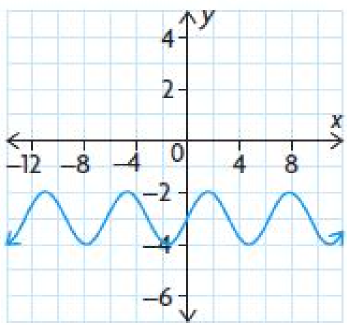
Chapter 1 Problem Set – Introduction to Functions

1.1 Functions #1 – 3, 5, 6, 7b-f, 9, 10, 12

1. State the domain and range of each relation. Then determine whether the relation is a function, and justify your answer.

a)



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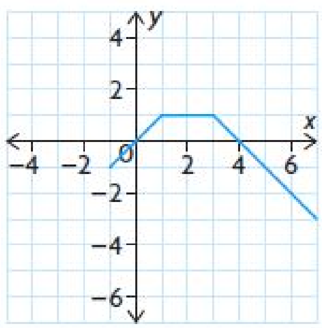
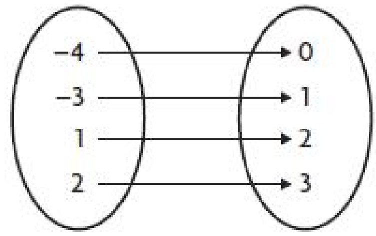


c) $\{(1, 4), (1, 9), (2, 7), (3, -5), (4, 11)\}$

d) $y = 3x - 5$

e)



f) $y = -5x^2$

2. State the domain and range of each relation. Then determine whether the relation is a function, and justify your answer.

a) $y = -2(x + 1)^2 - 3$

b) $y = \dfrac{1}{x + 3}$

c) $y = 2^{-x}$

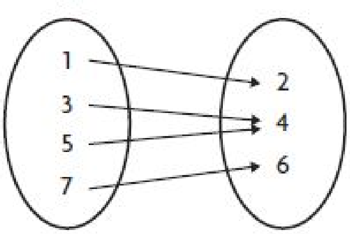
d) $y = \cos x + 1$

e) $x^2 + y^2 = 9$

f) $y = 2 \sin x$

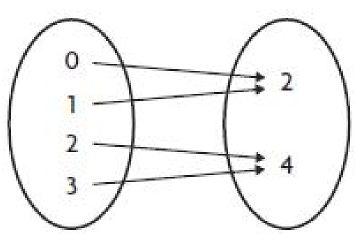
3. Determine whether each relation is a function, and state its domain and range.

a)



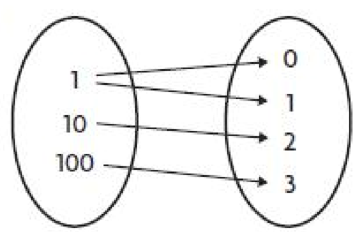
b) $\{(2, 3), (1, 3), (5, 6), (0, -1)\}$

c)



d) $\{(2, 5), (6, 1), (2, 7), (8, 3)\}$

e)

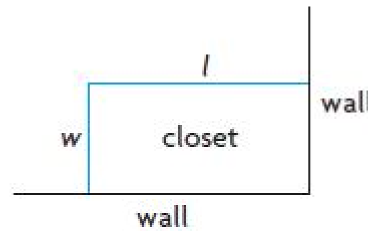


f) $\{(1, 2), (2, 1), (3, 4), (4, 3)\}$

5. Determine the equations that describe the following function rules:

a) The input is 3 less than the output.
b) The output is 5 less than the input multiplied by 2.
c) Subtract 2 from the input and then multiply by 3 to find the output.
d) The sum of the input and output is 5.

6. Martin wants to build an additional closet in a corner of his bedroom. Because the closet will be in a corner, only two new walls need to be built. The total length of the two new walls must be 12 m. Martin wants the length of the closet to be twice as long as the width, as shown in the diagram.



a) Explain why $l = 2w$.
b) Let the function $f(l)$ be the sum of the length and the width. Find the equation for $f(l)$.
c) Graph $y = f(l)$.
d) Find the desired length and width.

7. **A** The following table gives Tina's height above the ground while riding a Ferris wheel, in relation to the time she was riding it.

