ km}$$
$$q = 6.2 \text{ km}$$
$$r = 3.2 \text{ km}$$

I knew three sides and no angles.

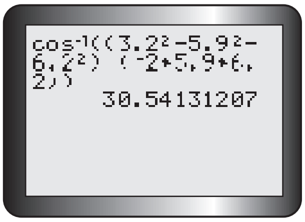
$$r^2 = p^2 + q^2 - 2pq\cos R$$
$$3.2^2 = 5.9^2 + 6.2^2 - 2(5.9)(6.2)\cos R$$
$$3.2^2 - 5.9^2 - 6.2^2 = -2(5.9)(6.2)\cos R$$

I chose the cosine law. I wrote the formula in terms of $\angle R$.

$$\frac{3.2^2 - 5.9^2 - 6.2^2}{-2(5.9)(6.2)} = \left(\frac{\cancel{-2(5.9)(6.2)}^{\,1}}{\cancel{-2(5.9)(6.2)}_{\,1}}\right)\cos R$$

To solve for $\angle R$, I divided both sides of the equation by $-2(5.9)(6.2)$.

$$\frac{3.2^2 - 5.9^2 - 6.2^2}{-2(5.9)(6.2)} = \cos R$$



I used the inverse cosine function on a calculator to evaluate.

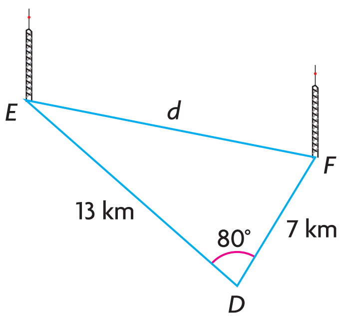
$$\angle R \doteq 31°$$

The measure of $\angle R$ is about 31°.

EXAMPLE 3 Solving a problem by using the cosine law

Ken's cell phone detects two transmission antennas, one 7 km away and the other 13 km away. From his position, the two antennas appear to be separated by an angle of 80°. How far apart, to the nearest kilometre, are the two antennas?

Chantal's Solution



I drew a sketch and labelled the distance between the two antennas as d.

$e = 7$ km

$f = 13$ km

$\angle D = 80°$

The triangle doesn't have a 90° angle. I knew two sides and the contained angle, so I chose the cosine law.

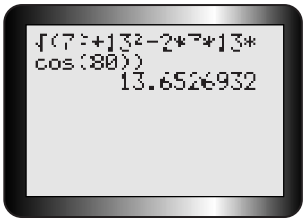
$$d^2 = e^2 + f^2 - 2ef \cos D$$

I wrote the formula in terms of $\angle D$.

$$d^2 = 7^2 + 13^2 - 2(7)(13)\cos 80°$$

$$d = \sqrt{7^2 + 13^2 - 2(7)(13)\cos 80°}$$

To solve for d, I took the square root of both sides of the equation.



I used a calculator to evaluate.

$d \doteq 14$ km

The two antennas are about 14 km apart.

In Summary

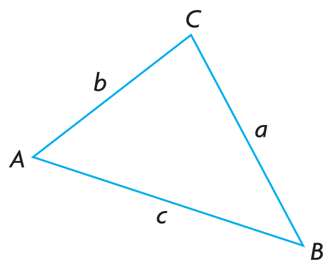
Key Idea

- The cosine law states that in any $\triangle ABC$,

$$a^2 = b^2 + c^2 - 2bc\cos A$$

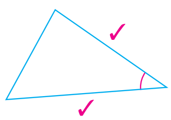
$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

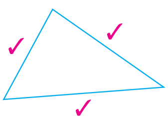


Need to Know

- The cosine law can be used only when you know
  - two sides and the contained angle or



  - all three sides



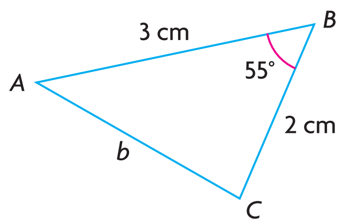
- The contained angle in a triangle is the angle between two known sides.

CHECK Your Understanding

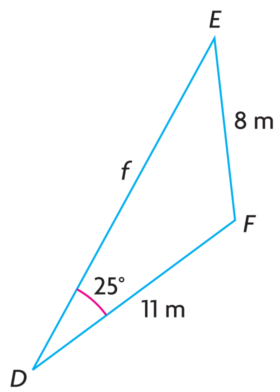
1. **i)** In which triangle is it necessary to use the cosine law to calculate the third side? Justify your answer.

ii) State the formula you would use to determine the length of side b in $\triangle ABC$.

a)

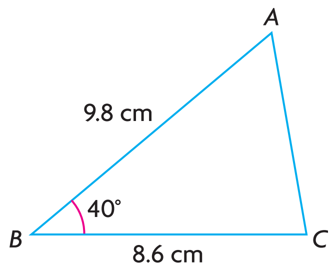


b)

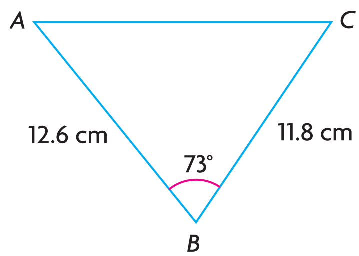


2. Determine the length of each unknown side to the nearest tenth of a centimetre.

a)

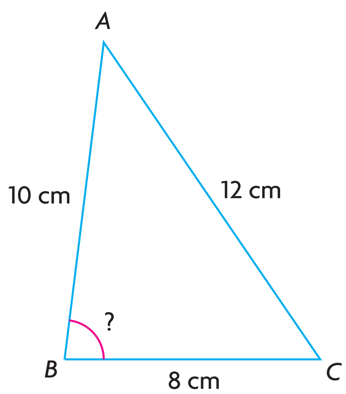


b)

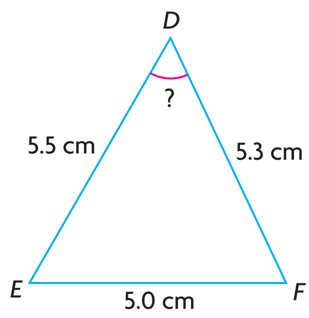


3. Determine each indicated angle to the nearest degree.

a)

