Functions 11

Teacher

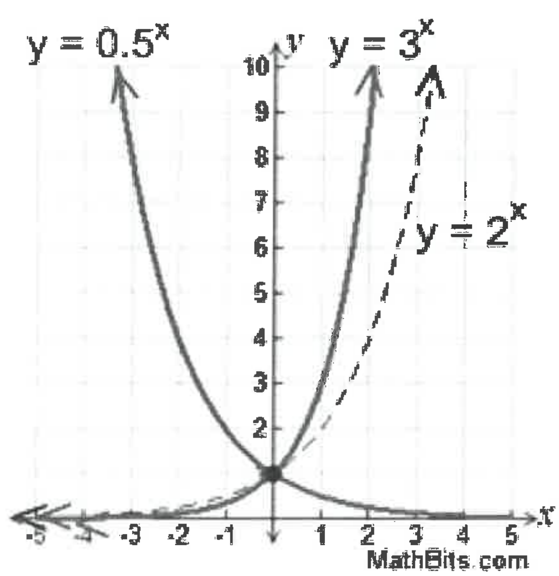
Course Notes

Unit 4 – Exponential Functions

LOCAL TITLE

We are learning to

- *understand the meaning of a zero, and learn how to find them algebraically*
- *determine the max or min value of a quadratic algebraically and graphically*
- *sketch parabolas (using transformations, zeroes, the vertex and y-intercept)*
- *solve real-world problems, including linear-quadratic systems*



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Chapter 4 – Exponential Functions

Contents with suggested problems from the Nelson Textbook. You are welcome to ask for help, from myself or your peers, with any of the following problems. They will be handed in on the day of the Unit Test as a homework check.

Section 4.2

Pg. 222 – 223 #5 – 8, 13

Section 4.3

Pg. 229: #2de, 3cdef, 4cd, 5, 6, 8, 10 (*a question of awesomeness*), 12 (*we may take up next day*)

Section 4.4

Pg. 236 – 237 #2acef, 4acdf, 5, 6, 7ac (simplify BEFORE substituting!), 9ad

Section 4.5/4.6

Pg. 251 – 253 #1 – 3, 5bcd (write each transformed function "properly), 8 – 10 (*for #10 please see example 4 on page 250*)

Section 4.7

READ Example 2 on pages 256 – 257 (which method do you prefer: Guess and Check, or Graphing Calculator?)

READ Example 4 on pages 259 – 260.

Pg. 261 – 263 #1, 3 – 9, 12 – 16 (you have two days for these problems)

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Chapter 4 – Exponential Functions

4.2 – Integer Exponents

Learning Goal: We are learning to work with integer exponents.

Before beginning, we should quickly review (*ominous music plays*):

THE POWER LAWS

Consider a typical "power" a^n. We call "*a*" the base. We call "*n*" the exponent and the entire expression a^n is called a power

The Laws: Given the powers a^m and a^n, with exponents *m* and *n*, and the number $\frac{a}{b}$, then $a, b \neq 0$

1) $1^m = 1$

2) $a^1 = a$

3) $a^0 = 1$

same base

4) $a^m \cdot a^n = a^{m+n}$

5) $(a \cdot b)^m = a^m \cdot b^m$

6) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Same base

7) $\frac{a^m}{a^n} = a^{m-n}$

8) $(a^m)^n = a^{m \cdot n}$

Until now, for the most part, the exponents you've been working with have always been NATURAL NUMBERS. But, we now will examine **INTEGER EXPONENTS**!!

ADDITIONAL POWER LAWS: * negative exponents reciprocate the base

9) $a^{-n} = \left(\frac{1}{a}\right)^{+n} = \frac{1^n}{a^n} = \frac{1}{a^n}$

10) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{+n} = \frac{b^n}{a^n}$

11) $\frac{a^{-m}}{b^{-n}} = \frac{b^{+n}}{a^{+m}}$

Example 4.2.1

Write each expression as a single power with a positive exponent:

a) $(4)^{-5}$

$= \left(\frac{1}{4}\right)^5$

$= \frac{1}{4^5}$

b) $\left(\frac{3}{2}\right)^{-4}$

$= \left(\frac{2}{3}\right)^4$

c) $\frac{7^3}{7^9} = 7^{3-9}$

$= 7^{-6}$

$= \left(\frac{1}{7}\right)^6$

Example 4.2.2

Find the number!