Time (s)	0	20	40	60	80	100	120	140	160	180	200	220	240
Height (m)	5	10	5	0	5	10	5	0	5	10	5	0	5

b) What is the domain?
c) What is the range?
d) Is this relation a function? Justify your answer.
e) Another student sketched a graph, but used height as the independent variable. What does this graph look like?
f) Is the relation in part e) a function? Justify your answer.

9. Explain why a relation that fails the vertical line test is not a function.

10. Consider the relation between x and y that consists of all points (x, y) such that the distance from (x, y) to the origin is 5.

a) Is (4, 3) in the relation? Explain.
b) Is (1, 5) in the relation? Explain.
c) Is the relation a function? Explain.

12. **T** The factors of 4 are 1, 2, and 4. The sum of the factors is $1 + 2 + 4 = 7$. The sum of the factors is called the sigma function. Therefore, $f(4) = 7$.

a) Find $f(6)$, $f(7)$, and $f(8)$.
b) Is $f(15) = f(3) \times f(5)$?
c) Is $f(12) = f(3) \times f(4)$?
d) Are there others that will work?

1.2 Properties of Functions – #5, 7 – 11

5. For each function, determine $f(-x)$ and $-f(-x)$ and compare it with $f(x)$. Use this to decide whether each function is even, odd, or neither.

a) $f(x) = x^2 - 4$

b) $f(x) = \sin x + x$

c) $f(x) = \frac{1}{x} - x$

d) $f(x) = 2x^3 + x$

e) $f(x) = 2x^2 - x$

f) $f(x) = |2x + 3|$

7. Identify a parent function whose graph has the given characteristics.

a) The domain is not all real numbers, and $f(0) = 0$.

b) The graph has an infinite number of zeros.

c) The graph is even and has no sharp corners.

d) As x gets negatively large, so does y. As x gets positively large, so does y.

8. Each of the following situations involves a parent function whose graph has been translated. Draw a possible graph that fits the situation.

a) The domain is $\{x \in \mathbf{R}\}$, the interval of increase is $(-\infty, \infty)$, and the range is $\{f(x) \in \mathbf{R} \mid f(x) > -3\}$.

b) The range is $\{g(x) \in \mathbf{R} \mid 2 \leq g(x) \leq 4\}$.

c) The domain is $\{x \in \mathbf{R} \mid x \neq 5\}$, and the range is $\{h(x) \in \mathbf{R} \mid h(x) \neq -3\}$.

9. Sketch a possible graph of a function that has the following characteristics:

- $f(0) = -1.5$
- $f(1) = 2$
- There is a vertical asymptote at $x = -1$.
- As x gets positively large, y gets positively large.
- As x gets negatively large, y approaches zero.

10. T

a) $f(x)$ is a quadratic function. The graph of $f(x)$ decreases on the interval $(-\infty, -2)$ and increases on the interval $(2, \infty)$. It has a y-intercept at $(0, 4)$. What is a possible equation for $f(x)$?

b) Is there only one quadratic function, $f(x)$, that has the characteristics given in part a)?

c) If $f(x)$ is an absolute value function that has the characteristics given in part a), is there only one such function? Explain.

11. $f(x) = x^2$ and $g(x) = |x|$ are similar functions. How might you describe the difference between the two graphs to a classmate, so that your classmate can tell them apart?

