# Functions 11

Teacher

*Course Notes*

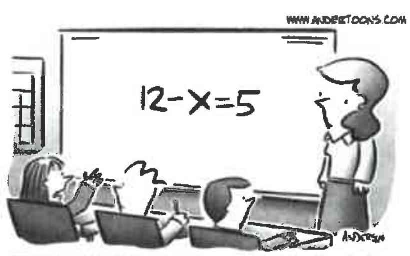
# Chapter 1 – Introduction to Functions

***TWO NUMBERS CAN DESCRIBE THE UNIVERSE***

*We will learn*

- *the meaning of the term Function and how to use function notation to calculate and represent functions*
- *the meanings of the terms domain and range, and how a function's structure affects domain and range*
- *how to use transformations to represent and sketch graphs*
- *how to determine the inverse of a function*

change cartoon




"This time X is 7. Last time it was 4. And the time before that it was 12. Maybe X should take some time to reflect and decide for itself what it really wants to be."

A∞Ω
MATH@TD

# Chapter 2 – Polynomial & Rational Expressions

*Contents with suggested problems from the Nelson Textbook. These problems are not going to be checked, but you can ask me any questions about them that you like.*

**Section 1.1**

Pg. 10 – 12 #1, 2 (*no ruler needed…*), 6, 7, (*no need for the VLT, but do sketch graphs even if you use Desmos to do the sketching*!), 9, 11, 12 (*think carefully about the idea that the domain and range are "limited"*)

**Section 1.2**

Page 23 #1-2, 5, 8b, 10, 11cd, 15, 16, challenge #17

**Section 1.3/1.4**

READ Examples 3 and 4 on pages 32 – 34 in your text

Pg. 35 – 37 #2 (also: *which are functions?*), 9bce, 11 (*use a graphing calculator, or Desmos if you want!*), 12, 14 (*calculate the functional values for each given domain value*)

**Section 1.5**

Pg. 47 – 49 #1, 8, 10, 16, 17

Also, determine the inverse (your method of choice) of:

a) $f(x)=2\sqrt{x-3}+5$ b) $g(x)=\dfrac{1}{x+3}$ c) $h(x)=\dfrac{1}{2}(x+3)^2-1$

**Section 1.6-1.8**

Handout (which will be handed in) and Pg. 70 #18

OR

Pg. 70 – 73 #4 (*state the transformations*), 5bd, 6 (*state the transformations*), 7b, 8c, 9a, 10 (*state the transformations*), 16, 17, 18, 19ac

# Chapter 1 – Introduction to Functions

## 1.1 Relations and Functions (*This is a KEY lesson*!)

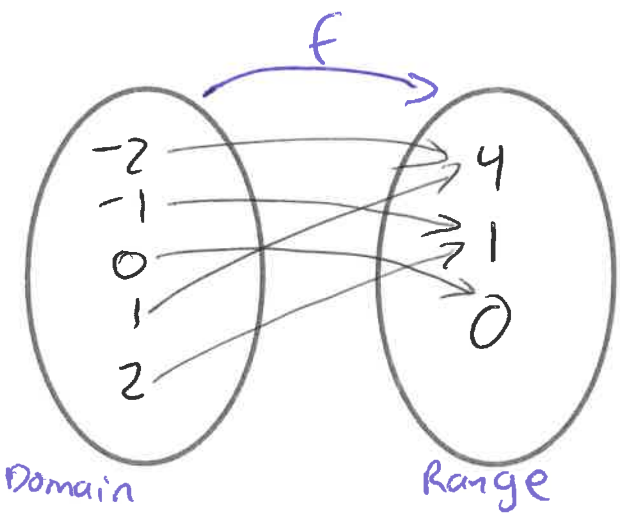
**Learning Goal:** We are learning to recognize functions in various representations.

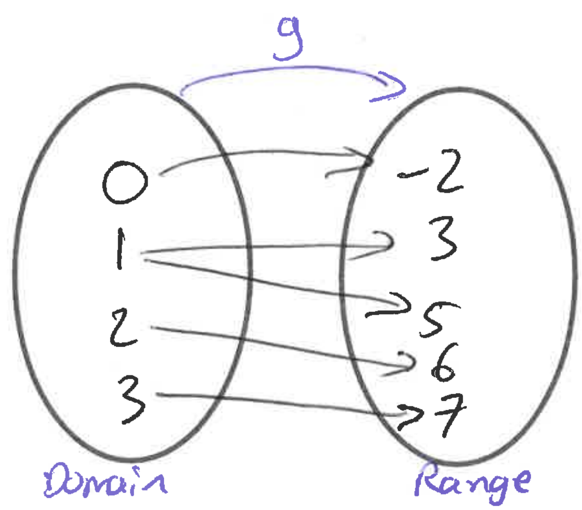
This course is called FUNCTIONS, so it seems rather important that you know what a function actually is. Thus you need to know, very well, the following (algebraic) definition:

**Definition 1.1.1**

A FUNCTION is an algebraic rule which connects two sets of numbers in a special way. A Fn assigns exactly one member (#) in the set called RANGE to each member (#) in the set called DOMAIN

We can visualize what a function is (and **isn't**) by using so-called "**arrow diagrams**":




$D_f: \{-2, -1, 0, 1, 2\}$

$R_f: \{0, 1, 4\}$

f IS a Fn, Each domain value gets exactly ONE range value. (is assigned)

$D_g: \{0, 1, 2, 3\}$

$R_g: \{-2, 3, 5, 6, 7\}$

we call g a relation.

g IS NOT a Fn. The domain value "1" is assigned two range values the set of real #'s

We need a few more definitions before moving on, so that we can "speak the language" of functions (and that language is mathematics!)

e.g.: $\{x \in \mathbb{R} \mid \underbrace{x \geq -3}\}$

belongs to

Think # line. Represents the sets

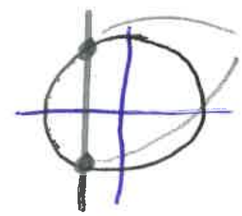
Note: This set has infinite members, greater than/equal to -3.

**Definition 1.1.2**

A SET is is a collection of objects. In math, we talk about sets of numbers.

**Definition 1.1.3**

A RELATION is a relationship between domain & range which is not a function. A domain value may have more than one range value.



two points w/ same domain value.

**Definition 1.1.4**

The DOMAIN of a function (or a relation) is

The set of numbers we are "allowed" to put into our function/relation

$f(x) = \sqrt{x}$

$f(4) = \sqrt{4} = 2$ yay

$f(-4) = \sqrt{-4}$ = Can't do! Break the universe!