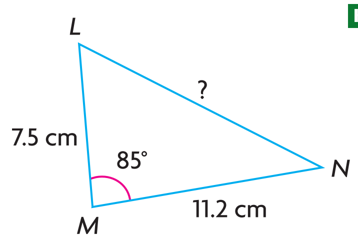


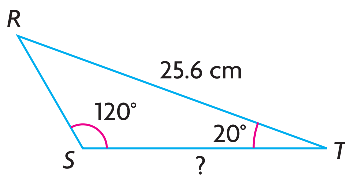
b)

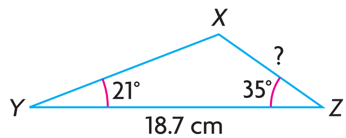


PRACTISING

4. Determine each indicated length to the nearest tenth of a centimetre.
K

a) 

b) 

c) 

5. Solve each triangle. Round each length to the nearest tenth of a centimetre and each angle to the nearest degree.

a) $\triangle ABC$: $\angle A = 68°$, $b = 10.1$ cm, $c = 11.1$ cm

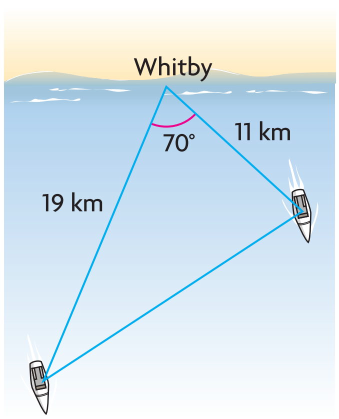
b) $\triangle DEF$: $\angle D = 52°$, $e = 7.2$ cm, $f = 9.6$ cm

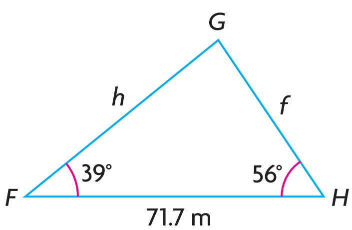
c) $\triangle HIF$: $\angle H = 35°$, $i = 9.3$ cm, $f = 12.5$ cm

d) $\triangle PQR$: $p = 7.5$ cm, $q = 8.1$ cm, $r = 12.2$ cm

6. A triangle has sides that measure 5 cm, 6 cm, and 10 cm. Do any of the angles in this triangle equal 30°? Explain.

7. Two boats leave Whitby harbour at the same time. One boat heads
A
19 km to its destination in Lake Ontario. The second boat heads on a course 70° from the first boat and travels 11 km to its destination. How far apart, to the nearest kilometre, are the boats when they reach their destinations?

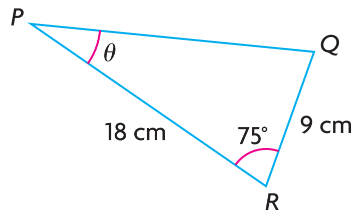




8. Louis says that he has no information to use the sine law to solve $\triangle FGH$ shown at the left and that he must use the cosine law instead. Is he correct? Describe how you would solve for each unknown side and angle.

9. Driving a snowmobile across a frozen lake, Sheldon starts from the most westerly point and travels 8.0 km before he turns right at an angle of 59° and travels 6.1 km, stopping at the most easterly point of the lake. How wide, to the nearest tenth of a kilometre, is the lake?

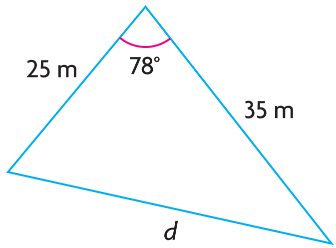
10. What strategy would you use to calculate angle θ in $\triangle PQR$ shown at the right? Justify your choice and then use your strategy to solve for θ to the nearest degree.



11. T A clock with a radius of 15 cm has an 11 cm minute hand and a 7 cm hour hand. How far apart, to the nearest centimetre, are the tips of the hands at each time?

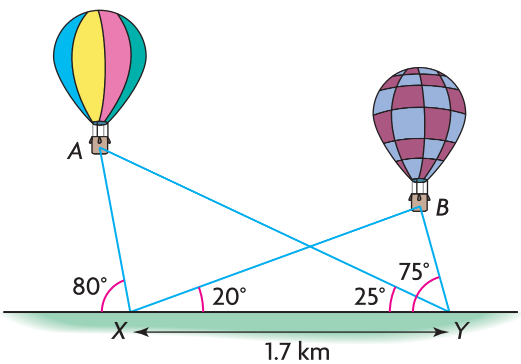
a) 3:30 p.m. b) 6:38 a.m.

12. C Use the triangle shown at the right to create a problem that involves its side lengths and interior angles. Then describe how to determine d.



Extending

13. Two observers standing at points X and Y are 1.7 km apart. Each person measures angles of elevation to two balloons, A and B, flying overhead as shown. For each question, round your answer to the nearest tenth of a kilometre.

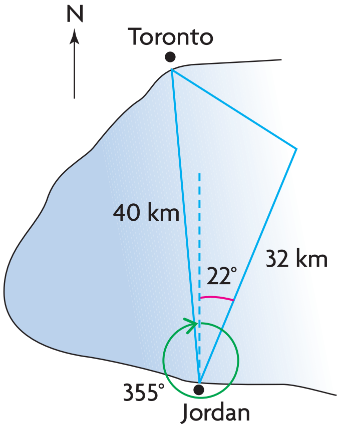


a) How far is balloon A from point X? From point Y?
b) How far is balloon B from point X? From point Y?
c) How far apart are balloons A and B?

5.5 Solving Problems by Using Acute-Triangle Models

GOAL

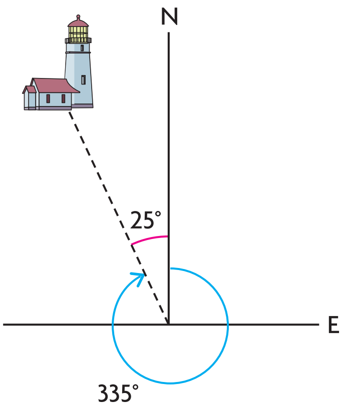
Solve problems involving the primary trigonometric ratios and the sine and cosine laws.