1.3 Transformations of Functions Review – Hand outs.

1.4 Inverses of Functions – #2 – 4, 7, 9, 12, 13, 15

2. Given the domain and range of a function, state the domain and range of the inverse relation.

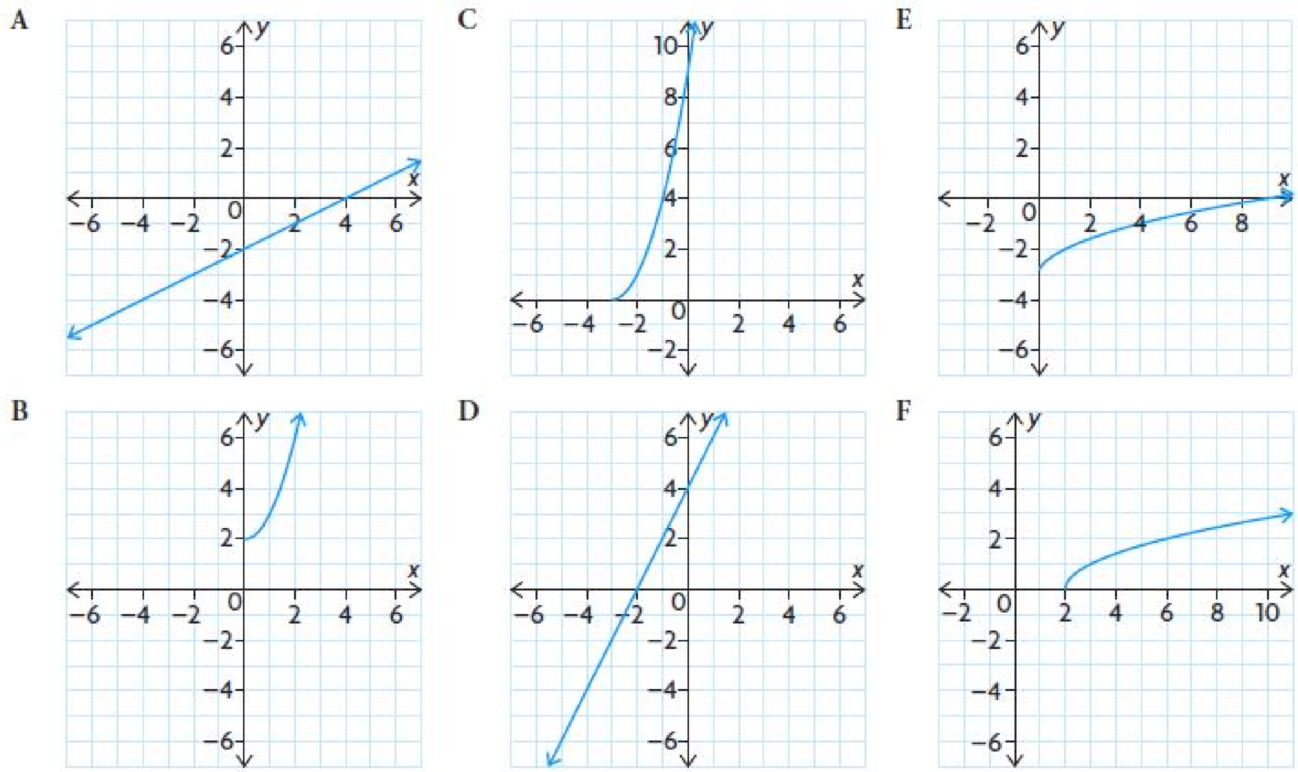
a) $D = \{x \in \mathbf{R}\}, R = \{y \in \mathbf{R}\}$

b) $D = \{x \in \mathbf{R} \mid x \geq 2\}, R = \{y \in \mathbf{R}\}$

c) $D = \{x \in \mathbf{R} \mid x \geq -5\}, R = \{y \in \mathbf{R} \mid y < 2\}$

d) $D = \{x \in \mathbf{R} \mid x < -2\}, R = \{y \in \mathbf{R} \mid -5 < y < 10\}$

3. Match the inverse relations to their corresponding functions.



4. Consider the function $f(x) = 2x^3 + 1$.

K

a) Find the ordered pair $(4, f(4))$ on the function.

b) Find the ordered pair on the inverse relation that corresponds to the ordered pair from part a).

c) Find the domain and range of f.

d) Find the domain and the range of the inverse relation of f.

e) Is the inverse relation a function? Explain.

7. a) The equation $F = \frac{9}{5}C + 32$ can be used to convert a known Celsius temperature, C, to the equivalent Fahrenheit temperature, F. Find the inverse of this relation, and describe what it can be used for.

A

b) Use the equation given in part a) to convert 20 °C to its equivalent Fahrenheit temperature. Use the inverse relation to convert this Fahrenheit temperature back to its equivalent Celsius temperature.