Simplify, then $\boxed{\text{evaluate}}$ each expression and state your answers in rational form:

a) $3^5(3^{-2})$

$= 3^{5+(-2)}$

$= 3^3$

$= 27$

b) $\left(2^{-3}(2^4)\right)^{-5}$

$= \left(2^{(-3)+4}\right)^{-5}$

$= (2^1)^{-5}$

$= 2^{-5}$

$= \frac{1}{2^5} = \frac{1}{32}$

c) $\frac{5^{-3}}{(5^2)^{-2}} = \frac{5^{-3}}{5^{-4}}$

$= 5^{(-3)-(-4)}$

$= 5^1$

$= 5$

Example 4.2.3 As a fraction

Evaluate and express in rational form:

a) $3^2(6^{-3})$

$= \frac{3^2}{6^3}$

$= \frac{3^2}{(3\cdot 2)^3}$

$= \frac{\cancel{3^2}}{\cancel{3^3}\cdot 2^3}$

$= \frac{1}{3\cdot 8}$

$= \frac{1}{24}$

or

$= \frac{3^2}{6^3}$

$= \frac{9}{216}$

$= \frac{1}{24}$

b) $2^{-3}+10^{-3}-3(5^{-3})$

$= \frac{1}{2^3}+\frac{1}{10^3}-\frac{3}{5^3}$ use C.D.

$= \frac{1}{2^3}+\frac{1}{2^3\cdot 5^3}-\frac{3}{5^3}$

$= \frac{5^3+1-3\cdot 2^3}{2^3\cdot 5^3}$

$= \frac{125+1-24}{1000} = \frac{102}{1000} = \frac{51}{500}$

c) $13^{-5}\times\left(\frac{13^2}{13^8}\right)^{-1}$

$= 13^{-5}\cdot\left(\frac{13^8}{13^2}\right)$

$= 13^{-5+8-2}$

$= 13^1$

$= 13$

Example 4.2.4

Evaluate using the laws of exponents (the power rules):

a) $3^2\times 9^{-3}\div 3^{-7}$

$= 3^2\cdot(3^2)^{-3}\div\frac{1}{3^7}$

$= 3^2\cdot 3^{-6}\cdot 3^7$

$= 3^{2+(-6)+7}$

$= 3^3$

$= 27$

b) $\dfrac{4^{-2}+3^{-1}}{5^{-1}+2^{-2}}$

$= \dfrac{\frac{1}{4^2}+\frac{1}{3}}{\frac{1}{5}+\frac{1}{2^2}}$

$= \dfrac{\frac{1}{16}+\frac{1}{3}}{\frac{1}{5}+\frac{1}{4}}$ C.D.

$= \dfrac{\frac{3+16}{48}}{\frac{4+5}{20}}$

$= \dfrac{\frac{19}{48}}{\frac{9}{20}}$

$= \frac{19}{\cancel{48}_{12}}\cdot\frac{\cancel{20}^{5}}{9}$

$= \frac{95}{108}$

Success Criteria:

- I can apply the exponent laws
- I can recognize that a negative exponent represents a reciprocal expression

Chapter 4 – Exponential Functions

4.3 – Rational Exponents

Learning Goal: We are learning to work with powers involving rational (fractional) exponents and to evaluate expressions containing them.

A **RATIONAL EXPONENT** can be a **FRACTION**. For example, we can consider the number $(16)^{\frac{3}{4}}$. Of course, the question we need to ask is:

What the rip is that thing??

As you know, a fraction has two parts: a numerator, and a denominator. When a fraction is used as an exponent, the two parts of the fraction carry two related (but different) meanings in terms of "powers".

Recall that 4^3 means $4 \times 4 \times 4$. Now $4^{\frac{1}{2}}$ does not mean $4 \div 4$! Your text has a nice explanation of the meaning of numbers like $4^{\frac{1}{2}}$. See (i.e. **READ** examples 1 and 2 on pages 224 and 225. For now, we will simply take the meaning of

root $4^{1/2} = \sqrt{4} = 2$

$27^{1/3} = \sqrt[3]{27} = 3$

Definition 4.3.1

Given a power with a "rational" (fractional) exponent $a^{\frac{m}{n}}$, the numerator of the exponent is a "power" in the usual sense, and the denominator represents a "root" or "radical".