LEARN ABOUT the Math

Steve leaves the marina at Jordan on a 40 km sailboat race across Lake Ontario intending to travel on a **bearing** of 355°, but an early morning fog settles. By the time it clears, Steve has travelled 32 km on a bearing of 22°.

bearing
the direction in which you have to move in order to reach an object. A bearing is a clockwise angle from magnetic north. For example, the bearing of the lighthouse shown is 335°.

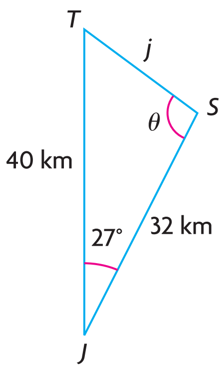


? In which direction must Steve head to reach the finish line?

EXAMPLE 1 Solving a problem by using the sine and cosine laws

Determine the direction in which Steve should head to reach the finish line.

Liz's Solution



I drew a sketch and labelled the interior angle I needed to calculate as θ.

$\angle J = 5° + 22°$

$= 27°$

A bearing of 355° means that Steve's destination should have been 5° west of north. But he drifted 22° east of north. So I added 5° and 22° to get $\angle J$.

$s = 40$ km

$t = 32$ km

$j^2 = s^2 + t^2 - 2st\cos J$

$j^2 = 40^2 + 32^2 - 2(40)(32)\cos 27°$

I didn't have information to be able to use the sine law to determine angle θ. So I used the cosine law to first calculate side j.

$$j = \sqrt{40^2 + 32^2 - 2(40)(32)\cos 27°}$$

To solve for j, I took the square root of both sides of the equation.

$$j \doteq 18.521 \text{ km}$$

I used a calculator to evaluate.

$$\frac{\sin \theta}{40} = \frac{\sin 27°}{18.521}$$

I now had information to be able to use the sine law.

$$\cancel{40}^{1} \times \frac{\sin \theta}{\cancel{40}_{1}} = 40 \times \frac{\sin 27°}{18.521}$$

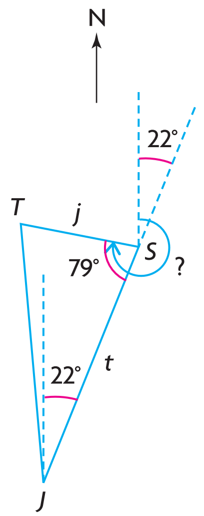
To solve for angle θ, I multiplied both sides of the equation by 40.

$$\sin \theta = 40 \times \frac{\sin 27°}{18.521}$$

$$\theta = \sin^{-1}\left(40 \times \frac{\sin 27°}{18.521}\right)$$

$$\theta \doteq 79°$$

I used the inverse sine function on a calculator to evaluate.



I now knew angle θ, but I needed to determine the bearing to T from S. I sketched the triangle with side t extended. I labelled the angles I knew and the angle I needed to determine.

$$\text{bearing} = 22° + 180° + 79°$$
$$= 281°$$

Steve must head at a bearing of about 281° to reach the finish line.

The bearing is measured from due north. So I added all the angles from due north to side j. I knew that the angle along the extended side t is 180°.

Reflecting

A. Why didn't Liz first use the sine law in her solution?

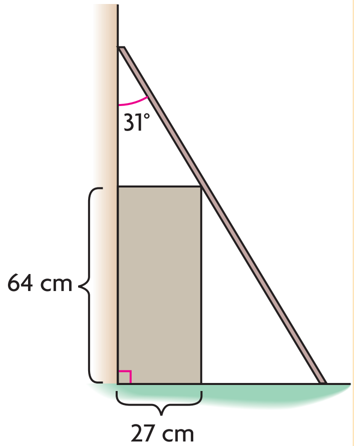
B. How is a diagram of the situation helpful? Explain.

C. Is it possible to solve this problem *without* using the sine law or the cosine law? Justify your answer.

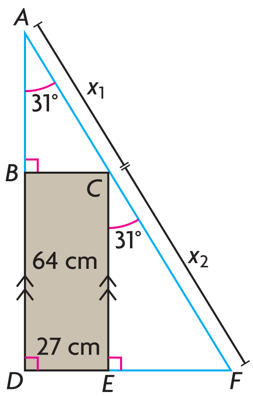
APPLY the Math

EXAMPLE 2 Solving a problem by using primary trigonometric ratios

A ladder leaning against a wall makes an angle of 31° with the wall. The ladder just touches a box that is flush against the wall and the ground. The box has a height of 64 cm and a width of 27 cm. How long, to the nearest centimetre, is the ladder?



Denis's Solution



The box and ladder form two right triangles. I labelled the two parts of the ladder as x_1 and x_2. The total length of the ladder is the sum of x_1 and x_2.

Since CE is parallel to AD, $\angle ECF$ is equal to $\angle BAC$, which is 31°.

In $\triangle ABC$:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 31° = \frac{BC}{x_1}$$

$$\sin 31° = \frac{27}{x_1}$$

$$x_1 \times \sin 31° = \overset{1}{\cancel{x_1}} \times \frac{27}{\underset{1}{\cancel{x_1}}}$$

$$x_1 \times \sin 31° = 27$$

In $\triangle CEF$:

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 31° = \frac{CE}{x_2}$$

$$\cos 31° = \frac{64}{x_2}$$

$$x_2 \times \cos 31° = \overset{1}{\cancel{x_2}} \times \frac{64}{\underset{1}{\cancel{x_2}}}$$

$$x_2 \times \cos 31° = 64$$

In $\triangle ABC$, $BC = 27$ cm; in $\triangle CEF$, $CE = 64$ cm.

In each triangle, I knew a side and an angle and I needed to calculate the hypotenuse. So I used primary trigonometric ratios:

- sine, since BC is opposite the 31° angle
- cosine, since CE is adjacent to the 31° angle

To solve for x_1, I multiplied both sides of the equation by x_1 and divided both sides by sin 31°.