**Definition 1.1.5**

The RANGE of a function (or a relation) is

The set of numbers calculated using the equation describing the function/relation

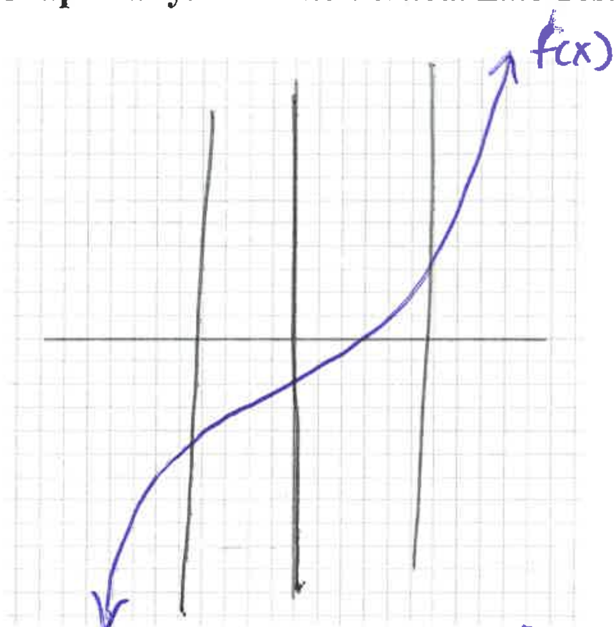
Two other important terms to know are:

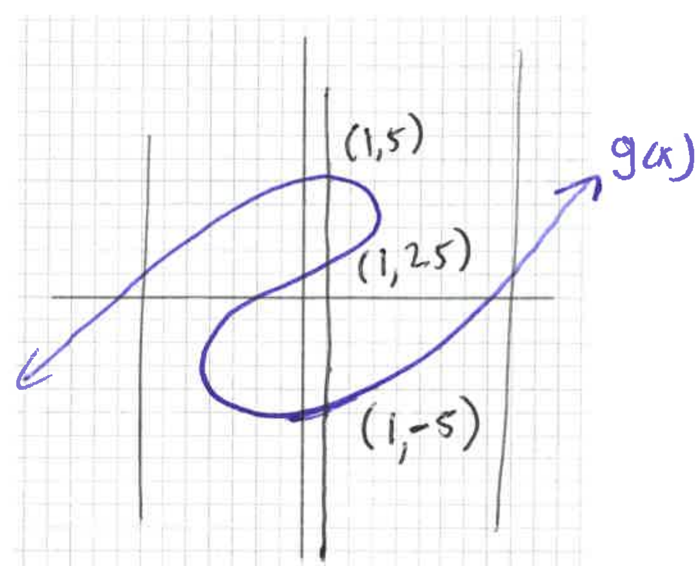
1) The **INDEPENDENT VARIABLE** is the domain "x variable"

2) The **DEPENDENT VARIABLE** is the range "f(x)" or "y"

# Knowing when a Relation is, and isn't, a Function

**Graphically**: *The Vertical Line Test* (VLT)




IS a Fn. Passes the VLT
Each Domain has only 1 range value.

NOT a fn. Fails the V.L.T.
A given domain value has multiple range values.

**Algebraically**: (*NOTE: this is a "rough" way of thinking about the problem*)

If the Dependent Variable is raised to an even power, the relation is NOT a function
(the "y")

e.g.

$y^3 = x - 5$
FN!
$y = \sqrt[3]{x-5}$

$y^2 + x = 2$
NOT a Fn.
$\Rightarrow y^2 = -x + 2$
$y = \pm\sqrt{-x+2}$

**Success Criteria:**

- I can determine the domain of a relation or function as the set of all values of the independent variable
- I can determine the range of a relation or function as the set of all values of the dependent variable
- I can apply the vertical line test to determine if a graph is a function
- I can recognize whether a relation is a function from its equation

# Chapter 1 – Introduction to Functions

## 1.2 Function Notation

**Learning Goal:** We are learning to use function notation to represent linear and quadratic functions

Here we learn a **NEW AND IMPROVED WAY** for describing a function, algebraically. You have been using the following form for functions (in this example, for a quadratic):

$$y = 3(x-2)^2 + 1$$

A much more useful way of writing function is to use **FUNCTION NOTATION**. The above quadratic (***which we call a "function of x" because the domain is given as x-values***) can be written as:

$f(x) = 3(x-2)^2+1$ $\qquad y = f(x)$

This new notation is so useful because the "symbol"

shows **BOTH** the **DOMAIN** and the **RANGE** values. Because of that, the function notation shows us **points** on the graph of the function. See next example

Let's do some examples (from your text on pages 23 – 24)

**Example 1.2.1**

**4.** Evaluate $f(-1)$, $f(3)$, and $f(1.5)$ for

**a)** $f(x) = (x-2)^2 - 1$ **b)** $f(x) = 2 + 3x - 4x^2$

$f(-1) = ((-1)-2)^2 - 1$
$= (-3)^2 - 1$
$\boxed{f(-1) = 8}$

This gives the point
$\boxed{(-1, 8)}$

$f(3) = (3-2)^2 - 1$
$= (1)^2 - 1$
$f(3) = 0$

$(3, 0)$

$f(1.5) = 2+3(1.5)-4(1.5)^2$
$= 2 + 4.5 - 9$
$= -2.5$

$(1.5, -2.5)$

$f(0) = (0-2)^2 - 1$

can't go further

Discrete Function (no line)
we can simply list the values

**Example 1.2.2**

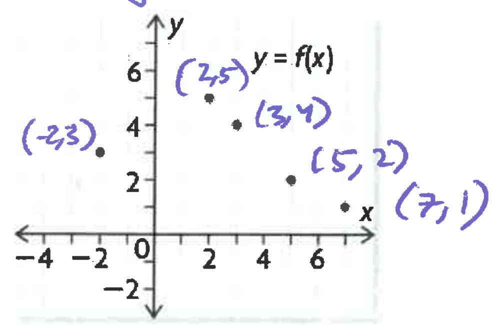
**6.** The graph of $y = f(x)$ is shown at the right.

**a)** State the domain and range of $f$.

**b)** Evaluate.

**i)** $f(3)$ **iii)** $f(5 - 3)$

**ii)** $f(5)$ **iv)** $f(5) - f(3)$




$D_f: \{-2, 2, 3, 5, 7\}$

$R_f: \{3, 5, 4, 2, 1\}$

i) $f(3) = 4$

ii) $f(5) = 2$

iii) $f(5-3) = f(2) = 5$

iv) $f(5) - f(3)$
$= 2 - 4$
$= -2$

Points on a graph look like
(domain, range)
$(x, f(x))$

**Example 1.2.3**

**11.** For $g(x) = 4 - 5x$, determine the input for $x$ when the output of $g(x)$ is

Domain Range

**a)** $-6$ **b)** $2$

These ARE $g(x)$

a) $g(x) = 4 - 5x$
$-6 = 4 - 5x$
$-10 = -5x$
$\frac{-10}{-5} = \frac{-5x}{-5}$
$2 = x$
$(2, -6)$

$2 = 4 - 5x$
$-2 = -5x$
$\frac{-2}{-5} = x$
$\frac{2}{5} = x$
$(\frac{2}{5}, 2)$

**Example 1.2.4**

7. For $h(x) = 2x - 5$, determine

a) $h(a)$ c) $h(3c - 1)$

b) $h(b + 1)$ d) $h(2 - 5x)$

a) $h(a) = 2a - 5$

b) $h(b+1) = 2(b+1) - 5$
$= 2b + 2 - 5$
$= 2b - 3$

c) $h(3c-1) = 2(3c-1) - 5$
$= 6c - 2 - 5$
$= 6c - 7$

d) $h(2-5x) = 2(2-5x) - 5$
$= 4 - 10x - 5$
$= -10x - 1$

**Example 1.2.5**

12. A company rents cars for \$50 per day plus \$0.15/km.

a) Express the daily rental cost as a function of the number of kilometres travelled. "C" "d" for distance

b) Determine the rental cost if you drive 472 km in one day.

c) Determine how far you can drive in a day for \$80.

a) $C(d)$ = kms + the \$50 / day

$\boxed{C(d) = 0.15d + 50}$ (Assuming a 1-day rental)

b) $C(472) = 0.15(472) + 50$
$= \$120.80$

c) $C(d) = 80 = 0.15d + 50$
$30 = 0.15d$
$200 = d$
∴ we can drive 200 km

**Success Criteria:**

- I can evaluate functions using function notation, by substituting a given value for x in the equation for f(x)
- I can recognize that f(x) = y corresponds to the coordinate (x, y)
- I can, given y = f(x), determine the value of x

