Functions 11

Teacher

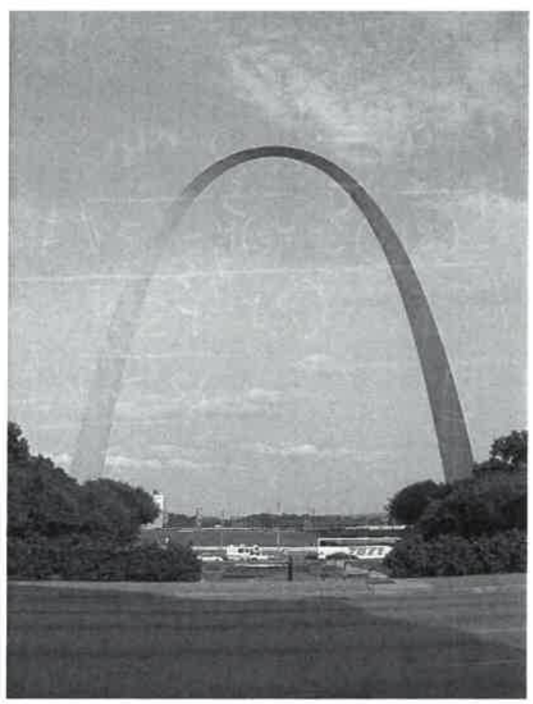
Course Notes

Unit 3 – Quadratic Functions

FUNCTIONS TO THE MAX (OR MIN...AND SOMETIMES ZERO)

We will learn

- *the meaning of a zero, and how to find them algebraically*
- *to determine the max or min value of a quadratic algebraically and graphically*
- *to sketch parabolas (using transformations, zeroes, the vertex and y-intercept)*
- *to solve real-world problems, including linear-quadratic systems*



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MATH@TD

Chapter 3 – Quadratic Functions

Contents with suggested problems from the Nelson Textbook. You are welcome to ask for help, from myself or your peers, with any of the following problems. They will be handed in on the day of the Unit Test as a homework check.

Section 3.1

Pg. 145 – 147 #3, 4, 5bc, 6 (*expand*!), 7, 8, 9de, 12 (*tricky*!)

Section 3.2

Pg. 153 #1, 3, 4abc (*one method is fine*), 6 (*Desmos*), 7bc, 8, 9 (*try Partial Factoring*), 11 (*ask for help on c if you feel the need!*)

Section 3.4

Pg. 167 – 168 #3 – 5abc, 6 – 7acef, 8 - 13

Section 3.5

Pg 177 – 178 #1bc, 2bcd, 4abef, 6cd, 7 (*Hint: what is the height of the ball when it is on the ground?*), 9, 11 (#9 and 11 are tricky – ask for help!), 14

Section 3.6

Pg 185 – 186 #1 – 3abc, 4, 6 – 9 (*these are a bit tricky…ask for help!*), 15

Section 3.7

Page 192 #4 – 6, 8 – 10

Section 3.8

Pg198 – 199 #1ab, 2ab, 3, 4bcd, 6, 8, 11 (*tangent means touching at one point!*), 12

Chapter 3 – Quadratic Functions

3.1 – Properties of Quadratic Functions

Learning Goal: We are learning to represent and interpret quadratic functions in three different forms.

This lesson is a review of some of what we learned about quadratics in Grade 10. In Grade 10 we studied the **THREE FORMS** of quadratic functions and the **information** they give:

1) Standard Form - $f(x) = ax^2 + bx + c$

Information

a is the stretch factor.

If $a < 0$, the parabola opens down

If $a > 0$, " " " up

c is the y-intercept

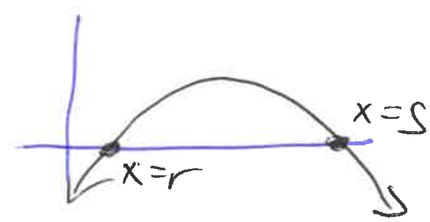
$(0, f(0)) = (0, c)$

2) Zeros (or Factored) Form - $f(x) = a(x-r)(x-s)$

Information

a is the stretch factor (same as above)

$\left.\begin{array}{l} x=r \\ x=s \end{array}\right\}$ Are the zeros of the quadratic, where the parabola crosses the x-axis.



3) Vertex Form - $f(x) = a(x-h)^2 + k$

Information

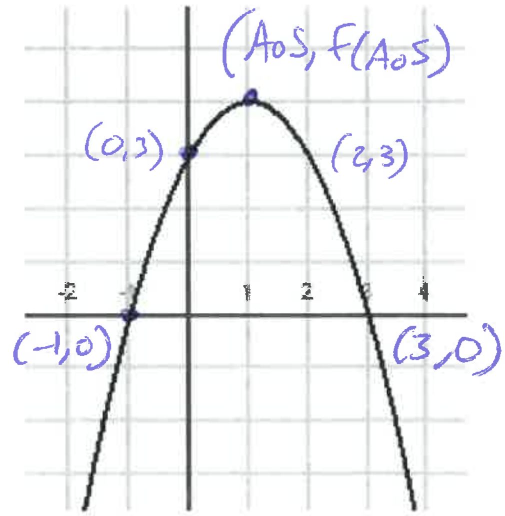
a = is the stretch factor

The point (h, k) is the vertex.

You can only read h if the equation is in $(x-h)$ form.

Recall the concept of the axis of symmetry. AoS

The AoS is a vertical line passing through the vertex. Therefore we can write the coordinates of the vertex as (AoS, f(AoS))



How to Calculate

The AoS is the "average" of the zeros.

(Or any two pts w/ the same y-value.

$$x = \frac{(-1)+(3)}{2} = 1$$

$$x = \frac{0+2}{2} = 1$$

Example 3.1.1

Given the quadratic function $f(x) = \frac{1}{2}(x+3)^2 - 1$, state:

vertex form

a) The direction the parabola opens
b) The coordinates of the vertex
c) The equation of the axis of symmetry

a) opens up since $a = +\frac{1}{2}$

b) $= (-3, -1)$

$f(x) = \frac{1}{2}(x-(\underline{-3}))^2 \underline{-1}$

c) AoS $=$ ~~(-3)+(-1)/2~~

$x = -3$

Given by the x-coordinate of the vertex.