To solve for x_2, I multiplied both sides of the equation by x_2 and divided both sides by cos 31°.

$$\frac{x_1 \times \cancel{\sin 31°}}{\cancel{\sin 31°}} = \frac{27}{\sin 31°} \qquad \frac{x_2 \times \cancel{\cos 31°}}{\cancel{\cos 31°}} = \frac{64}{\cos 31°}$$

$$x_1 = \frac{27}{\sin 31°} \qquad x_2 = \frac{64}{\cos 31°}$$

$$x_1 \doteq 52.423\,308\,71 \qquad x_2 \doteq 74.664\,537\,42$$

I used a calculator to evaluate.

$$\begin{aligned} \text{length} &= x_1 + x_2 \\ &= 52.423\,308\,71 + 74.664\,537\,42 \\ &\doteq 127 \text{ cm} \end{aligned}$$

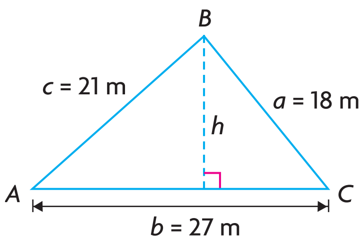
I added x_1 and x_2 to determine the length of the ladder.

The ladder is about 127 cm long.

EXAMPLE 3 Selecting a strategy to calculate the area of a triangle

Jim has a triangular backyard with side lengths of 27 m, 21 m, and 18 m. His bag of fertilizer covers 400 m². Does he have enough fertilizer?

Barbara's Solution



I drew a sketch with the longest side at the bottom. I labelled all the given information.

$$a^2 = b^2 + c^2 - 2\,bc \cos A$$
$$18^2 = 27^2 + 21^2 - 2(27)(21)\cos A$$
$$18^2 - 27^2 - 21^2 = -2(27)(21)\cos A$$

To determine the area, I needed the height, h, of the triangle. To determine h, I had to calculate an angle first, so I chose $\angle A$. I used the cosine law to determine $\angle A$.

$$\frac{18^2 - 27^2 - 21^2}{-2(27)(21)} = \left(\frac{\cancel{-2(27)(21)}}{\cancel{-2(27)(21)}}\right)\cos A$$

$$\cos A = \frac{18^2 - 27^2 - 21^2}{-2(27)(21)}$$

To solve for $\angle A$, I divided both sides of the equation by $-2(27)(21)$.

$$\angle A = \cos^{-1}\left(\frac{18^2 - 27^2 - 21^2}{-2(27)(21)}\right)$$

$$\angle A \doteq 41.75°$$

I used the inverse cosine function on a calculator to evaluate.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

Since I knew $\angle A$, I used a primary trigonometric ratio to calculate h.

$$\sin A = \frac{h}{21}$$

$$\sin 41.75° = \frac{h}{21}$$

$$21 \times \sin 41.75° = \overset{1}{\cancel{21}} \times \frac{h}{\underset{1}{\cancel{21}}}$$

To solve for h, I multiplied both sides of the equation by 21.

$$21 \times \sin 41.75° = h$$

$$13.98 \text{ m} \doteq h$$

I used a calculator to evaluate.

$$A = \frac{1}{2} b \times h$$

$$= \left(\frac{1}{2}\right) 27 \times 13.98$$

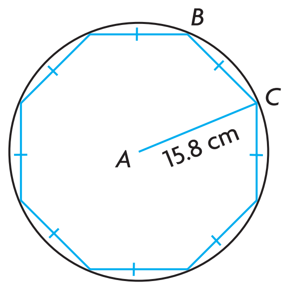
$$\doteq 189 \text{ m}^2$$

I substituted the values for b and h into the formula for the area of a triangle.

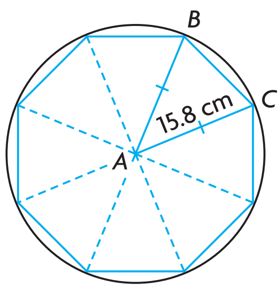
Since 189 is less than half of 400, Jim has enough fertilizer to cover his lawn twice and still have some fertilizer left over.

EXAMPLE 4 Solving a problem to determine a perimeter

A regular octagon is inscribed in a circle of radius 15.8 cm. What is the perimeter, to the nearest tenth of a centimetre, of the octagon?



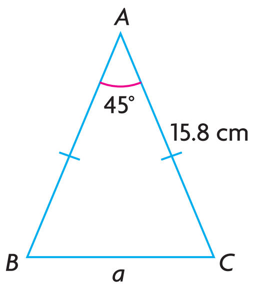
Shelley's Solution



An octagon is made up of eight identical triangles, each of which is isosceles because two sides are the same length (radii of the circle).

$$\angle A = 360° \div 8$$
$$= 45°$$

A circle measures 360°. $\angle A$ is $\frac{1}{8}$ of 360°. So I divided 360° by 8 to get $\angle A$.



I labelled the base of $\triangle BAC$ as a.

$$\angle A + \angle B + \angle C = 180°$$
$$45° + 2\angle B = 180°$$
$$2\angle B = 180° - 45°$$
$$2\angle B = 135°$$
$$\angle B = 67.5°$$

Since $\triangle BAC$ is isosceles, $\angle B = \angle C$. The sum of the interior angles is 180°, so I calculated $\angle B$.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{a}{\sin 45°} = \frac{15.8}{\sin 67.5°}$$

I knew an angle and the side opposite that angle. So I used the sine law to calculate a.

$$\cancel{\sin 45°}^{1} \times \frac{a}{\cancel{\sin 45°}_{1}} = \sin 45° \times \frac{15.8}{\sin 67.5°}$$

To solve for a, I multiplied both sides of the equation by sin 45°.

$$a = \sin 45° \times \frac{15.8}{\sin 67.5°}$$
$$a \doteq 12.09 \text{ cm}$$

I used a calculator to evaluate.

$$\text{perimeter} = 12.09 \times 8$$
$$\doteq 96.7 \text{ cm}$$

The octagon is made up of eight sides of length equal to a. So I multiplied a by 8 to get the perimeter.

The perimeter of the octagon is about 96.7 cm.