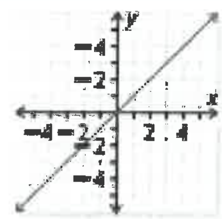
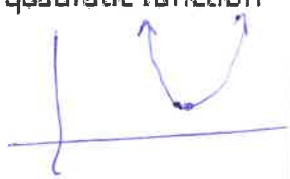
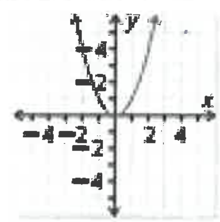
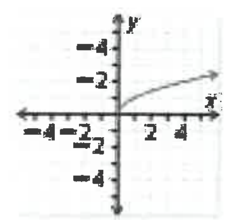
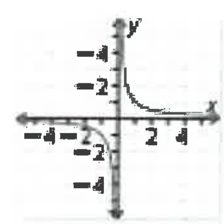
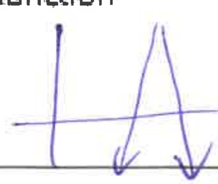
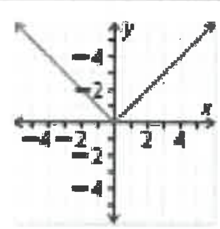
Points = (domain, range)
( x, f(x))

# Chapter 1 – Introduction to Functions

## 1.3 and 1.4 Parent Functions and Domain and Range

**Learning Goal:** We are learning the graphs and equations of five basic functions; and using their tables, graphs, or equations to find their domains and ranges.

We will be closely studying **5 TYPES OF FUNCTIONS** (*Actually we'll study more than the following five, but for now….the big five are:* )

| Equation of Function | Name of Function | Sketch of Graph |
|---|---|---|
| $f(x) = x$ <br> $g(x) = 3x-2$ | linear function |  |
| $f(x) = x^2$ <br> $h(x) = 2(x-5)^2+1$ <br> vertex @ (5, 1) | quadratic function <br>  |  |
| $f(x) = \sqrt{x}$ <br> $p(x) = -\frac{1}{2}\sqrt{3x+7}-5$ | square root function |  |
| $f(x) = \frac{1}{x}$ <br> $r(x) = \frac{3}{2(x+1)}+7$ | reciprocal function |  |
| $f(x) = \lvert x\rvert$ <br> $q(x) = -3\lvert 2x-5\rvert+11$ | absolute value function <br>  |  |

# DOMAIN AND RANGE

Two **INCREDIBLY IMPORTANT** aspects of functions are their Domain + Range

"where stuff happens"

what is happening

Again, the Domain is the set of input values (independent)

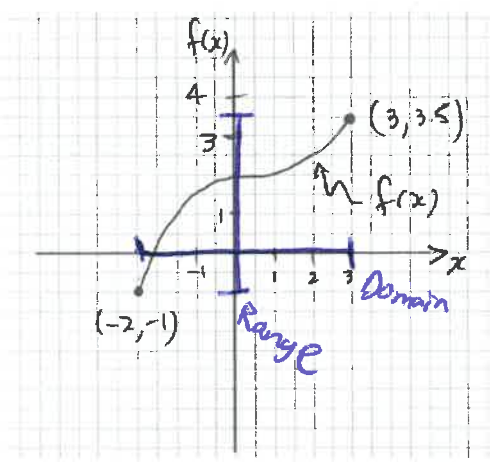
And, the Range is the set of output values (dependent

**Example 1.4.1**

Given the **SKETCH OF THE GRAPH** of the **RELATION** determine: the domain, the range of the relation, and whether the relation is, or is not, a function.

a)

fn = ✓
passes VLT

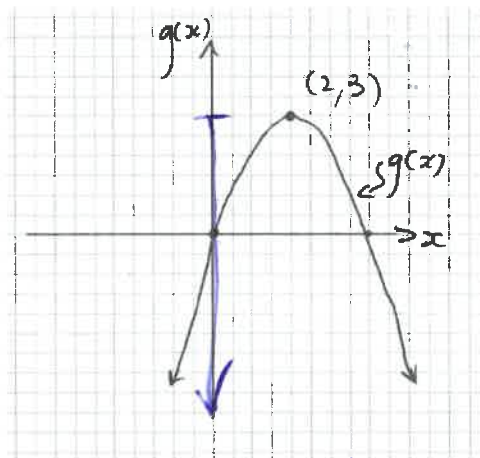



$D_f: \{x \in \mathbb{R} \mid -2 \le x \le 3\}$

$R_f: \{f(x) \in \mathbb{R} \mid -1 \le f(x) \le 3.5\}$

b)

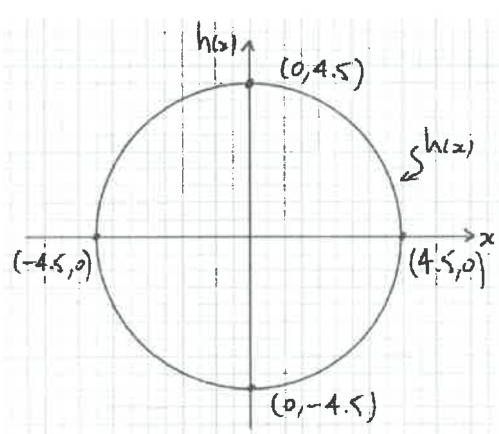
fn = ✓




$D_g: \{x \in \mathbb{R}\}$

$R_g: \{g(x) \in \mathbb{R} \mid g(x) \le 3\}$

c)




Not a Fn.
Is a relation.

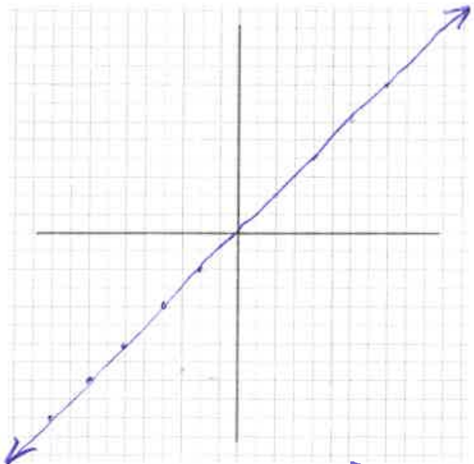
$D_h: \{x \in \mathbb{R} \mid -4.5 \le x \le 4.5\}$

$R_h: \{h(x) \in \mathbb{R} \mid -4.5 \le h(x) \le 4.5\}$

THE PARENT FUNCTIONS (*for Grade 11*)

Together we will explore (graphically) basic properties of the five $\boxed{\textit{parent}}$ functions:

a) Linear $f(x)=x$

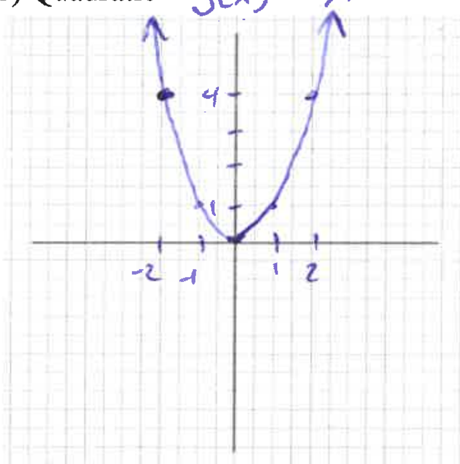


| $x$ | $f(x)$ | Points |
|---|---|---|
| $-2$ | $-2$ | $(-2,-2)$ |
| $-1$ | $-1$ | $(-1,-1)$ |
| $0$ | $0$ | $(0,0)$ |
| $1$ | $1$ | $(1,1)$ |
| $2$ | $2$ | $(2,2)$ |