Two ways

e.g. For the number $16^{\frac{3}{4}}$

$= (16^3)^{1/4}$

$= (4096)^{1/4}$

Difficult!

OR

$= (16^{1/4})^3$

$= (2)^3$

$= 8$

The rule $(a^m)^n = a^{m \cdot n}$

So we can break up rational exponent into a power of a power.

Tip: Easier to do the root first

Example 4.3.1

These are in radical form.
Need rational exponents

From your text: Pg. 229 #2.

Write in exponent form, and then evaluate:

a) $\sqrt[9]{512}$

$= (512)^{1/9}$

$= 2$

c) $\sqrt[3]{27^2}$

$= (27^2)^{1/3}$

$= (27^{1/3})^2$

$= 3^2$

$= 9$

b) $\sqrt[3]{-27}$

$= (-27)^{1/3}$

$= -3$

Note: We ***CANNOT*** take an **Even Root** of a **negative radicand**.
We *CAN* take an **Odd Root** of a negative radicand, however.

f) $\sqrt[4]{\left(\frac{16}{81}\right)^{-1}}$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

$= \left(\left(\frac{16}{81}\right)^{-1}\right)^{1/4}$

$= \left(\frac{81}{16}\right)^{1/4}$

$= \frac{81^{1/4}}{16^{1/4}}$

$= \frac{3}{2}$

DO NOT EVALUATE

Example 4.3.2

From your text: Pg. 229 #3

Write as a single power:

a) $\left(8^{\frac{2}{3}}\right)\left(8^{\frac{1}{3}}\right)$

$= 8^{\frac{2}{3}+\frac{1}{3}}$

$= 8^{3/3}$

$= 8^{1}$

$= 8$ (wrong. No exponent shown)

b) $8^{\frac{2}{3}} \div 8^{\frac{1}{3}}$

$= 8^{\frac{2}{3}-\frac{1}{3}}$

$= 8^{1/3}$

$= \sqrt[3]{8}$ (wrong. This is radical form)

Example 4.3.3

From your text: Pg. 229 #4

Write as a single power, then **evaluate**. Express answers in **rational form**.

$\sqrt[n]{a} = a^{\frac{1}{n}}$

$\sqrt{a} = a^{1/2}$

a) $\sqrt{5} \cdot \sqrt{5}$

$= (\sqrt{5})^2$

$= (5^{1/2})^2$

$= 5^{1}$

$= 5$

b) $\dfrac{\sqrt[3]{-16}}{\sqrt[3]{2}}$

$= \dfrac{(-16)^{1/3}}{2^{1/3}}$

$= \left(-\frac{16}{2}\right)^{1/3}$

$= (-8)^{1/3}$

$= -2$

Success Criteria:

- I can understand that the numerator of a fractional exponent is the power, while the denominator is the root.

Chapter 4 – Exponential Functions

4.4 – Simplifying Expressions Involving Exponents

Learning Goal: We are learning to simplify algebraic expressions involving powers and radicals.

Keep the EXPONENT RULES in your mind at all times.

One of the Keys of the exponent rules is "SAMENESS".

- When you have the SAME BASE, (but possibly different exponents) you can combine powers.

e.g. $\frac{x^3 \times x^4}{x^7}$ $= x^{3+4-7}$

$= x^0$

$= 1$

- When you have the SAME EXPONENT (but possibly different bases) you can "combine the bases under the same exponent".

e.g. $\frac{\sqrt[3]{12} \times \sqrt[3]{36}}{\sqrt[3]{16}}$ $= \frac{(12)^{1/3}(36)^{1/3}}{(16)^{1/3}}$

$= \left(\frac{(\overset{3}{\cancel{12}})(\overset{9}{\cancel{36}})}{(\underset{\cancel{4}\ 1}{\cancel{16}})}\right)^{1/3}$ Reduce!

$= 27^{1/3}$

$= 3$

Now we turn to problems involving both numbers and variables being exponentized (not a word, but it should be because of how awesome it sounds).