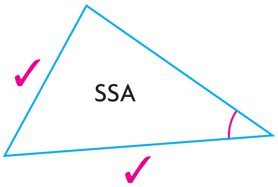
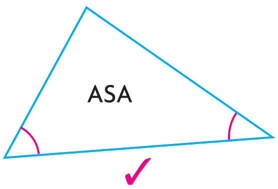
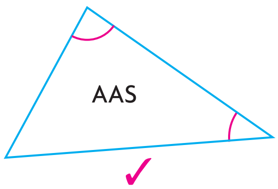
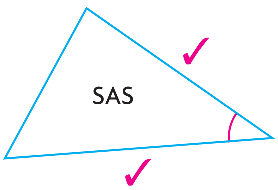
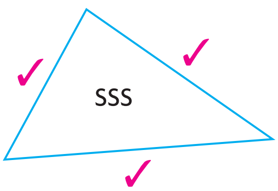
In Summary

Key Idea

- The primary trigonometric ratios, the sine law, and the cosine law can be used to solve problems involving triangles. The method you use depends on the information you know about the triangle and what you want to determine.

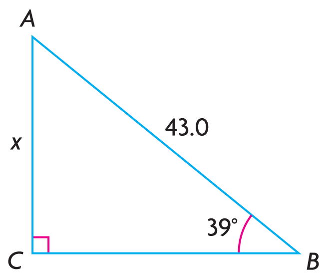
Need to Know

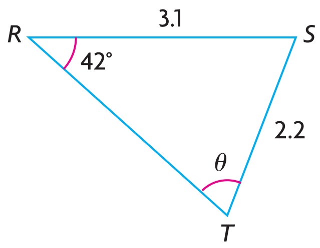
- If the triangle in the problem is a right triangle, use the primary trigonometric ratios.
- If the triangle is oblique, use the sine law and/or the cosine law.

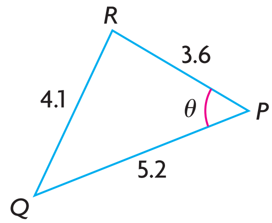
Given	Required to Find	Use
	angle	sine law
 or 	side	sine law
	side	cosine law
	angle	cosine law

CHECK Your Understanding

1. For each triangle, describe how you would solve for the unknown side or angle.

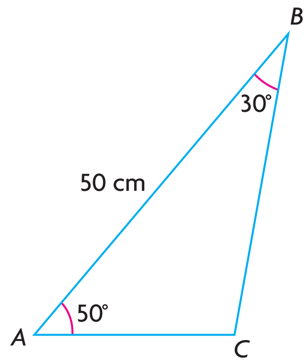
a) 

b) 

c) 

2. Complete a solution for each part of question 1. Round x to the nearest tenth of a unit and θ to the nearest degree.

PRACTISING



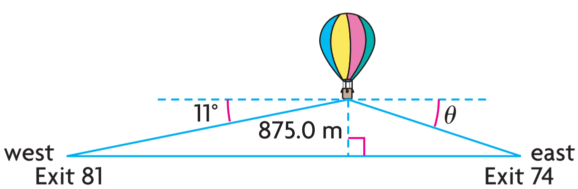
3. Determine the area of $\triangle ABC$, shown at the right, to the nearest square centimetre.
K

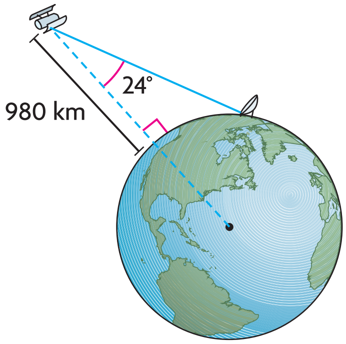
4. The legs of a collapsible stepladder are each 2.0 m long. What is the maximum distance between the front and rear legs if the maximum angle at the top is 40°? Round your answer to the nearest tenth of a metre.

5. To get around an obstacle, a local electrical utility must lay two sections of underground cable that are 371.0 m and 440.0 m long. The two sections meet at an angle of 85°. How much extra cable is necessary due to the obstacle? Round your answer to the nearest tenth of a metre.
A

6. A surveyor is surveying three locations (M, N, and P) for new rides in an amusement park around an artificial lake. $\angle MNP$ is measured as 57°. MN is 728.0 m and MP is 638.0 m. What is the angle at M to the nearest degree?

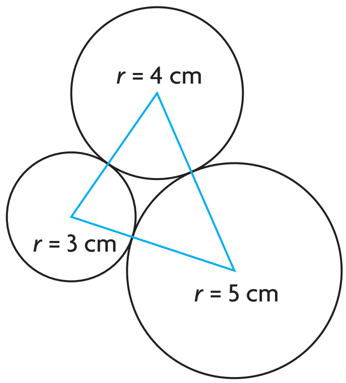
7. Mike's hot-air balloon is 875.0 m directly above a highway. When he is looking west, the angle of depression to Exit 81 is 11°. The exit numbers on this highway represent the number of kilometres left before the highway ends. What is the angle of depression, to the nearest degree, to Exit 74 in the east?

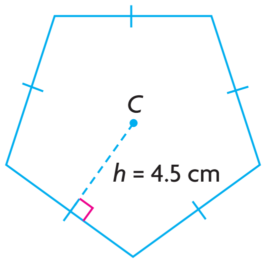




8. A satellite is orbiting Earth 980 km above Earth's surface. A receiving dish is located on Earth such that the line from the satellite to the dish and the line from the satellite to Earth's centre form an angle of 24° as shown at the left. If a signal from the satellite travels at 3×10^8 m/s, how long does it take to reach the dish? Round your answer to the nearest thousandth of a second.

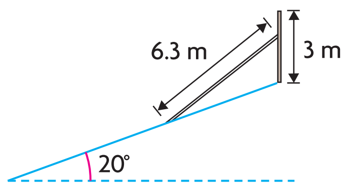
9. Three circles with radii of 3 cm, 4 cm, and 5 cm are touching each other as shown. A triangle is drawn connecting the three centres. Calculate all the interior angles of the triangle. Round your answers to the nearest degree.

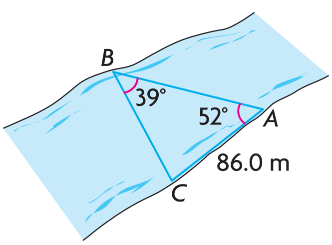




10. T Given the regular pentagon shown at the left, determine its perimeter to the nearest tenth of a centimetre and its area to the nearest tenth of a square centimetre.