$D_f: \{x \in \mathbb{R}\}$

$R_f: \{f(x) \in \mathbb{R}\}$

b) Quadratic $g(x)=x^2$

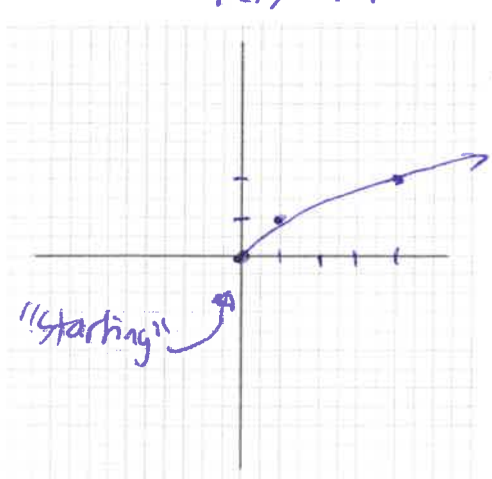



| $x$ | $x^2$ | Points |
|---|---|---|
| $-2$ | $4$ | $(-2,4)$ |
| $-1$ | $1$ | $(-1,1)$ |
| $0$ | $0$ | $(0,0)$ |
| $1$ | $1$ | $(1,1)$ |
| $2$ | $4$ | $(2,4)$ |

$D_g: \{x \in \mathbb{R}\}$

$R_g: \{g(x) \in \mathbb{R} \mid g(x) \geq 0\}$

c) Square Root $r(x)=\sqrt{x}$

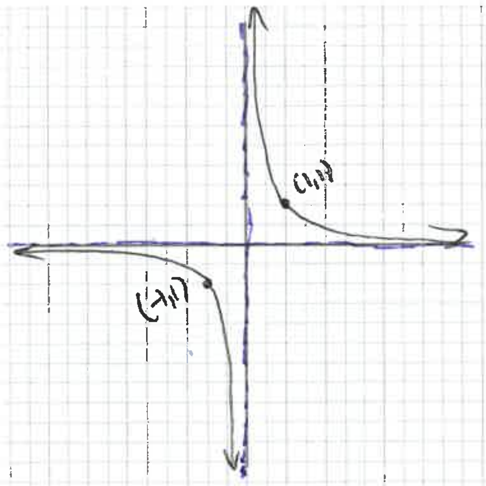



| $x$ | $\sqrt{x}$ | Points |
|---|---|---|
| $0$ | $0$ | $(0,0)$ |
| $1$ | $1$ | $(1,1)$ |
| $4$ | $2$ | $(4,2)$ |
| $9$ | $3$ | $(9,3)$ |

$D_r: \{x \in \mathbb{R} \mid x \geq 0\}$

$R_r: \{r(x) \in \mathbb{R} \mid r(x) \geq 0\}$

d) Reciprocal $h(x) = \frac{1}{x}$

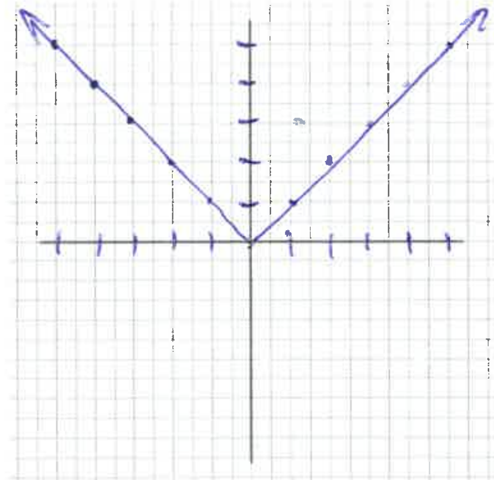



| $x$ | $\frac{1}{x}$ |
|---|---|
| $-2$ | $-\frac{1}{2}$ |
| $-1$ | $-1$ |
| $0$ | Error |
| $1$ | $1$ |
| $2$ | $\frac{1}{2}$ |

$D_h: \{x \in \mathbb{R} \mid x \neq 0\}$

$R_h: \{h(x) \in \mathbb{R} \mid h(x) \neq 0\}$

e) Absolute Value $m(x) = |x|$



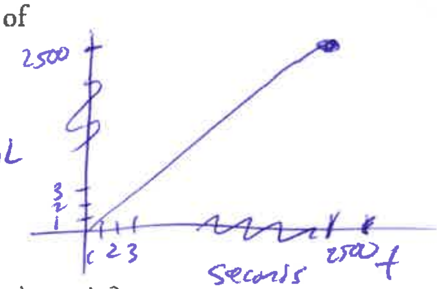
| $x$ | $\lvert x \rvert$ |
|---|---|
| $-2$ | $2$ |
| $-1$ | $1$ |
| $0$ | $1$ |
| $1$ | $1$ |
| $2$ | $2$ |

$D_m: \{x \in \mathbb{R}\}$

$R_m: \{m(x) \in \mathbb{R} \mid m(x) \geq 0\}$

**Example 1.4.2** (*From Pg. 36 in your text*)

**8.** Write a function to describe coffee dripping into a 10-cup carafe at a rate of 1 mL/s. State the domain and range of the function (1 cup = 250 mL).




Let $t$ be time (domain)

Let $V(t)$ be range

then, $V(t) = t$

250 mL × 10 cups = 2500 mL and 2500 seconds total

$D_V = \{t \in \mathbb{R} \mid 0 \leq t \leq 2500\}$

$R_V = \{V(t) \in \mathbb{R} \mid 0 \leq t \leq 2500\}$

**Example 1.4.3** (*From Pg. 37 in your text…use Desmos*)

**9.** Determine the domain and range of each function. Show Graph

a) $f(x) = -3x + 8$

$D: \{x \in \mathbb{R}\}$

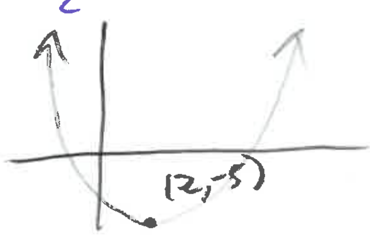
$R: \{f(x) \in \mathbb{R}\}$

All linear functions have full domain/range (except horizontal/vertical lines)