Note: this is the equation of a vertical line.

Example 3.1.2

Given the quadratic function $g(x)=-2(x+3)(x-1)$, state

a) The direction the parabola opens ↳ zeros form

b) The zeros of the quadratic

c) The equation of the axis of symmetry

d) The coordinates of the vertex

e) The function in vertex form

Sketch the graph of the function.

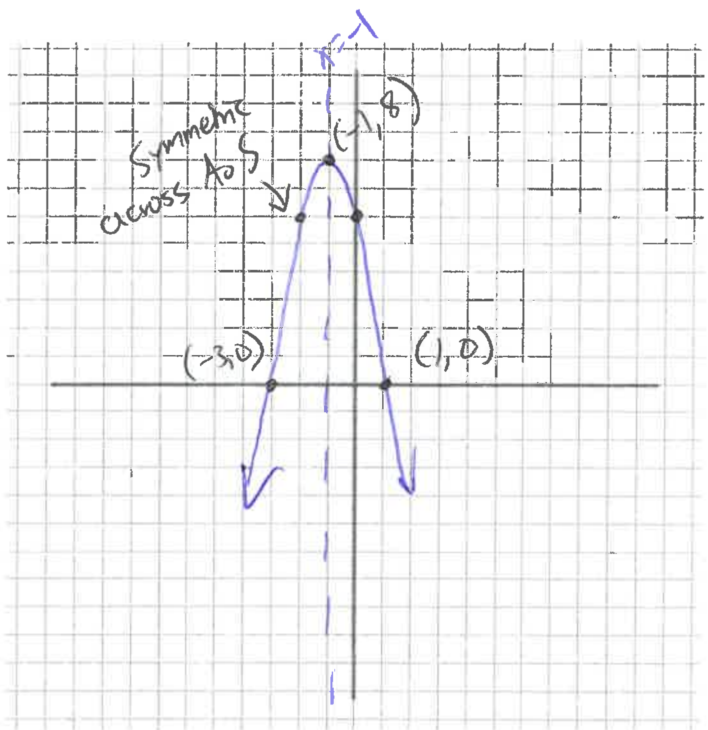
y-int @ $g(0)=6$
$(0,6)$

a) down, since $a=-2$

b) $x=-3$, $x=1$

c) AoS is avg of zeroes

$\frac{(-3)+1}{2}=\boxed{-1}$



d) vertex as (AoS, F(AoS))

$=(-1, g(-1))$

$=(-1, +8)$

e) $g(x)=a(x-h)^2+k = -2(x+1)^2+8$

To sketch I expect a min of 3 calculated pts. 5 is better.

Example 3.1.3

Given the two points $(4,7)$, $(-5,7)$ which are on a parabola, determine the equation of the axis of symmetry.

Same y-value means these are equidistant from the AoS. Find the average.

AoS $=\frac{(4+-5)}{2}=-\frac{1}{2}$

$\boxed{x=-\frac{1}{2}}$

Note: In real life we must restrict the domain & range.

Example 3.1.4 (From Pg. 147 in your text)

11. The height of a rocket above the ground is modelled by the quadratic function $h(t) = -4t^2 + 32t$, where $h(t)$ is the height in metres t seconds after the rocket was launched.

a) Graph the quadratic function.
b) How long will the rocket be in the air? How do you know?
c) How high will the rocket be after 3 s?
d) What is the maximum height that the rocket will reach?

$$h(t) = -4t^2 + 32t$$
$$= \underbrace{(-4t)(t-8)}_{\text{zeros form}}$$

Zeros @ $t = 0, t = 8$

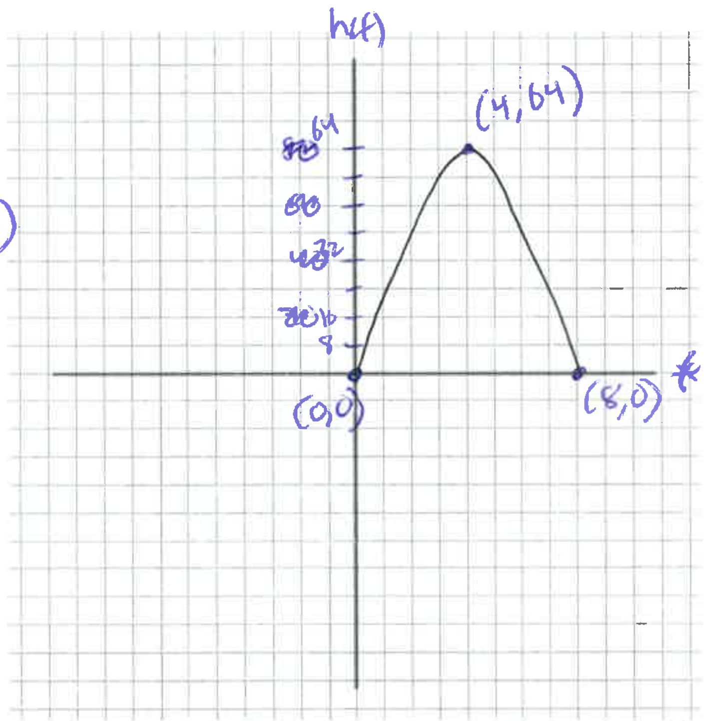
AoS: $x = \frac{0+8}{2} = 4$

vertex: $(4, h(4))$
$= (4, 64)$

b) 8 seconds

c) $h(3)$
$= -4(3)^2 + 32(3)$
$= 60\,m$

d) 64 m



$D: \{t \in \mathbb{R} \mid 0 \le t \le 8\}$ No negative time

$R: \{h(t) \in \mathbb{R} \mid 0 \le h(t) \le 64\}$

Success Criteria:

- I can recognize a quadratic function in standard, factored, and vertex form
- I can determine the zeros, direction of opening, axis of symmetry, vertex, domain and range from the graph of a parabola
- I can determine the equation of quadratic function from its parabola

Chapter 3 – Quadratic Functions

3.2 – The Maximum or Minimum of Quadratic Functions

Learning Goal: We are learning to determine the maximum/minimum value of a quadratic function.