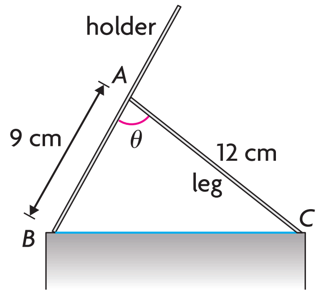
11. A 3 m high fence is on the side of a hill and tends to lean over. The hill is inclined at an angle of 20° to the horizontal. A 6.3 m brace is to be installed to prop up the fence. It will be attached to the fence at a height of 2.5 m and will be staked downhill from the base of the fence. What angle, to the nearest degree, does the brace make with the hill?





12. A surveyor wants to calculate the distance BC across a river. He selects a position, A, so that CA is 86.0 m. He measures $\angle ABC$ and $\angle BAC$ as 39° and 52°, respectively, as shown at the left. Calculate the distance BC to the nearest tenth of a metre.

13. For best viewing, a document holder for people who work at computers should be inclined between 61° and 65° ($\angle ABC$). A 12 cm support leg is attached to the holder 9 cm from the bottom. Calculate the minimum and maximum angle θ that the leg must make with the holder. Round your answers to the nearest degree.

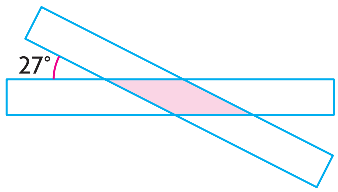


14. C Match each method with a problem that can be solved by that method. Describe how each method could be used to complete a solution.

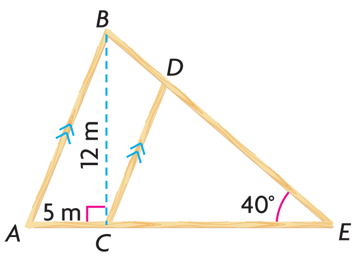
Method	Problems
Cosine law	Chris lives in a U-shaped building. From his window, he sights Bethany's window at a bearing of 328° and Josef's window at a bearing of 19°. Josef's window is 54 m from Bethany's and both windows are directly opposite each other. How far is each window from Chris's window?
Sine law	When the Sun is at an angle of elevation of 41°, Martina's treehouse casts a shadow that is 11.4 m long. Assuming that the ground is level, how tall is Martina's treehouse?
Primary trigonometric ratios	Ken walks 3.8 km west and then turns clockwise 65° before walking another 1.7 km. How far does Ken have to walk to get back to where he started?

Extending

15. Two paper strips, each 2.5 cm wide, are laying across each other at an angle of 27°, as shown at the right. What is the area of the overlapping paper? Round your answer to the nearest tenth of a square centimetre.



16. The diagram shows a roofing truss with AB parallel to CD. Calculate the total length of wood needed to construct the truss. Round your answer to the nearest metre.

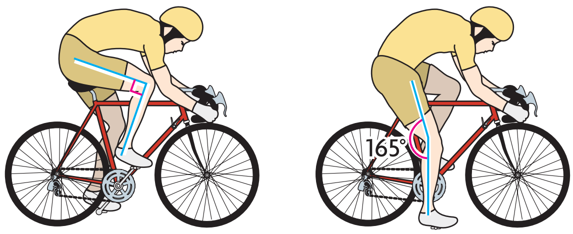


17. Lucas takes a 10.0 m rope and creates a triangle with interior angles of 30°, 70°, and 80°. How long, to the nearest tenth of a metre, is each side?

Curious Math

Cycling Geometry

Competitive cyclists pay great attention to the geometry of their legs and of their bikes. To provide the greatest leverage and achieve optimum output, a cyclist's knees must form a right angle with the pedals at the top of a stroke and an angle of 165° at the bottom of a stroke.

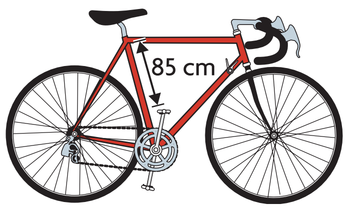


To achieve these conditions, cyclists adjust the seat height to give the appropriate distances.

1. When Mark is standing, the distance from his hips to his knees is 44 cm and from his hips to the floor is 93 cm. When Mark's knees form each angle listed, how far are his hips from his heels? Round your answers to the nearest centimetre.

a) 90° **b)** 165°

2. On Mark's bike, one pedal is 85 cm from the base of the seat. For Mark to achieve optimum output, by how much should he raise his seat? Round your answer to the nearest centimetre.



3. The table below lists the measurements of other cyclists in Mark's riding club. Would these cyclists be able to ride Mark's bike? What seat height(s) would they require? Round your answers to the nearest centimetre.

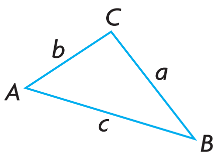
Name	Hips to Knees	Hips to Floor
Terry	38 cm	81 cm
Colleen	45 cm	96 cm
Sergio	41 cm	89 cm

5 Chapter Review

FREQUENTLY ASKED Questions

Q: What is the cosine law, and how do you use it to determine angles and sides in triangles?

A: The cosine law is a relationship that is true for *all* triangles:



$$a^2 = b^2 + c^2 - 2\,bc \cos A$$
$$b^2 = a^2 + c^2 - 2\,ac \cos B$$
$$c^2 = a^2 + b^2 - 2\,ab \cos C$$

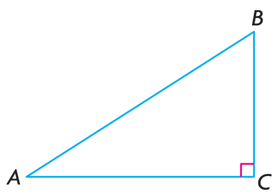
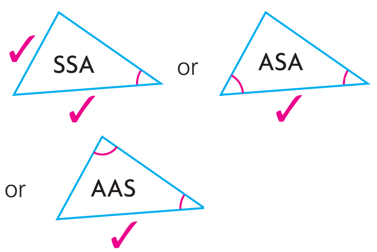
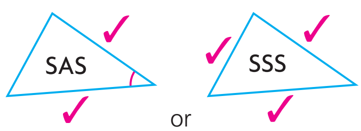
If you don't know a side and an angle opposite it, use the cosine law. The sine law can be used only if you know a side and the angle opposite that side. You can use the cosine law in combination with the sine law and other trigonometric ratios to solve a triangle.