d) $p(x) = \frac{2}{3}(x - 2)^2 - 5$

$D: \{x \in \mathbb{R}\}$

$R: \{p(x) \in \mathbb{R} \mid p(x) \geq -5\}$

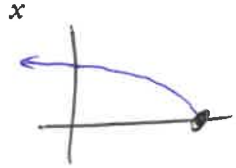



f) $r(x) = \sqrt{5 - x}$

$\sqrt{\text{positive}}$

$5 - x \geq 0$

$5 \geq x$



$D_r: \{x \in \mathbb{R} \mid x \leq 5\}$

$R_r: \{r(x) \in \mathbb{R} \mid r \geq 0\}$

**Example 1.4.4**

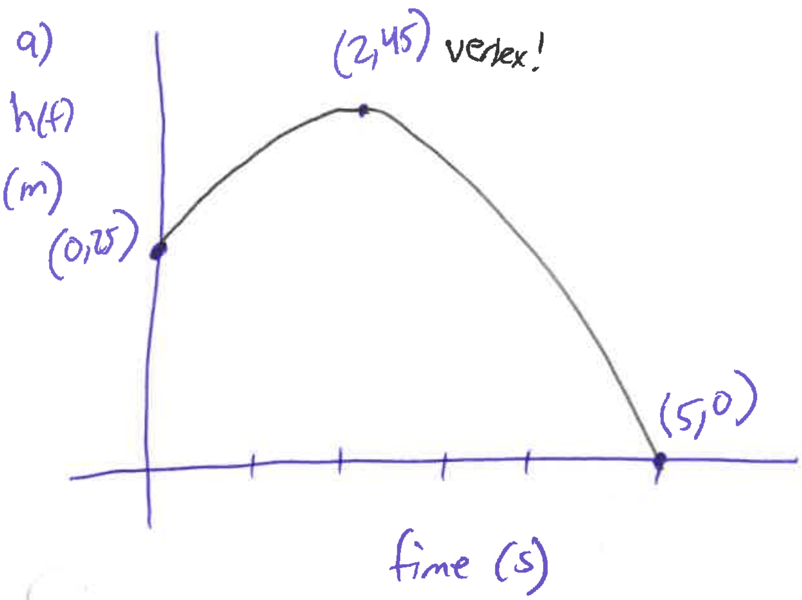
**10.** A ball is thrown upward from the roof of a 25 m building. The ball reaches a height of 45 m above the ground after 2 s and hits the ground 5 s after being thrown.

**a)** Sketch a graph that shows the height of the ball as a function of time.

**b)** State the domain and range of the function.

**c)** Determine an equation for the function.

a)




b) $D_h = \{t \in \mathbb{R} \mid 0 \le t \le 5\}$

$R_h = \{h(t) \in \mathbb{R} \mid 0 \le h(t) \le 45\}$

c) $h(t) = a(t-h)^2 + k$

where $(h,k)$ is the vertex
$a$ is some unknown stretch factor

$h(t) = a(t-2)^2 + 45$

To find $a$, use a non-vertex point $(5,0)$

$0 = a(5-2)^2 + 45$

$-45 = a(9)$

$-5 = a$

$\therefore h(t) = -5(t-2)^2 + 45$

**Success Criteria:**

- I can identify the unique characteristics of five basic types of functions
- I can identify the domain and ranges of five basic types of functions
- I can identify when there are restrictions given real-world situations

# Chapter 1 – Introduction to Functions

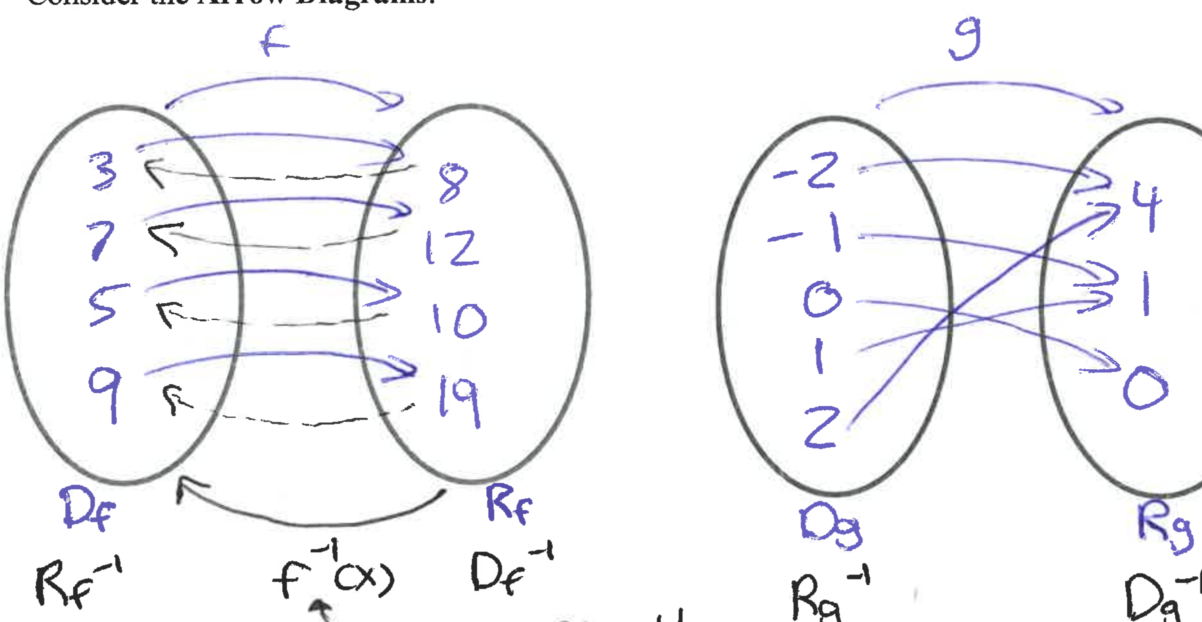
## 1.5: Inverses of Functions

**Learning Goal:** We are learning to determine inverses of functions and investigate their properties.

**Definition 1.5.1** (*very rough definition!*)

Given a function $f(x)$, the inverse of the function (which we write as $f^{-1}(x)$) can be considered to "**undo**" what $f(x)$ originally did.

Consider the **Arrow Diagrams**:




not an exponent!

Note: $g^{-1}(x)$ is NOT a fn!

**Big Idea**

Switch DOMAIN & RANGE

(switch "x" and "y")

**Example 1.5.1**

Given the graph of $f(x)$ determine: $D_f, R_f, f^{-1}(x), D_{f^{-1}}, R_{f^{-1}}$

$f(x) = \{(2,3),(4,2),(5,6),(6,2)\}$ . Is $f^{-1}(x)$ a function?

$D_f: \{2, 4, 5, 6\}$

$R_f: \{3, 2, 6\}$

$f^{-1}(x) = \{(3,2),(2,4),(6,5),(2,6)\}$

$D_{f^{-1}} = \{3,2,6\} = R_f!$

$R_{f^{-1}} = \{2,4,5,6\} = D_f!$

$f^{-1}(x)$ is NOT a fn! "2" has two range values.

## Determining the Inverse of a Function

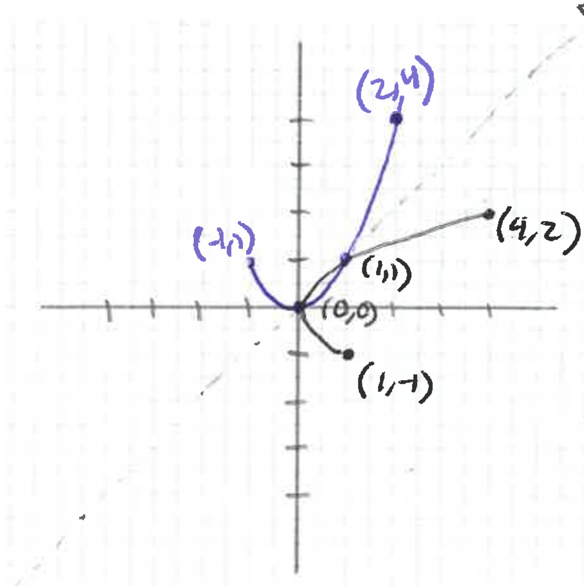
We can determine the inverse of some given function in either of two ways: Graphically and Algebraically.