One very important aspect of quadratic functions is that they all have either a maximum (if the associated parabola opens down) or a minimum (if the parabola opens up). **Max/Min's** have so many **applications** in the real world that it's **ridiculous**.

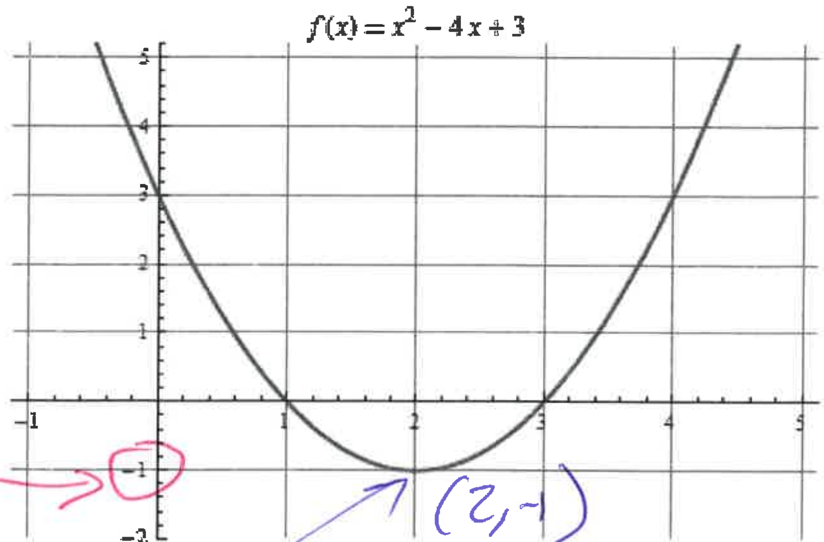
The **BIG QUESTION** we are faced with is this:

How do we find the Maximum or Minimum Value for some given Quadratic?

Example 3.2.1

To find a minimum (or maximum) of a quadratic, you are NOT allowed to

(Blue)



That is the min.
The range value!

There it is

Figure 3.2.1

Figure 3.2.1b

So, we do need to find the vertex, but we also need to **KEEP IN MIND WHAT THE NUMBERS ASSOCIATED WITH THE VERTEX MEAN.**

In order to find the vertex using algebra, we will consider three techniques:

1) **USING THE ZEROS, TO FIND THE AXIS OF SYMMETRY**, and then the vertex **(this is the easiest technique,** ***assuming we can factor the quadratic*****)**.
2) **COMPLETING THE SQUARE** to find the vertex (this is the toughest technique, but it's nice because you *end up with the quadratic in vertex form*).
3) ☆ **USE PARTIAL FACTORING TO FIND THE AXIS OF SYMMETRY**, and then the vertex.

Note: We can also use graphing calculators to find the max/min of a quadratic!

Example 3.2.2

Expect a max ☹ (−) not ☺ (+a)

Determine the max or min value for the function $f(x) = -3x^2 - 12x + 15$ by finding **THE ZEROS** of the quadratic.

Factor $f(x) = -3(x^2+4x-5)$

$= -3(x+5)(x-1)$

So zeroes @ $x = -5, 1$

AoS @ $\left(\frac{-5+1}{2}\right) = -2$

$\therefore$ vertex at $(x, f(x)) = (-2, 27)$

(AoS, f(AoS))

$\therefore$ the max is 27 @ $x=-2$.

Example 3.2.3

"Create a perfect square"

COMPLETE THE SQUARE to find the vertex of the quadratic and state **where** the max (min) is and **what** the max (min) is.

$g(x) = 2x^2 + 8x - 5$

① Factor a out of the first two terms.

$g(x) = 2(x^2+4x) - 5$

② Take $\frac{1}{2}$ of the "x" coefficient & square it.

$\left(\frac{4}{2}\right)^2 = 4$

③ Add that number on and subtract it away

$g(x) = 2(\underbrace{x^2+4x+4}_{\text{perfect square. Factor it.}}-4) - 5$

$g(x) = 2((x+2)^2 - 4) - 5$

④ Distribute "a" & collect like terms.

$= 2(x+2)^2 - 8 - 5$

$g(x) = 2(x+2)^2 - 13$

* This is vertex form!

So, the min is -13 at $x = -2$!! ☺

Much better than completing the square.

Gives us two symmetric points.

Example 3.2.4

Using PARTIAL FACTORING determine the axis of symmetry. Then find the vertex and state the min or max value.

$$h(x) = \cancel{5}3x^2 + 15x - 3 \quad \text{"c"}$$

① Consider the first two terms. Common factor them.

$h(x) = 3x(x+5) \; -3$

② Set factors to zero. they have y-values of "c"

$x=0$ + $x=-5$

$h(0) = -3 \quad (0,-3)$

$h(-5) = -3 \quad (-5,-3)$

Are symmetric points

③ Find AoS

$x = \frac{0 + -5}{2} = -\frac{5}{2} = -2.5$

④ Find y-value of vertex

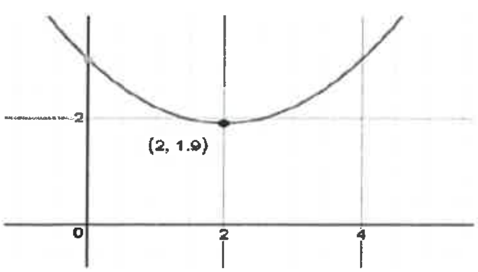
vertex: $(AoS, f(AoS)) = (-2.5, -21.75)$

$\therefore$ A min of -21.75 @ $x=-2.5$

Example 3.2.5

Using graphing technology, determine the max/min value of the quadratic

$$f(x) = 0.3x^2 - 1.2x + 3.1$$



Graph on Desmos

See answer is ~~f~~ 1.9 @ $x=2$

P.F.