Keep in mind 1) Exponent Rules
2) BEDMAS

Example 4.4.1

Simplify, leaving you answer with only positive exponents:

a) $(x^3)^2(x^{-8})$

$= x^6 \cdot x^{-8}$

$= x^{6-8}$

$= x^{-2}$

$= \left(\frac{1}{x}\right)^2$

$= \frac{1}{x^2}$

b) $\frac{z^4(z^{-2})}{z^{-3}}$

$= z^{4+(-2)-(-3)}$

$= z^5$

c) $\sqrt{36x^{-6}}$

$(ab)^n$
$a^n b^n$

$= (36x^{-6})^{1/2}$

$= 36^{1/2}(x^{-6})^{1/2}$

$= 6x^{-3}$ only affects the x

$= \frac{6}{x^3}$

d) $\left(\frac{(3x^2y)^{-1}(x^3y^{-4})}{(x^3y^2)^{-2}}\right)^{-2}$

stays

$= \left(\frac{(x^3y^2)^2(x^3y^{-4})}{3x^2y}\right)^{-2}$

$= \left(\frac{(x^6y^4)(x^3y^{-4})}{3x^2y}\right)^{-2}$ Combine the exponents

$= \left(\frac{x^{6+3-2}\,y^{4+(-4)-1}}{3}\right)^{-2}$

$= \left(\frac{x^7y^{-1}}{3}\right)^{-2}$

$= \left(\frac{x^7}{3y}\right)^{-2}$

$= \left(\frac{3y}{x^7}\right)^2$

$= \frac{9y^2}{x^{14}}$

e) $\left(\dfrac{\sqrt{16a^6}}{(a^3)^{-1}}\right)^{\frac{3}{2}}$

$= \left(\dfrac{(16a^6)^{1/2}}{a^{-3}}\right)^{3/2}$

$= \left(\dfrac{4a^3}{a^{-3}}\right)^{3/2}$

$= \left(4a^{3-(-3)}\right)^{3/2}$

$= (4a^6)^{3/2}$

$= \left((4a^6)^{1/2}\right)^3$

$= (2a^3)^3$

$= 8a^9$

f) $\left(\dfrac{(6x^3)^2(6y^3)}{(9xy)^6}\right)^{-\frac{1}{3}}$

$= \left(\dfrac{(6^2x^6)(6^1y^3)}{9^6x^6y^6}\right)^{-1/3}$

Combine like bases

$= \left(\dfrac{6^3 \cancel{x^{6-6}} y^{3-6}}{9^6}\right)^{-1/3}$

$= \left(\dfrac{6^3}{9^6y^3}\right)^{-1/3}$

$= \left(\dfrac{9^6y^3}{6^3}\right)^{1/3}$

$= \dfrac{(9^6)^{1/3} \cdot (y^3)^{1/3}}{(6^3)^{1/3}}$

$= \dfrac{9^2 y}{6}$

$= \dfrac{81y}{6}$

$= \dfrac{27y}{2}$

Success Criteria:

- I can simplify algebraic expressions containing powers by using the exponent laws
- I can simplify algebraic expressions involving radicals

Chapter 4 – Exponential Functions

4.5 – 4.6 – Properties and Transformations of Exponential Functions

Learning Goal: We are learning to identify the characteristics and transformations of the graphs and equations of exponential functions.

Exponential Functions are of the (basic) form: the variable is in the exponent!

$$f(x) = b^{x}$$

(of course, we can apply transformations to this basic, or parent, function!! Fun Times are – a – coming!!)

In the basic exponential function $f(x) = b^x$, b is the base. **THE BASE OF AN EXPONENTIAL FUNCTION IS JUST A NUMBER.** For example, we might have the functions

$f(x) = 2^x$ $\quad$ $g(x) = 3^x$

$h(x) = \left(\frac{1}{3}\right)^x$

What the Base of an Exponential Function tells you

whether the function is describing

Growth OR Decay

when $b > 1$ $\quad$ when $0 < b < 1$

(we only care about positive bases)