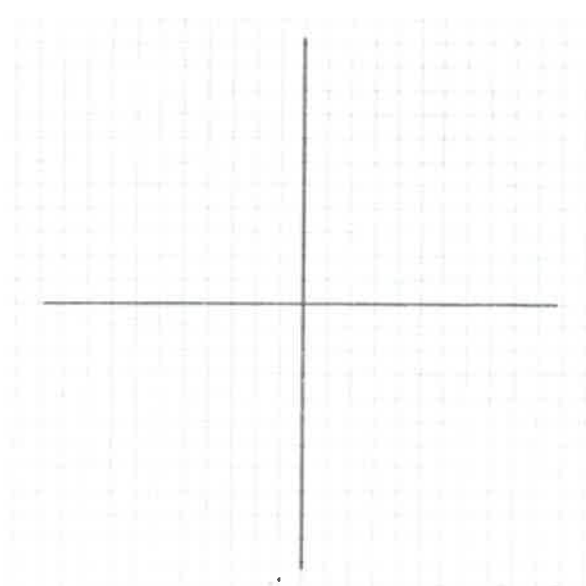
*Note: Finding a function inverse graphically is not a very useful method, but it can be instructive.*

### Function Inverses Graphically

This line is unchanged under inversion. (It is invariant)

y=x acts like a mirror!



Note: $f^{-1}(x)$ is NOT a fn. Fails VLT.

## Function Inverses Algebraically

Determining algebraic representations of inverse relations for given functions can be done in (at least) two ways:

1) Use algebra in a "brute force" manner (keeping in mind the Big Idea)
2) Use Transformations (keeping in mind "inverse operations")

**Example 1.5.2**

Determine the inverse of

a) $f(x)=2x-5$ b) $g(x)=\frac{1}{2}\sqrt{x-1}+2$.

State the domain and range of both the function and its inverse.

Here we will use "brute force".

Method:

1) Switch $x$ and $f(x)$, and call "$f(x)$", $f^{-1}(x)$.
   "x" "y"
2) Solve for $f^{-1}(x)$

a) f(x) linear $D_f=\{x\in\mathbb{R}\}$

$R_f=\{f(x)\in\mathbb{R}\}$

$y=2x-5$

$\downarrow$

$x=2y-5$

$x+5=2y$

$\frac{x+5}{2}=y$

$$\boxed{f^{-1}(x)=\frac{x}{2}+\frac{5}{2}}$$

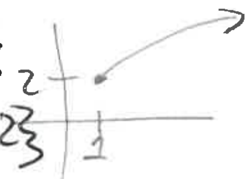
$D_{f^{-1}}=\{x\in\mathbb{R}\}$

$R_{f^{-1}}=\{f^{-1}(x)\in\mathbb{R}\}$

* The inverse of a linear function is linear.

b. $D_f:\{x\in\mathbb{R}\mid x\geq 1\}$

$R_f:\{f(x)\in\mathbb{R}\mid f(x)\geq 2\}$




$y=\frac{1}{2}\sqrt{x-1}+2$

$\downarrow$

$x=\frac{1}{2}\sqrt{y-1}+2$

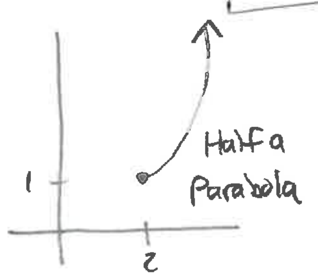
$x-2=\frac{1}{2}\sqrt{y-1}$

$2(x-2)=\sqrt{y-1}$

$2(x-2)^2=y-1$

$2(x-2)^2+1=y$

$$\boxed{g^{-1}(x)=2(x-2)^2+1}$$




$D_{g^{-1}}:\{x\in\mathbb{R}\mid x\geq 2\}$

$R_{g^{-1}}:\{g^{-1}(x)\in\mathbb{R}\mid g^{-1}(x)\geq 1\}$

**Example 1.5.3**

Using transformations determine the inverse of $f(x)=2\sqrt{\frac{1}{3}(x-1)}+2$.

① Hor transformations become vertical
② vertical " " horizontal
③ The type of function inverses
(squares become quadratics)

Other: $+$ becomes $-$
$\div$ becomes $\times$
and vice-versa

So, f(x) is square root

| V | | H | |
|---|---|---|---|
| Stretch | $\times 2$ | Stretch | $\div 3$ |
| Shift | $+2$ | Shift | $-1$ |

$f^{-1}(x)$ is quadratic

| V | | H | |
|---|---|---|---|
| Stretch | $\times 3$ | Stretch | $\div 2$ |
| Shift | $+1$ | Shift | $-2$ |

So, $f^{-1}(x) = 3\left(\frac{1}{2}(x-2)\right)^2 + 1$

when solving reciprocal functions, use brute force to find $f^{-1}(x)$

$f(x) = \frac{1}{x}$

$\downarrow$

$x = \frac{1}{y}$

$xy = 1$

$y = \frac{1}{x}$

$\boxed{f^{-1}(x) = \frac{1}{x}}$

Inverses of reciprocals are reciprocals!

**Success Criteria:**

- I can determine the inverse of a function using various techniques
- I can determine the inverse of a coordinate (a , b) by switching the variables: (b , a)
- I can recognize that the domain of an inverse is the range of the original function
- I can recognize that the range of an inverse is the domain of the original function
- I can understand that the inverse of a function is a reflection along the line y = x

# Chapter 1 – Introduction to Functions

## 1.6 – 1.8: Transformations of Functions *(Part 1)*

**Learning Goal:** We are learning to apply combinations of transformations in a systematic order to sketch graphs of functions.

To **TRANSFORM** something is to CHANGE (or move)

**TRANSFORMATIONS OF FUNCTIONS** can be seen in two ways: algebraically, and graphically. We'll begin by examining transformations graphically.

But before we do, we need to remember that the **GRAPH OF A FUNCTION**, $f(x)$, is given by:

$$f(x)=\left\{\left(x,f(x)\right)\middle|\, x\in D_f\right\}$$

So, for functions we have two things (NUMBERS!) to "transform". We can apply transformations to

1) **Domain** values (which we call **HORIZONTAL TRANSFORMATIONS**)
2) **Range** values (which we call **VERTICAL TRANSFORMATIONS**)