Study *Aid*

- See Lesson 5.4, Examples 1, 2, and 3.
- Try Chapter Review Questions 7 and 8.

Q: How do you know which strategy to use to solve a trigonometry problem?

A:

Given	Use
A right triangle with any two pieces of information 	Primary trigonometric ratios: $\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$
A triangle with information about sides and opposite angles 	The sine law: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ or $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$
A triangle with no information that allows you to use the sine law 	The cosine law: $a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2\,ab \cos C$

Study *Aid*

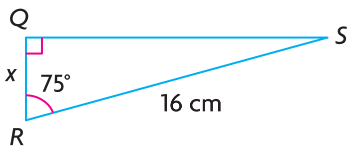
- See Lesson 5.5, Examples 1 to 4.
- Try Chapter Review Questions 9 and 10.

PRACTICE Questions

Lesson 5.1

1. Determine x to the nearest unit and angle θ to the nearest degree.

a)



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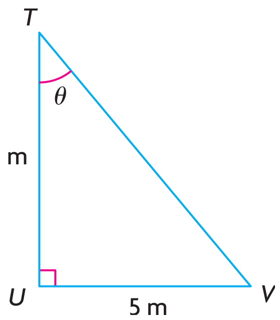


2. From Tony's seat in the classroom, his eyes are 1.0 m above ground. On the wall 4.2 m away, he can see the top of a blackboard that is 2.1 m above ground. What is the angle of elevation, to the nearest degree, to the top of the blackboard from Tony's eyes?

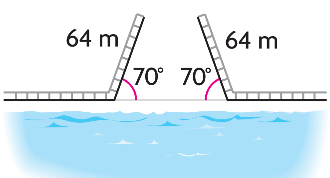
Lesson 5.2

3. A triangular garden has two equal sides 3.6 m long and a contained angle of 80°.

a) How much edging, to the nearest metre, is needed for this garden?

b) How much area does the garden cover? Round your answer to the nearest tenth of a square metre.

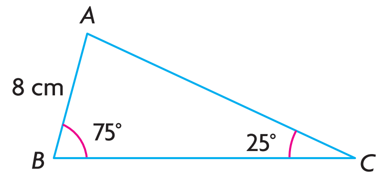
4. A Bascule bridge is usually built over water and has two parts that are hinged. If each part is 64 m long and can fold up to an angle of 70° in the upright position, how far apart, to the nearest metre, are the two ends of the bridge when it is fully open?



Lesson 5.3

5. Use the sine law to solve each triangle. Round each length to the nearest centimetre and each angle to the nearest degree.

a)



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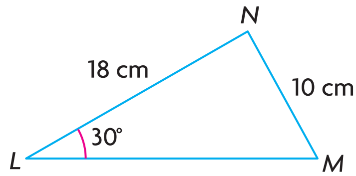

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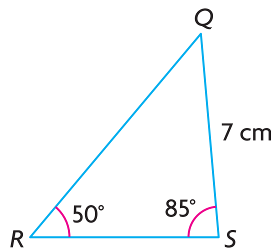
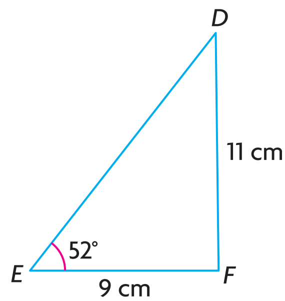
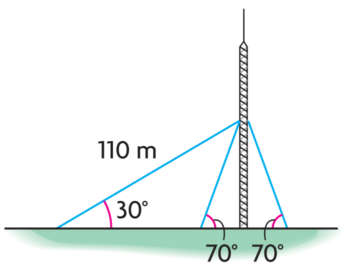


d)



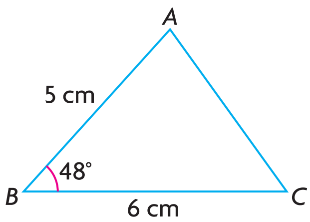
6. A temporary support cable for a radio antenna is 110 m long and has an angle of elevation of 30°. Two other support cables are already attached, each at an angle of elevation of 70°. How long, to the nearest metre, is each of the shorter cables?



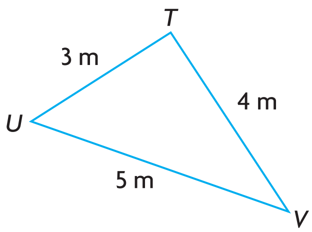
Lesson 5.4

7. Use the cosine law to calculate each unknown side length to the nearest unit and each unknown angle to the nearest degree.

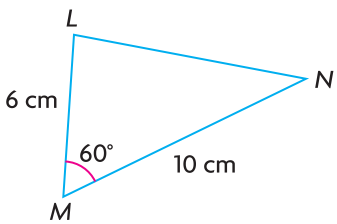
a)



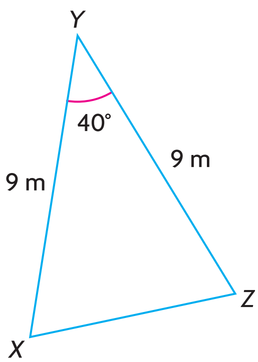
b)



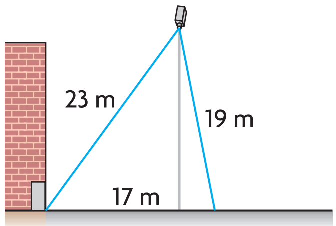
c)



d)



8. A security camera needs to be placed so that both the far corner of a parking lot and an entry door are visible at the same time. The entry door is 23 m from the camera, while the far corner of the parking lot is 19 m from the camera. The far corner of the parking lot is 17 m from the entry door. What angle of view for the camera, to the nearest degree, is required?