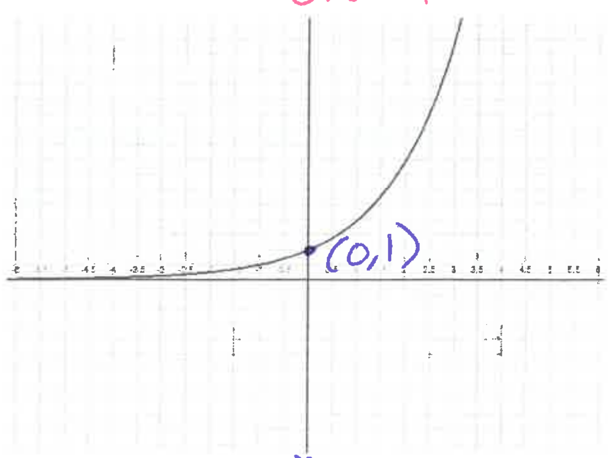
- population
- virus growth
- investments

- radioactive decay
- death of populations
- temperature

Domain and Range of the Basic Exponential Function

Consider the sketches:

Growth

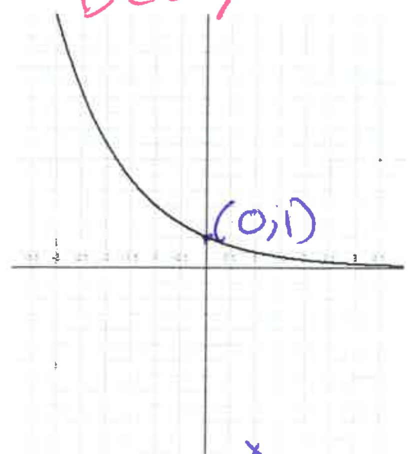


$f(x) = 2^x$

$D_f: \{x \in \mathbb{R}\}$

$R_f: \{f(x) \in \mathbb{R} \mid f(x) > 0\}$

Decay



$g(x) = \left(\frac{1}{2}\right)^x$

$D_g: \{x \in \mathbb{R}\}$

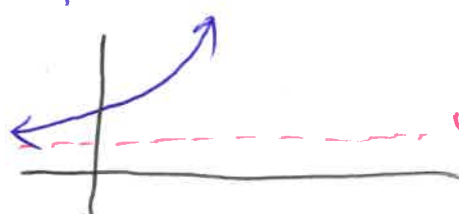
$R_g: \{g(x) \in \mathbb{R} \mid g(x) > 0\}$

y = 0 is a horizontal asymptote

ALL Exponential Functions have a Horizontal ASYMPTOTE (Basic Exponential Functions have $y = 0$ as their Horizontal Asymptote. The Horizontal Asymptote of a Transformed Exponential Function depends on the vertical shift

eg: $f(x) = 2^x + 1$

H.A.



ALL BASIC Exponential Functions pass through the point $(0,1)$.

Transformed Exponential Functions will have a y-intercept, but depends on vertical shift, horizontal shift, vertical stretch, and horizontal stretch.

we need to calculate it using $x = 0$.

The Transformed Exponential Function

The general form of an exponential function is:

$$f(x)=a\cdot b^{k(x-d)}+c$$

Where: a = vertical stretch

c = vertical shift
(horizontal asymptote)

k = horizontal stretch. Factored ~~[illegible]~~ out of

d = horizontal shift

$f(x) = 2\cdot 3^x$

This is base 3, w/ stretch factor of 2.

Example 4.6.1

State the transformations applied to the parent function $f(x)=3^x$. Also state the y-intercept, and the equation of the horizontal asymptote of the transformed function.

Factor k!

$g(x)=-2\cdot 3^{3x+3}+4$

$= -2\cdot 3^{3(x+1)} + 4$

Vertical
- Flip
- Stretch x 2
- Shift up 4

Horizontal
~~Shift~~
Stretch $\times \frac{1}{3}$
Shift 1 Left

time elapsed

temp of the liquid

Example 4.6.2

From your Text: Page 252 #7

7. A cup of hot liquid was left to cool in a room whose temperature was 20 °C. The temperature changes with time according to the function $T(t) = 80\left(\frac{1}{2}\right)^{\frac{t}{30}} + 20$. Use your knowledge of transformations to sketch this function. Explain the meaning of the y-intercept and the asymptote in the context of this problem.

time is "chunked" into 30 minute intervals when the temperature is recorded.