There are **THREE BASIC FUNCTIONAL TRANSFORMATIONS**

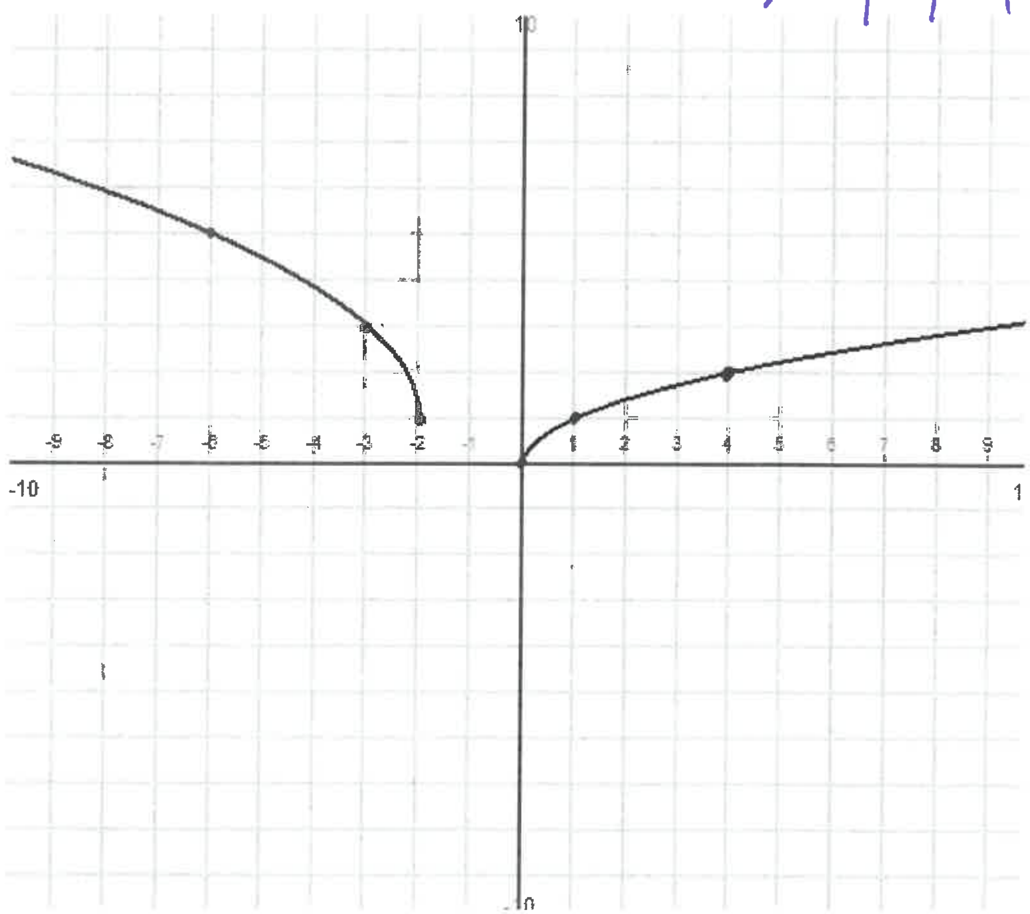
1) Flips (*Reflections "around" an axis*) } multiplying
2) Stretches (*Dilations*) }
3) Shifts (*Translations*) – addition

So, we can have **Horizontal** flips, stretches and/or shifts, and **Vertical** flips, stretches and/or shifts. Now let's take a look at how transformations can be applied to functions.

***Note*:** We'll (mostly) be applying transformations to our so-called "parent functions" (although applying transformations to linear functions can seem pretty silly!)

**Example 1.8.1**

Consider, and make observations concerning the sketch of the graph of the parent function $f(x)=\sqrt{x}$ and the transformed function $g(x)=2\sqrt{-x-2}+1$. $= 2\sqrt{-(x+2)}+1$




**Horizontal Transformations**

Flip

Shift left 2

**Vertical Transformations**

Stretch

Shift up 1

***Note***: In the above example we can **algebraically** describe $g(x)$ as a transformed $f(x)$ with the functional equation $g(x)=2f(-x-2)+1$

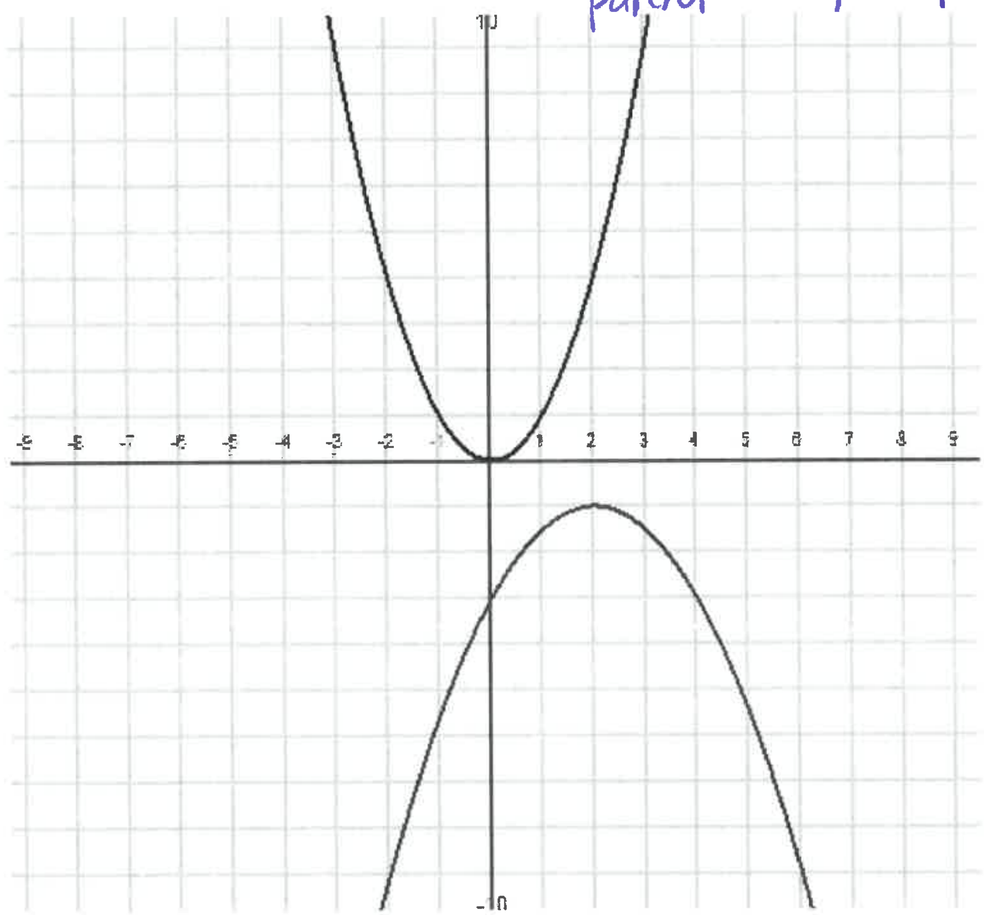
Horizontal

Vertical

Example 1.8.2

Consider, and make observations concerning the sketch of the graph of the parent function $f(x)=x^2$ and the transformed function $g(x)=-\frac{1}{2}(x-2)^2-1$

Parent




**Horizontal Transformations**

Shift Right 2

**Vertical Transformations**

Flip

Stretch (Flattened)

Shift Down 1

***Note***: In the above example we can **algebraically** describe $g(x)$ as a transformed $f(x)$ with the functional equation

$g(x) = -\frac{1}{2} f(x-2) - 1$

# Chapter 1 – Introduction to Functions

## 1.6 – 1.8: Transformations of Functions *(Part 2)*

We now turn to examining Transformations of Functions from an algebraic point of view (although a geometric perspective will still shine though!)

**Definition 1.8.1**

Given a function $f(x)$ we can obtain a related function through functional transformations as

"k" must be factored out

$$g(x)=af\left(k(x-d)\right)+c \text{, where}$$

a is the vertical stretch/dilation. If a < 0, we also have a vertical flip (reflection)

c is the vertical shift (translation)

k is the horizontal stretch with stretch factor of $\frac{1}{k}$. If k < 0, we also have a horizontal flip.

d is the horizontal translation/shift

k + d do the opposite of what you expect.

**Example 1.8.3**

Consider the given function. State its parent function, and all transformations.

$$f(x)=3\sqrt{-x+2}-1$$

Parent: $g(x)=\sqrt{x}$

$$=3\sqrt{-(x-2)}-1$$

Always factor k!

**Horizontal Transformations**

Flip? Yes!

Stretch ×1

Shift 2 right

~~Always~~ −(+2)

**Vertical Transformations**

Flip? No

Stretch ×3

Shift down 1

**Example 1.8.4**

The basic absolute value function $f(x)=|x|$ has the following transformations applied to it: **Vertical Stretch** $-3$, **Vertical Shift 1 up, Horizontal Shift 5 right.**

a c d

Determine the equation of the transformed function.

$$g(x) = -3|x-5| + 1$$

## Back to a geometric point of view

Sketching the graph of a transformed function can be relatively easy if we know:

1) The shape of the parent function AND a few (3 or 4) points on the parent.
2) How transformations affect the points on the parent
   i) Horizontal transformations affect the domain values (***OPPOSITE!!!!!!***)
   ii) Vertical transformations affect the range values

Note: Given a point on some parent function which has transformations applied to it is called an ***IMAGE POINT*** on the transformed function.