$f(x) = \underbrace{0.3x(x-4)}_{\text{set to zero}} + 3.1$

So $x=0$, $x=4$

AoS: $x = \frac{0+4}{2} = 2$

$\therefore$ vertex is $(2, f(2)) = (2, 1.9)$

Success Criteria:

- I can recognize when a function has a maximum or minimum value (based on "*a*")
- I can find the max/min (vertex) value using various methods (*partial factoring* ☺)

Chapter 3 – Quadratic Functions

3.4 – Operations with Radical Numbers

Learning Goal: We are learning to simplify and perform operations on radicals.

First we need to understand that **RADICALS** (*square roots, cube roots, etc*) **ARE NUMBERS**, and working with them should not induce any kind of fear in your spirit. So, **FEAR NOT!**

A COUPLE OF THINGS TO REMEMBER:

1) The square root of a square number is a nice integer.

e.g. $\sqrt{25} =$ 5

$\sqrt{49} =$ 7

2) The cube root of a cubed number is a nice integer

e.g. $\sqrt[3]{27} =$ 3

$\sqrt[3]{125} =$ 5

Now, if we don't have a radical with a perfect square (or cube as the case may be) we could use a calculator to find the root.

Exact value

e.g. $\sqrt{24} = \underbrace{4.898979485566356196394568114941 18...}_{\text{Decimal Expansion}}$

BUT the "**DECIMAL EXPANSION**" is **unending** and **doesn't repeat** and so we can only **APPROXIMATE THE VALUE** of $\sqrt{24}$ because of the need to **ROUND-OFF**. "**EXACT NUMBERS**" like $\sqrt{24}$ are sometimes preferred in mathematical solutions and so **we do need to know how to work with these radical NUMBERS.** Working with radical numbers means we'll be:

- adding/subtracting
- multiplying/dividing them.

Before beginning, there is one thing to keep in mind:

COEFFICIENTS WITH COEFFICIENTS, RADICALS WITH RADICALS

e.g. The number $2\sqrt{5}$ has a coefficient part of 2 and a radical part of $\sqrt{5}$ (like polynomials)

Such a number (with both a coefficient and a radical part) is called a mixed radical

Example 3.4.1

Multiply the following:

a) $\sqrt{5}\times\sqrt{3} = \sqrt{5\times 3} = \sqrt{15}$

b) $-2\sqrt{7}\times 3\sqrt{6} = -2\cdot 3\cdot\sqrt{7\cdot 6} = -6\sqrt{42}$

c) $5\sqrt{10}\times\sqrt{5} = 5\sqrt{10\cdot 5} = 5\sqrt{50}$

d) $\sqrt{2}\times\sqrt{2} = \sqrt{4} = 2$

$\left(\sqrt[5]{29}\right)^5 = 29$

$\left(\sqrt{2}\right)^2 = 2$ "squaring + square rooting" are inverse operations. They cancel.

radical symbol

$\sqrt{a}$

radicand

Radical

Example 3.4.2

Simplify the following:

a) $\sqrt{50} = \sqrt{25\cdot 2}$
$= \sqrt{25}\cdot\sqrt{2}$
$= 5\sqrt{2}$

* Look for a perfect square factor of the RADICAND

b) $-3\sqrt{27} = -3\sqrt{9\cdot 3}$
$= -3\sqrt{9}\sqrt{3}$
$= -3\cdot 3\sqrt{3}$
$= -9\sqrt{3}$

c) $2\sqrt{50}\times\left(-3\sqrt{24}\right)$

$= \left(2\sqrt{25\cdot 2}\right)\left(-3\sqrt{4\cdot 6}\right)$
$= \left(2\sqrt{25}\sqrt{2}\right)\left(-3\sqrt{4}\sqrt{6}\right)$
$= \left(10\sqrt{2}\right)\left(-6\sqrt{6}\right)$
$= -60\sqrt{2\cdot 6}$
$= -60\sqrt{12}$ $\quad 4\cdot 3$
$= -60\sqrt{4}\sqrt{3}$
$\boxed{= -120\sqrt{3}}$

Add/subtract the coefficients

Note: **We can only ADD OR SUBTRACT "LIKE" RADICALS.**
e.g. $2\sqrt{3}$ and $-5\sqrt{3}$ **ARE LIKE, but** $2\sqrt{5}$ and $3\sqrt{20}$ **ARE NOT** (*or aren't they?.....*)

similar to rules for polynomials!

Example 3.4.3

Add the following:

a) $3\sqrt{2} + 7\sqrt{2}$ $\neq \cancel{10\sqrt{4}}$

$= 10\sqrt{10}$

b) $5\sqrt{7} - 3\sqrt{5} - 7\sqrt{7}$

$= -2\sqrt{7} - 3\sqrt{5}$ Done!

c) $2\sqrt{5} - 3\sqrt{20}$

$= 2\sqrt{5} - 3\sqrt{4 \cdot 5}$

$= 2\sqrt{5} - 3 \cdot 2\sqrt{5}$

$= 2\sqrt{5} - 6\sqrt{5}$ $\boxed{= -4\sqrt{5}}$

d) $-3\sqrt{300} + \sqrt{243}$

$= -3\sqrt{100 \cdot 3} + \sqrt{81 \cdot 3}$

$= -30\sqrt{3} + 9\sqrt{3}$

$= -21\sqrt{3}$

Distributive Property!

Example 3.4.4

Simplify:

a) $2\sqrt{3}\,(3\sqrt{2} - 5\sqrt{6})$

$= 6\sqrt{3\cdot 2} - 10\sqrt{3\cdot 6}$

$= 6\sqrt{6} - 10\sqrt{18}$ $\quad 18 = 9\cdot 2$

$= 6\sqrt{6} - 10\sqrt{9\cdot 2}$

$= 6\sqrt{6} - 30\sqrt{2}$

b) $(3\sqrt{12} - 5\sqrt{2})(2\sqrt{3} + 3\sqrt{2})$ FOIL

$= 6\sqrt{36} + 9\sqrt{24} - 10\sqrt{6} - 15\sqrt{4}$

$(4\cdot 6)$

$= 6(6) + 9\sqrt{4\cdot 6} - 10\sqrt{6} - 15(2)$

$= 36 + 18\sqrt{6} - 10\sqrt{6} - 30$

$= 6 + 8\sqrt{6}$

c) $(5 - 2\sqrt{2})^2$

$= (5 - 2\sqrt{2})(5 - 2\sqrt{2})$

$= 25 - 10\sqrt{2} - 10\sqrt{2} + 4\sqrt{4}$

$= 25 - 20\sqrt{2} + 4(2)$

$= 33 - 20\sqrt{2}$

Success Criteria:

- I can recognize "like" radicals. Totally awesome dude!
- I can write a radical in simplest form
- I can simplify radicals by adding, subtracting, multiplying, and dividing
- I can appreciate that a radical is an EXACT answer and therefore SUPERIOR to decimals

Chapter 3 – Quadratic Functions

3.5 – Solving Quadratic Equations

Learning Goal: We are learning to solve quadratic functions in different ways.