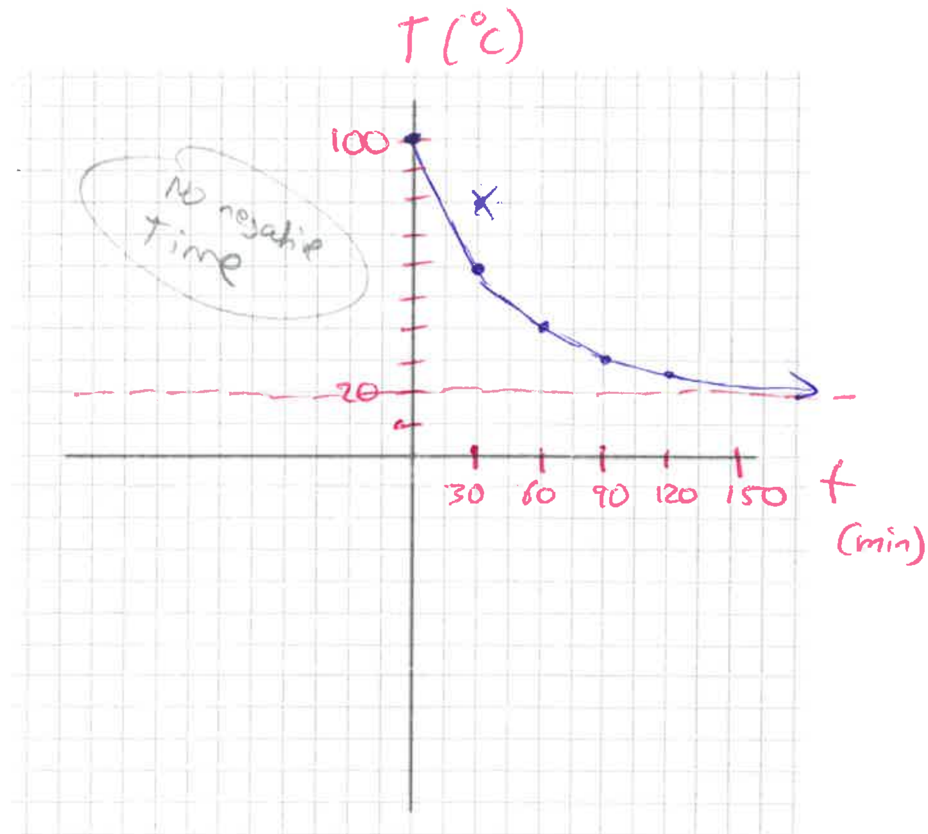
Analysis

1) Decay pattern $\left(b = \frac{1}{2}\right)$

2) y-int $(t=0)$

$T(0) = 100.$

3) H.A. $y = 20$



Points

t	$T(t)$
30	$80\left(\frac{1}{2}\right)^{\frac{30}{30}} + 20 = 60$
60	40
90	30
120	25

Domain is restricted

$D_f = \{t \in \mathbb{R} \mid t \geq 0\}$

$R_T = \{T(t) \in \mathbb{R} \mid 20 < T(t) \leq 100\}$

The liquid cannot go cooler than room temperature.

The y-int is the initial temp of the liquid.

Example 4.6.3

Parent function

From your Text: Page 252 #5a

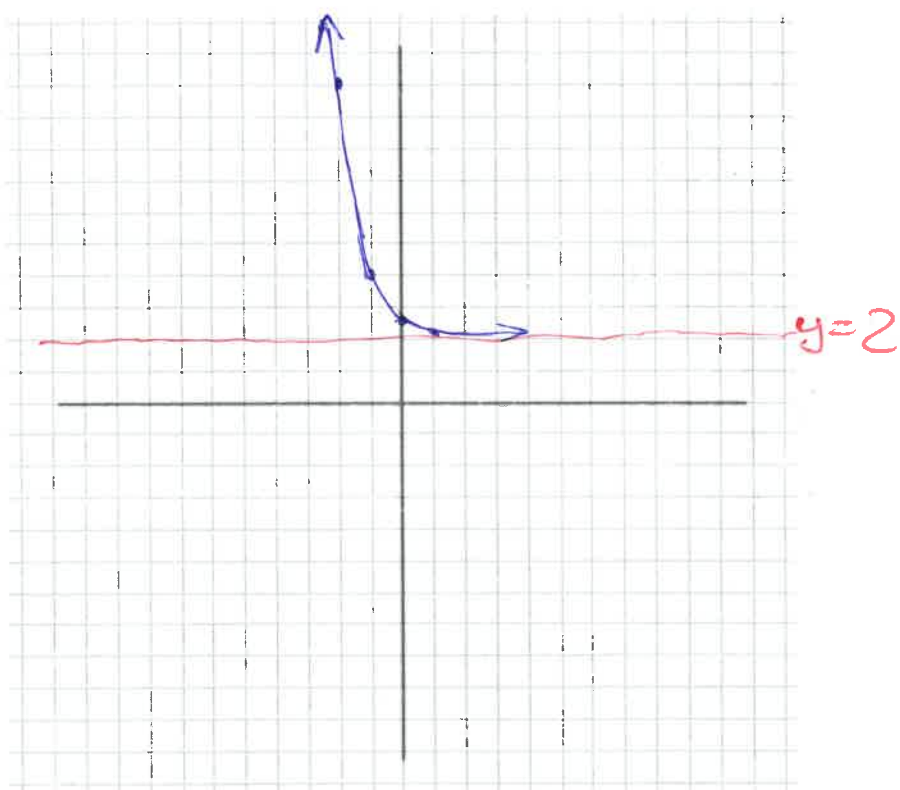
Let $f(x)=4^x$. For the function which follows,