Lesson 5.5

9. Sketch and solve each triangle. Round your answers to the nearest degree and to the nearest tenth of a centimetre.

a) $\triangle ABC$: $\angle B = 90°$, $\angle C = 33°$, $b = 4.9$ cm

b) $\triangle DEF$: $\angle E = 49°$, $\angle F = 64°$, $e = 3.0$ cm

c) $\triangle GHI$: $\angle H = 43°$, $g = 7.0$ cm, $i = 6.0$ cm

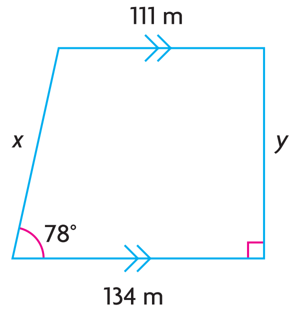
d) $\triangle JKL$: $j = 17.0$ cm, $k = 18.0$ cm, $l = 21.0$ cm

10. Two sides of a parallelogram measure 7.0 cm and 9.0 cm. The longer diagonal is 12.0 cm long.

a) Calculate all the interior angles, to the nearest degree, of the parallelogram.

b) How long is the other diagonal? Round your answer to the nearest tenth of a centimetre.

5 Chapter Self-Test



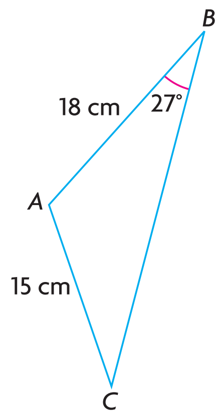
1. A 3 m ladder can be used safely only at an angle of 75° with the horizontal. How high, to the nearest metre, can the ladder reach?

2. A road with an angle of elevation greater than 4.5° is steep for large vehicles. If a road rises 61 m over a horizontal distance of 540 m, is the road steep? Explain.

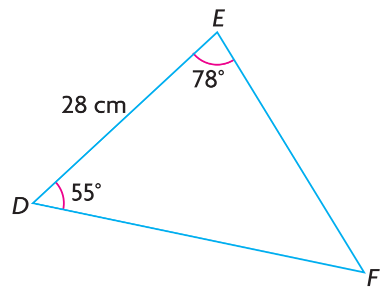
3. A surveyor has mapped out a property as shown at the left. Determine the length of sides x and y to the nearest metre.

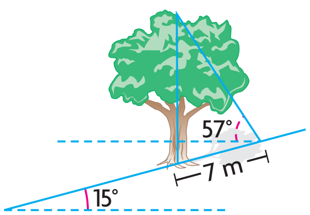
4. Solve each triangle. Round each length to the nearest centimetre and each angle to the nearest degree.

a)



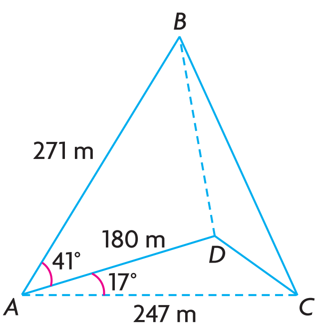
b)





5. A 5.0 m tree is leaning 5° from the vertical. To prevent it from leaning any farther, a stake needs to be fastened 2 m from the top of the tree at an angle of 60° with the ground. How far from the base of the tree, to the nearest metre, must the stake be?

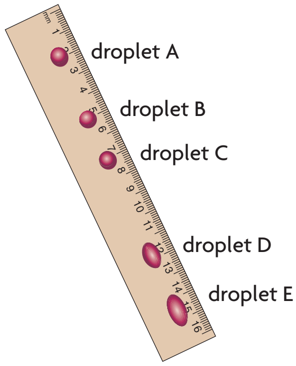
6. A tree is growing vertically on a hillside that is inclined at an angle of 15° to the horizontal. The tree casts a shadow uphill that extends 7 m from the base of its trunk when the angle of elevation of the Sun is 57°. How tall is the tree to the nearest metre?



7. Charmaine has planned a nature walk in the forest to visit four stations: A, B, C, and D. Use the sketch shown at the left to calculate the total length, to the nearest metre, of the nature trail, from A to B, B to C, C to D, and D back to A.

8. A weather balloon at a height of 117 m has an angle of elevation of 41° from one station and 62° from another. If the balloon is directly above the line joining the stations, how far apart, to the nearest metre, are the two stations?

Crime Scene Investigator

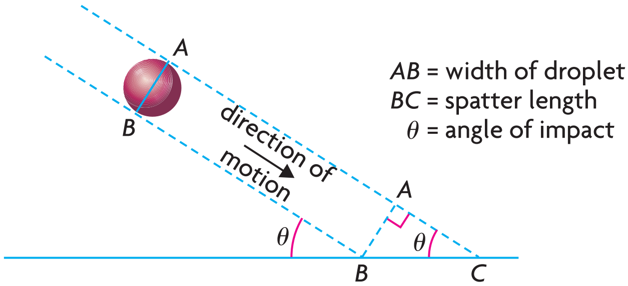


A great deal of mathematics and science is used in crime scene investigation. For example, a blood droplet is spherical when falling. If the droplet strikes a surface at 90°, it will form a circular spatter. If it strikes a surface at an angle, the spatter spreads out.

Suppose blood droplets, each 0.8 cm wide, were found on the ruler shown at the right.

? How far above the ruler, to the nearest centimetre, did each droplet originate?

A. Use the diagram below to determine a relationship to calculate the angle of impact, θ, given a droplet of width AB.

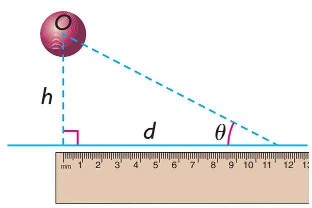


B. Which droplet appears most circular? What is the length of that droplet on the ruler, to the nearest tenth of a centimetre? What would be the angle of impact for that droplet, to the nearest degree? Create a table like the one shown and record your results.

Droplet	Original Width	Length	sin θ	θ
A	0.8 cm			
B	0.8 cm			

C. Measure the length of the other four droplets. Use your relationship from part A to determine θ for each droplet.

D. If a droplet strikes the ruler head-on, the droplet originated from distance h. If the droplet strikes the ruler at some angle of impact, the length of the droplet on the ruler would be d. Use the diagram at the right to determine the distance h for droplets A to E.



E. How far above the ruler did each droplet originate? Explain what happened with droplet B. Where appropriate, round to the nearest tenth of a centimetre.

Task Checklist

- ✔ Did you draw correct sketches for the problem?
- ✔ Did you show your work?
- ✔ Did you provide appropriate reasoning?
- ✔ Did you explain your thinking clearly?