Note that last day we looked at section 3.6. We now go back to 3.5 as this is a better order for the concepts.

Before beginning we should look at the difference between a Quadratic FUNCTION and a Quadratic EQUATION. A function such as $f(x) = 3x^2 - 5x + 1$ has a graph with **infinitely** many points. On the other hand, a quadratic equation (in standard form) looks like:

$$3x^2 - 5x + 1 = 0$$

(*What is the difference between the function and the equation?*) Looking only for solutions when $y = 0$

In section 3.6 we saw how to find the **zeroes** of quadratic functions, using the techniques of factoring, the quadratic formula or using graphing technology. As it turns out, solving a quadratic equation is **Exactly** the same as finding the zeros of a quadratic function.

Quadratic equations, therefore can have 2, 1, or 0 SOLUTIONS. "roots"

4 Techniques
1. Factoring
2. Vertex + algebra
3. Quadratic formula
4. Graphing

Example 3.5.1

Solve the equations:

a) $x^2 - 5x - 14 = 0$

x −14
+ −5
$(-7)(+2)$

$(x-7)(x+2) = 0$

∴ The roots/solutions are $x = 7$, $x = -2$

(When is each factor equal to zero)

b) $2x^2 + 5x = 2x + 4$

Goal: Stuff = 0. Rearrange eqn!

$2x^2 + 3x - 4 = 0$

x −4
+ 3
D.N.F.

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-4)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{9 + 32}}{4}$$

$$= \frac{-3 \pm \sqrt{41}}{4}$$

$\therefore x = \frac{-3 + \sqrt{41}}{4}$, $x = \frac{-3 - \sqrt{41}}{4}$ (Exact Form)

OR

$x = 0.851, -2.35$

When given decimals, you may work in decimals.

Example 3.5.2

Solve $-2.3x^2 - 1.32x = -1.45$

$-2.3x^2 - 1.32x + 1.45 = 0$

$\therefore x = \dfrac{-(-1.32) \pm \sqrt{(-1.32)^2 - 4(-2.3)(1.45)}}{2(-2.3)}$

$= \dfrac{1.32 \pm 3.88}{-4.6}$

$\therefore x = \dfrac{1.32 + 3.88}{-4.6}$

$= -1.13$

$x = \dfrac{1.32 - 3.88}{-4.6}$

$= +0.56$

Example 3.5.3 (From your text: Pg. 178 #6a)

means profit $= 0$

6. Determine the break-even quantities for each profit function, where x is the number sold, in thousands.

a) $P(x) = -x^2 + 12x + 28$

$\dfrac{0}{-1} = \dfrac{-x^2 + 12x + 28}{-1}$

$0 = x^2 - 12x - 28$

$0 = (x - 14)(x + 2)$

x -28
+ -12
-14 $+2$

$\therefore x = 14$ or $\cancel{x = -2}$ Inadmissible

Selling 14,000 breaks even!

Selling -2000 objects is ridiculous!

8. The population of a region can be modelled by the function $P(t) = 0.4t^2 + 10t + 50$, where $P(t)$ is the population in thousands and t is the time in years since the year 1995.

a) What was the population in 1995?

b) What will be the population in 2010?

c) In what year will the population be at least 450 000? Explain your answer.

a) In 1995, 0 years passed, so $P(0)$

$P(0) = 0.4(0)^2 + 10(0) + 50$

$= 50$ ← carries meaning. 50,000 people!

b) # of years? $2010 - 1995 = 15$ years $t = 15$

$P(15) = 0.4(15)^2 + 10(15) + 50$

$= 290$ ∴ 290,000 ppl

c) Find t! $P(t) = 450$

$0.4(t)^2 + 10(t) + 50 = 450$

$0.4(t)^2 + 10t - 400 = 0$

$\therefore x = \frac{-(10) \pm \sqrt{(10)^2 - 4(0.4)(-400)}}{2(0.4)}$

$= \frac{-10 \pm \sqrt{740}}{0.8}$

$x = \frac{-10 \pm 27.2}{0.8}$

$x = 21.5$ or $\cancel{x = -46.5}$

Negative years? Ridiculous

Population at 450,000 after 21.5 years. (22)

Success Criteria:

- I can solve quadratic functions by factoring, then setting each factor equal to zero
- I can solve quadratic functions by using the quadratic formula

Chapter 3 – Quadratic Functions

3.6 – Zeroes of Quadratic Functions

Learning Goal: We are learning to determine the number of zeros of a quadratic function.

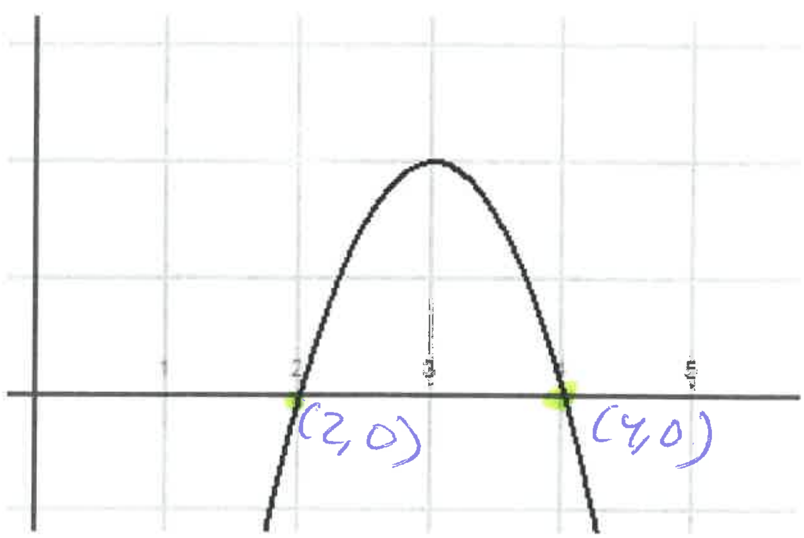
Before we begin, let's think about a couple of things…

Remember – **FUNCTIONS CAN BE DESCRIBED AS A SET OF ORDERED PAIRS**, where the "ordered pair" is a pair of numbers: a **domain value** and a **range value** which can look like $(x, f(x))$. We have talked about the vertex of a parabola. Consider a parabola opening down (which means it will have a maximum value.