**Example 1.8.5**

Given the sketch of the function $f(x)$ determine the image points of the transformed function $-2f\left(\frac{1}{3}(x+1)\right)+3$ and sketch the graph of the transformed function.

Vertical
- flip
- stretch x2
- shift 3 up

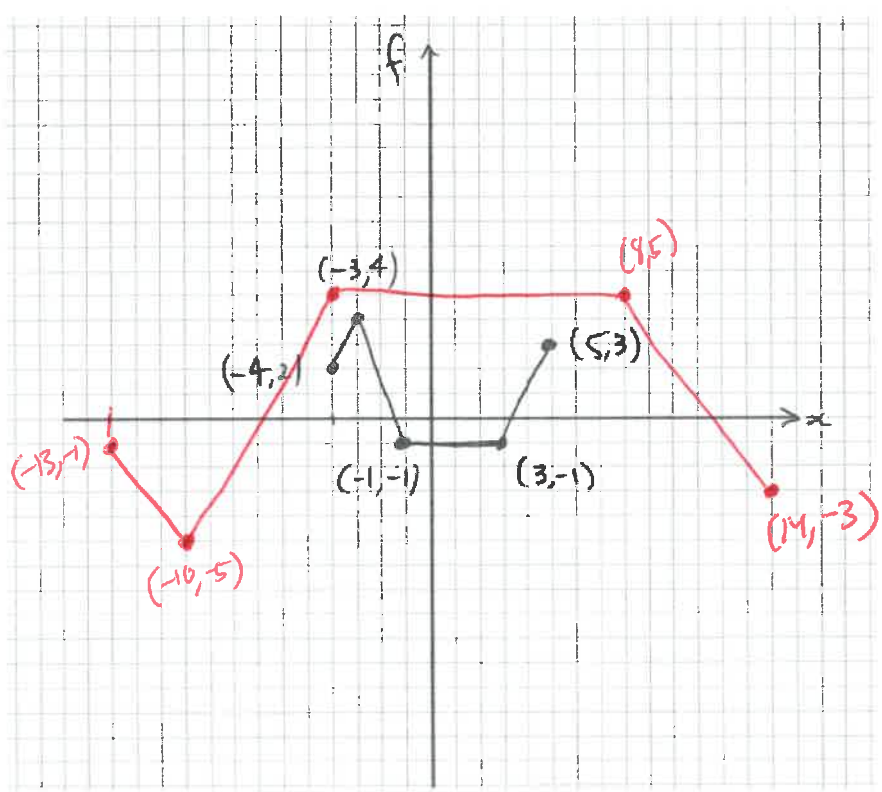
Horizontal
- stretch x3
- shift 1 left

Parent

| x | y |
|---|---|
| -4 | 2 |
| -3 | 4 |
| -1 | -1 |
| 3 | -1 |
| 5 | 3 |

Transformed

| 3x - 1 | -2y + 3 |
|---|---|
| -13 | -1 |
| -10 | -5 |
| -4 | 5 |
| 8 | 5 |
| 14 | -3 |




Note: we say (-13,-1) is the image of (-4,2)

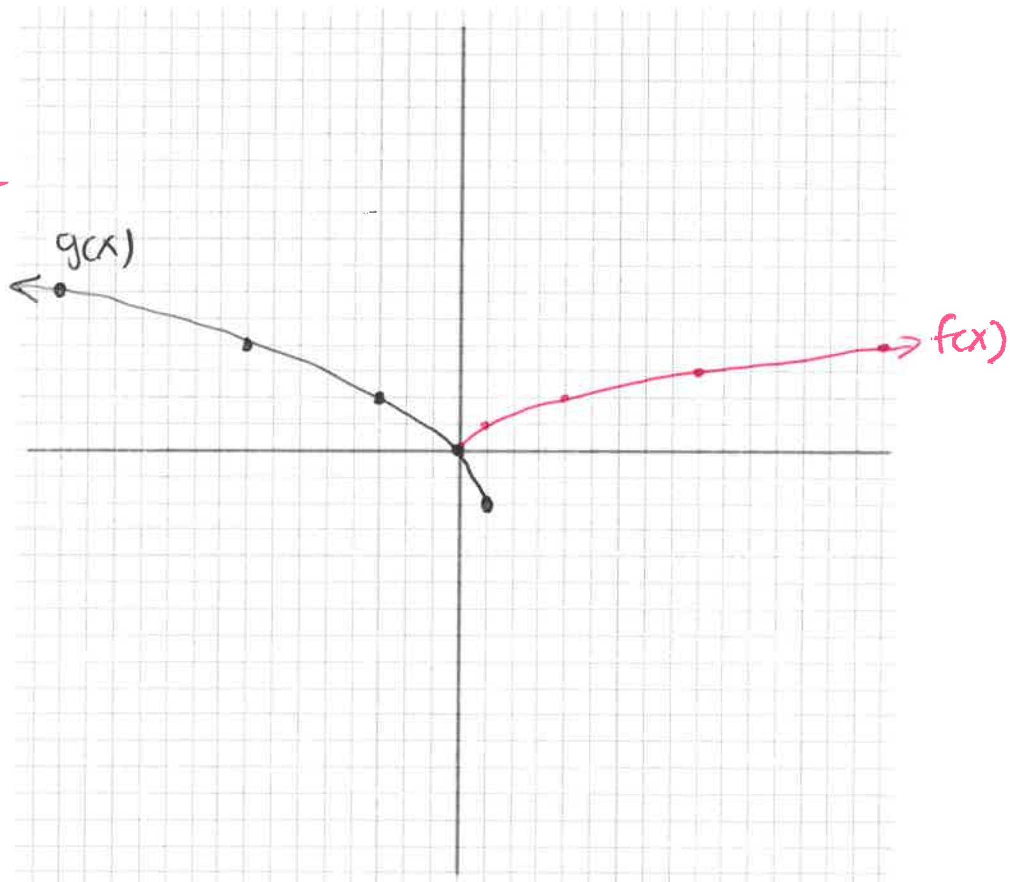
Factor k!

$g(x) = 2\sqrt{-(x-1)} - 2$

**Example 1.8.6**

On the same set of axes sketch the graphs of $f(x) = \sqrt{x}$ and $g(x) = 2\sqrt{-x+1} - 2$. Determine three points on the parent function and state the image points for each.

| Vertical | Hor. |
|---|---|
| Flip No | yes flip |
| Stretch x 2 | No stretch |
| Down 2 | Shift 1 right |




Parent

| x | f(x) |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 4 | 2 |
| 9 | 3 |
| 16 | 4 |

Transformed

| $-x+1$ | $2f-2$ |
|---|---|
| 1 | -2 |
| 0 | 0 |
| -3 | 2 |
| -8 | 4 |
| -15 | 6 |

**Success Criteria:**

- I can use the value of a to determine if there is a vertical stretch/reflection in the x-axis
- I can use the value of k to determine if there is a horizontal stretch/reflection in the y-axis
- I can use the value of d to determine if there is a horizontal translation
- I can use the value of c to determine if there is a vertical translation
- I can transform x coordinates by using the expression $\frac{1}{k}x + d$
- I can transform y coordinates by using the expression $ay + c$