- State the transformations applied to $f(x)$
- State the y-intercept, and the horizontal asymptote
- Sketch the transformed function, and write the function "properly"
- State the domain and range of the transformed function

$g(x)=0.5f(-x)+2$ Apply these to the parent.

$$g(x) = 0.5 \cdot 4^{-x} + 2$$

$$= 0.5\left(\frac{1}{4}\right)^x + 2$$



vertical

Stretch $\times \frac{1}{2}$

Shift $+2$

Horizontal

Flip $(-x)$

y-int @ $x=0$

$$g(0) = 0.5\left(\frac{1}{4}\right)^0 + 2$$

$$= \frac{5}{2} \text{ or } 2.5$$

H.A. @ $y=2$

x	g(x)
-2	10
-1	4
0	2.5
1	2.125

Success Criteria:

- I can identify the graph of an exponential function
- I can identify and apply the four transformations (a, k, d, c) to the equation of an exponential function

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Chapter 4 – Exponential Functions

4.7 – Applications of Exponential Functions

Learning Goal: We are learning to use exponential functions to solve problems involving exponential growth and decay.

Anything in the real world which grows, or decays can be "MODELED" (or in some sense "*DESCRIBED*") with words, or pictures or mathematics. **Mathematical models** are useful for getting solutions to problems, and making predictions.

So far in Mathematics 11U we have studied the basics of functions in general (chapter 1), we've done some algebra (chapter 2), and we've examined Quadratic functions (chapter 3). Part of our study of Quadratics was learning how to use the vertex of a parabola to answer questions about maxima and minima for some real word problems. For example we saw a question where we tried to maximize revenue for a school store. Quadratic MODELS are very useful for solving max/min problems.

In this lesson we want to work on LEARNING HOW TO SOLVE PROBLEMS DEALING WITH **GROWTH AND DECAY.** We have to decide what type of function will best model (or describe) the type of growth/decay seen in the problem (hint: for this lesson we'll be examining Exponential Growth and Decay, and therefore we expect that exponential functions will be used…shocking, I know)

Q. What is Exponential Growth or Decay?

Consider the following:

A single cell divides into two "daughter" cells. Both daughter cells divide resulting in four cells. Those four cells each divide and we now have a population of 8

Describe, using mathematics, how the cell population changes from generation to generation.

The population doubles !

Generation	G_0	G_1	G_2	G_3
# cells	1	2	4	8
Power of 2	2^0	2^1	2^2	2^3

So, for any generation x,

$g(x) = 2^x$

0.035

Example 4.7.1

Being a financial wizard, you deposit $1,000 into an account which pays 3.5% interest, Compounded annually.

a) Determine who much money is in your account after t =1,2,3, and 4 years.

b) Determine a mathematical model which can describe how the value of the account is changing from year to year.