The vertex of that parabola is NOT the maximum. Instead, the vertex is a POINT which is made up of two special numbers. The domain value is WHERE the max occurs and the functional value (the "y" value) is the maximum.

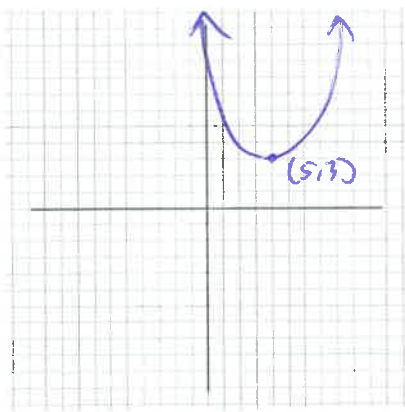
When we talk about the ZEROS of a quadratic we need to understand what we mean by that. Consider the sketch of the graph of the quadratic function $f(x) = -2(x-3)^2 + 2$

A zero has the point $(x_z, 0)$ where x_z is the zero. This value makes the function zero



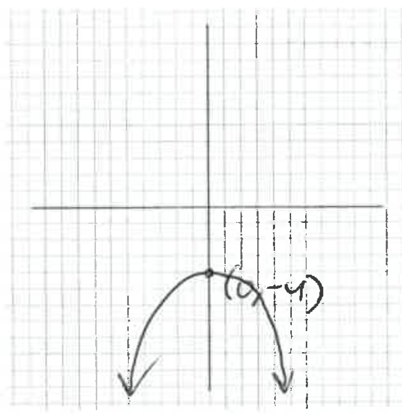
$\therefore$ For the parabola sketch, the zeros are at $x=2$, $x=4$

Q. Do all quadratics have 2 zeros? NO!!!!!!!



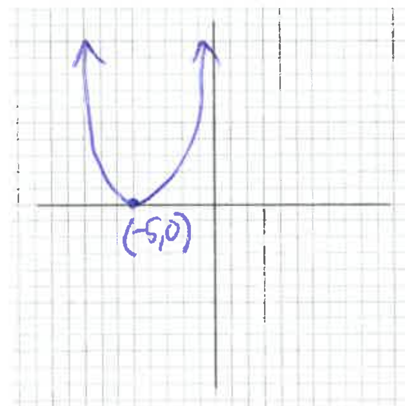
A positive min.

NO zeros



A negative max.

No zeros.



The vertex is on the x-axis.

One zero.

Finding the Zeros of a Quadratic

We can find those pesky zeros in a number of ways:

1) Writing the quadratic in zeros form (by factoring)
2) Writing the quadratic in vertex form, and doing some algebra (a bit nasty)
3) Using the quadratic formula (but the quadratic ***MUST BE IN STANDARD FORM*** - $f(x) = ax^2 + bx + c$)
4) Using graphing technology (lame, but legit)

Example 3.6.1

Determine the zeros:

a) $f(x) = x^2 - 3x - 4$

x-4
+ -3
(-4 +1)

$f(x) = (x-4)(x+1)$

zeros occur when $f(x) = 0$

$(x-4)(x+1) = 0$ when either factor is zero

$\therefore x = 4, \ x = -1$

b) $g(x) = 2x^2 + x - 1$

x -2
+ 1
2 -1

$g(x) = (2x+1)(x-1)$

$0 = (2x+1)(x-1)$

$2x+1 = 0$

$2x = -1$

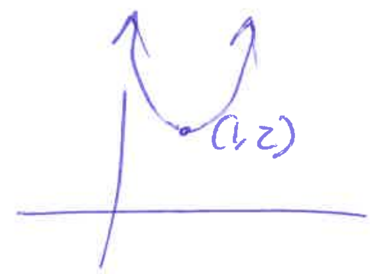
$\boxed{x = -\frac{1}{2}}$ or

$x - 1 = 0$

$\boxed{x = 1}$

Example 3.6.2

Determine why the quadratic $f(x) = 2(x-1)^2 + 2$ has no zeros.

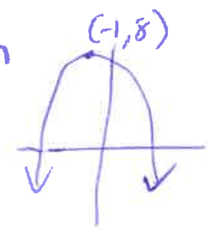


- vertex @ (1, 2)
- a is positive, so we have a min value. Parabola opens up.

No zeros

Example 3.6.3

Determine the zeros of $g(x) = -(x+1)^2 + 8$



The vertex is at (-1, 8). Graph opens down

$\therefore$ Are 2 zeros!

Techniques

1. Convert to Standard Form + Factor (or Quad. Formula)
2. Do the math

$$-(x+1)^2 + 8 = 0$$

$$\frac{-(x+1)^2}{-1} = \frac{-8}{-1}$$

$$\sqrt{(x+1)^2} = \sqrt{8}$$

$$x+1 = \pm\sqrt{8}$$

Don't forget $\pm$!

$$\therefore \; x = \pm\sqrt{8} - 1$$

Now $\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$

So

$\boxed{x = 2\sqrt{2} - 1 \quad \text{or} \quad x = -2\sqrt{2} - 1}$ Exact Form

$\boxed{x \doteq 1.83 \quad \text{or} \quad x \doteq -3.83}$ Approximate Form

Example 3.6.4

Using the quadratic formula, determine the zeros of the quadratic:

In case you've forgotten, the quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

a) $f(x) = 2x^2 + 3x - 7$

$a=2,\ b=3,\ c=-7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-7)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{65}}{4}$$

You must write out both zeros!

$$x = \frac{-3 + \sqrt{65}}{4} \quad \text{or} \quad x = \frac{-3 - \sqrt{65}}{4}$$

$x \doteq 1.27$ $\qquad$ $x \doteq -2.77$

b) $g(x) = 3x^2 - 2x + 4$

$a=3\ \ b=-2\ \ c=4$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(4)}}{2(3)}$$

$$= \frac{2 \pm \sqrt{-44}}{6}$$

$\therefore$ No zeros!

we can't take the square root of a negative!

The Discriminant

The Discriminant of the quadratic formula is called the **DISCRIMINANT** because it's very discriminating. "b^2-4ac" tells us HOW MANY zeros a function has!

The Discriminant is $b^2 - 4ac$

1) If $b^2 - 4ac > 0$ (+) $\rightarrow$ 2 zeros

2) If $b^2 - 4ac = 0$ $\rightarrow$ 1 zero

3) If $b^2 - 4ac < 0$ (negative) $\rightarrow$ 0 zeros

Example 3.6.5

Determine the number of zeros using the discriminant:

a) $f(x) = 2x^2 + 3x - 2$

$b^2-4ac = (3)^2 - 4(2)(-2)$
$= 9 + 16$
$= 25 > 0$ 2 zeros

b) $g(x) = -x^2 + 4x - 4$

$b^2-4ac = 4^2 - 4(-1)(-4)$
$= 16 - 16$
$= 0$ 1 zero

c) $h(x) = 3x^2 + 5x + 6$

$b^2-4ac = 5^2 - 4(3)(6)$
$= 25 - 72 < 0$

No zeros

Success Criteria:

- I can recognize that a quadratic function may have 0, 1, or 2 zeros
- I can use the discriminant of the quadratic formula to determine the number of zeros

Chapter 3 – Quadratic Functions

3.7 – Families of Quadratic Functions

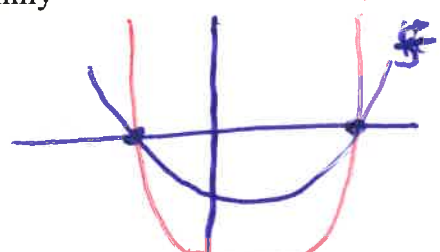
Learning Goal: We are learning the properties of families of quadratic functions.

Consider the two quadratic functions:

$f(x) = 2(x-3)^2 + 1$, and $g(x) = -3(x-3)^2 + 1$ **What's Different?** a

Clearly $f(x)$ and $g(x)$ are different functions, but they do share the same vertex, and the same axis of symmetry. These quadratics are said to be in the same "family" h

(some might say they are in the same vertex family)



Next, consider $h(x) = 3(x+2)(x-4)$, and $f(x) = \frac{2}{3}(x+2)(x-4)$. We see another kind of family here because $h(x)$ and $f(x)$ share the same zeros, and the same axis of symmetry.

(some might say these quadratics are in the same zeroes family) **What's Different?** a

Finally consider the third form of a quadratic. Consider

$f(x) = -3x^2 - 2x + \underline{7}$

$g(x) = 2x^2 + 8x + \underline{7}$

Here the y-intercept is the commonality.

Example 3.7.1

Determine the equation of the quadratic with zeros $x = 3$, and $x = -1$ and that passes through the point $(5,6)$. we don't know a!

$f(x) = a(x-3)(x+1)$

use $(5,6)$ to find a

$6 = a(5-3)(5+1)$

$6 = a(2)(6)$

$6 = 12a \Rightarrow a = \frac{1}{2}$

$\therefore f(x) = \frac{1}{2}(x-3)(x+1)$

Example 3.7.2

Determine the equation of the quadratic function $f(x)$ with a max value of 3 and axis of symmetry with equation $x = -5$ if $f(2) = -18$.

So a must be negative!

vertex: $(AoS, f(AoS))$ A max or min value

vertex $= (-5, 3) = (h, k)$ But.... we don't know a!

x max

use vertex form

$f(x) = a(x-(-5))^2 + 3$

use $f(2) = -18$ to find a

$(2, -18)$

$-18 = a((2)+5)^2 + 3$

$-18 = a(7)^2 + 3$

$-21 = 49a$

$-\frac{21}{49} = a \Rightarrow -\frac{3}{7}$

$\therefore f(x) = -\frac{3}{7}(x+5)^2 + 3$

Success Criteria:

- I can solve for "*a*" if given either the vertex or zeros

Chapter 3 – Quadratic Functions

3.8 – Linear-Quadratic Systems

Learning Goal: We are learning to solve problems involving the intersection of a linear and quadratic function.

Recall from Grade 10 that solving a SYSTEM OF LINEAR EQUATIONS could be interpreted to mean finding the point of intersection of the two lines. The solution to a **SoLE** is a point, (x, y). From an algebraic point of view, we have two techniques for solving a SoLE:

1) Substitution
2) Elimination

Example 3.8.1

Solve the SoLE

$2x+3y=7$ (1)

$x-2y=-7$ (2)

Algebraically

Substitution

③ $x = 2y - 7$

Sub ② into ①

$2(2y-7) + 3y = 7$

$4y - 14 + 3y = 7$

$7y = 21$

$\therefore y = 3$

Sub $y=3$ into ③

$x = 2(3) - 7$

$x = 6 - 7$

$x = -1$

$\therefore$ The POI is $(-1, 3)$

Elimination

we need the coefficients to match for one variable.