Year 0: 1000

Year 1: 1000 + interest
$= 1000 + (0.035)(1000)$
$= 1035$
$1000(1.035)$

Year 2: 1035 + interest
$= 1035 + (0.035)(1035)$
Factor
$= 1035(1 + 0.035)$
$= 1000(1.035)(1.035)$
$= 1000(1.035)^2$

Year 3: Year 2 + interest
$= 1000(1.035)^2 + 0.035(1000(1.035)^2)$
$= 1000(1.035)^2(1.035)$
$= 1000(1.035)^3$

Year 4
$A = 1000(1.035)^4$
amount @ end of the year

b) $A(t) = 1000(1.035)^t$
$(1 + 0.035)$

Definition 4.7.1

A function describing Exponential Growth is of the form:

$$A(x) = A_0(1+r)^x$$

growth rate as a decimal. $(0 < r < 1)$

A naught, means initial amount

A function describing Exponential Decay is of the form:

$$A(x) = A_0(1-r)^x$$

General Equations

%'s are not numbers.
Convert to decimals.

Example 4.7.2

From your text, Pg. 263

10. In each case, write an equation that models the situation described. Explain what each part of each equation represents.

a) the percent of colour left if blue jeans lose 1% of their colour every time they are washed

b) the population if a town had 2500 residents in 1990 and grew at a rate of 0.5% each year after that for *t* years

a) Decay in Colour

1% = 0.01

start w/ 100% colour, so 1

$$C_{(w)} = C_0(1-r)^w$$
$$= 1(1-0.01)^w$$
$$= 1(0.99)^w$$

b) Growth in Population

$$P_{(t)} = P_0(1+r)^t$$

r = 0.005 (0.5%)

$$P_{(t)} = 2500(1+0.005)^t$$
$$= 2500(1.005)^t$$

Example 4.7.3

A new car depreciates at a rate of 20% per year. Steve bought a new car for $26,000.

r = 0.20

Decay!

$V_{(t)}$ value based on time.

a) Write the equation that models this scenario.

$$V_{(t)} = 26{,}000(1-0.2)^t = 26000(0.8)^t$$

b) How much will Steve's car be worth in 3 years?

t = 3

$$W_{(3)} = 26000(0.8)^3$$
$$= \$13{,}312$$

c) When will Steve's car be worth $4000?

$$W_{(t)} = 4000 = 26000(0.8)^t$$

Try 6 $\quad f_{(6)} = \$6{,}815.74$

Try 8 $\quad f_{(8)} = \$4{,}362.08$

In about 8-9 years.

Additional Applications – *DOUBLING AND HALF-LIFE*

Thus far, we have only seen examples with single period rates: "yearly" "monthly" "daily"

Unfortunately, it's not always that simple…Our rates could be…

Every 3 years $a^{\frac{t}{3}}$ Every 6 hours $a^{\frac{t}{6}}$ Every 4 days $a^{\frac{t}{4}}$

How do we deal with the exponent in these cases?

$y = ab^{\frac{t}{p}}$ t = time elapsed
p = period for one complete cycle

t + p must be in the same units.

Example (Doubling)

A species of bacteria has a population of 300 at 9 am. It doubles every 3 hours.

$y = 2^x$

a) Write the function that models the growth of the population, P, at any hour, t

$P(t) = 300(2)^{\frac{t}{3}}$

b) How many will there be at 6 pm?

9am – 6pm = 9 hours

$P(9) = 300(2)^{9/3}$
$= 300(2)^3$
$= 2400$

c) How many will there be at 11 pm?

9am – 11 pm = 14 hours

$P(14) = 300(2)^{14/3}$
$= 7619.525$
$= 7620$ cells

d) Determine the time at which the population first exceeds 3000.

try 8pm = 11 hrs

$P(11) = 300(2)^{11/3}$
$= 3809.76$

Try 7pm = 10 hrs

$P(10) = 300(2)^{10/3}$
$= 3023.81$

Shortly before 7pm.

Example (Half-Life)

A 200g sample of radioactive material has a half-life of 138 days. How much will be left in 5 years?

A decay question $\left(\frac{1}{2}\right)$ is the factor)

5 years = 5 x 365

= 1825 days

$$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{138}}$$

$$A(1825) = 200g\left(\frac{1}{2}\right)^{\frac{1825}{138}}$$

$$= 0.02089 \text{ grams}$$

Success Criteria:

- I can differentiate between exponential growth and exponential decay
- I can use the exponential function $f(x) = ab^x$ to model and solve problems involving exponential growth and decay
    - Growth rate is $b = 1 + r$. Decay rate is $b = 1 - r$.
    - r is a DECIMAL, not a percent!!!!!