② × 2 $2x - 4y = -14$ ③

Eliminate by ③ − ①

$2x - 4y = -14$

$-[2x + 3y = 7]$

$-7y = -21$

$y = 3$

Sub $y=3$ into ②

$x - 2(3) = -7$

$x - 6 = -7$

$x = -1$

$\therefore$ PoI is $(-1, 3)$

Graphically

POI $= (-1, 3)$

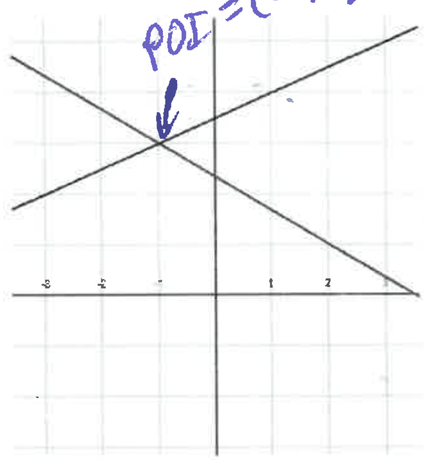


Figure 3.8.1

Illustrate graphically @ end

Solving a Linear-Quadratic System is more difficult, but we have the tools to succeed!
We will need to make use of (at least) one Property (or Rule) of Algebra:

The Transitive Property of Equality

Rule: Given three numbers (or more generally, three mathematical objects) a, b, and c, and **if** $c = a$ and $c = b$, **then** $a = b$.

We can say the two equations are equal.

Example: If $f(x) = -2x - 4$, and $g(x) = x^2 - 3x - 10$, and if $f(x) = g(x)$, then

$$x^2 - 3x - 10 = -2x - 4$$

> Note: **Solving a Linear-Quadratic System** is equivalent to **finding the solution(s) to a quadratic equation.** For L-QS`s we can therefore have 0, 1, or 2 solutions.
>
> **We will apply the techniques for solving quadratic equations!**

Example 3.8.2

Solve the Linear-Quadratic System given directly above.

At the PoI, the two equations have equal y-values!

$x^2 - 3x - 10 = -2x - 4$

$x^2 - x - 6 = 0$ (get "stuff = 0)

$(x-3)(x+2) = 0$

$\therefore$ $x = 3$ or $x = -2$
(two solutions)

But, we are not done!
Need the y-coordinate

$f(3) = -2(3) - 4$
$= -10$

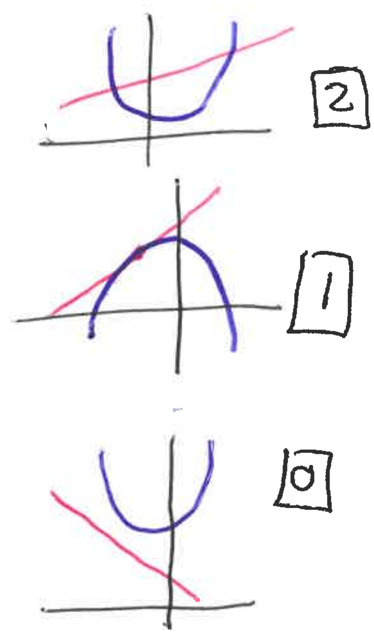
So $(3, -10)$

$f(-2) = -2(-2) - 4$
$= 0$

So $(-2, 0)$

These are the PoI's

Possible solutions



Example 3.8.3 (#2c, on Page 198 from your text)

Determine the point(s) of intersection of the two functions algebraically:

$f(x) = 3x^2 - 2x - 1,\ \ g(x) = -x - 6$

$\Rightarrow 3x^2 - 2x - 1 = -x - 6$

$3x^2 - x + 5 = 0$

Does not factor. Use quad formula

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(5)}}{2(3)}$$

$$= \frac{1 \pm \sqrt{-59}}{6}$$

can't do. ∴ No POI!

Example 3.8.4

Use the discriminant.

Determine the number of points of intersection without solving the System:

$f(x) = x^2 + 2x + 14,\ \ g(x) = 8x + 5$ (Hint: To solve this problem you must be very discriminating)

$x^2 + 2x + 14 = 8x + 5$

$\therefore x^2 - 6x + 9 = 0$

$b^2 - 4ac = (-6)^2 - 4(1)(9)$

$= 36 - 36$

$= 0$

∴ One solution

Example 3.8.5 (#9 on Page 199 in your text)

9. Determine the value(s) of k such that the linear function $g(x) = 4x + k$ does not intersect the parabola $f(x) = -3x^2 - x + 4$.

So no solutions. ∴ discriminant < 0

$\Rightarrow -3x^2 - x + 4 = 4x + k$

$\underbrace{-3x^2}_{a} \; \underbrace{-5x}_{b} \; \underbrace{+4-k}_{c}$

4-k is our constant

when is $b^2 - 4ac < 0$?

$(-5)^2 - 4(-3)(4-k) < 0$

$25 + 12(4-k) < 0$

$12(4-k) < -25$

$4 - k < \frac{-25}{12}$

$-k < \frac{-25}{12} - 4$

$k > \frac{25}{12} + 4\left(\frac{48}{12}\right)$

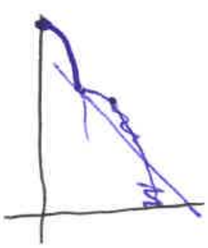
$k > \frac{73}{12}$

Example 3.8.6 (#10 in your text)

10. A daredevil jumps off the CN Tower and falls freely for several seconds before releasing his parachute. His height, $h(t)$, in metres, t seconds after jumping can be modelled by

$h_1(t) = -4.9t^2 + t + 360$ before he released his parachute; and
$h_2(t) = -4t + 142$ after he released his parachute.

How long after jumping did the daredevil release his parachute?



$-4.9t^2 + t + 360 = -4t + 142$

$-4.9t^2 + 5t + 218 = 0$

Q.F. $t = \frac{-(5) \pm \sqrt{5^2 - 4(-4.9)(218)}}{2(-4.9)}$

$= \frac{-5 \pm 65.56}{-9.8}$

Inadmissible

So $t_1 = \cancel{\frac{-5 + 65.56}{-9.8}}$ Neg time?

$t_2 = \frac{-5 - 65.56}{-9.8}$

$\doteq 7.2$ seconds

Success Criteria:

- I can solve for the points of intersection by
  1. Making the functions equal to each other
  2. Solving for the zeros (x-coordinates) of the resulting quadratic function
  3. Substituting the zeros into the linear equation to determine the corresponding y-values
- I can identify when solutions are inadmissible