9. If $f(x) = kx^3 - 1$ and $f^{-1}(15) = 2$, find k.

T

12. Determine the inverse of each function.

a) $f(x) = 3x + 4$
b) $h(x) = -x$
c) $g(x) = x^3 - 1$
d) $m(x) = -2(x + 5)$

13. A function g is defined by $g(x) = 4(x - 3)^2 + 1$.

a) Determine an equation for the inverse of $g(x)$.
b) Solve for y in the equation for the inverse of $g(x)$.
c) Graph $g(x)$ and its inverse using graphing technology.
d) At what points do the graphs of $g(x)$ and its inverse intersect?
e) State **restrictions** on the domain or range of g so that its inverse is a function.
f) Suppose that the domain of $g(x)$ is $\{x \in \mathbf{R} \mid 2 \leq x \leq 5\}$. Is the inverse a function? Justify your answer.

15. Do you have to restrict either the domain or the range of the function $y = \sqrt{x + 2}$ to make its inverse a function? Explain.

1.5 Piecewise Defined Functions – (Abs. Value.) Pg. 16 #2, 4 – 8, 10

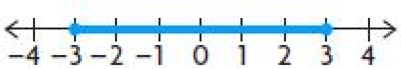
2. Evaluate.

a) $|-22|$
b) $-|-35|$
c) $|-5 - 13|$
d) $|4 - 7| + |-10 + 2|$
e) $\dfrac{|-8|}{-4}$
f) $\dfrac{|-22|}{|-11|} + \dfrac{-16}{|-4|}$

4. Graph on a number line.

a) $|x| < 8$
b) $|x| \geq 16$
c) $|x| \leq -4$
d) $|x| > -7$

5. Rewrite using absolute value notation.

a)


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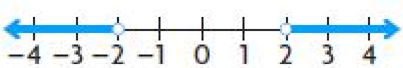

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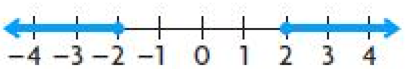

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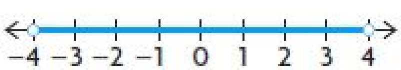


6. Graph $f(x) = |x - 8|$ and $g(x) = |-x + 8|$.

a) What do you notice?
b) How could you have predicted this?

7. Graph the following functions.

a) $f(x) = |x - 2|$

b) $f(x) = |x| + 2$

c) $f(x) = |x + 2|$

d) $f(x) = |x| - 2$

8. Compare the graphs you drew in question 7. How could you use transformations to describe the graph of $f(x) = |x + 3| - 4$?

10. Predict what the graph of $f(x) = 3 - |2x - 5|$ will look like. Verify your prediction using graphing technology.

(Piecewise) #1 – 5, 7 – 9

1. Graph each piecewise function.

a) $f(x) = \begin{cases} 2, \text{ if } x < 1 \\ 3x, \text{ if } x \geq 1 \end{cases}$

d) $f(x) = \begin{cases} |x + 2|, \text{ if } x \leq -1 \\ -x^2 + 2, \text{ if } x > -1 \end{cases}$

b) $f(x) = \begin{cases} -2x, \text{ if } x < 0 \\ x + 4, \text{ if } x \geq 0 \end{cases}$

e) $f(x) = \begin{cases} \sqrt{x}, \text{ if } x < 4 \\ 2^x, \text{ if } x \geq 4 \end{cases}$

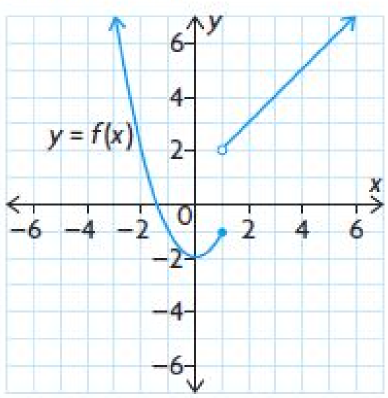
c) $f(x) = \begin{cases} |x|, \text{ if } x \leq -2 \\ -x^2, \text{ if } x > -2 \end{cases}$

f) $f(x) = \begin{cases} \dfrac{1}{x}, \text{ if } x < 1 \\ -x, \text{ if } x \geq 1 \end{cases}$

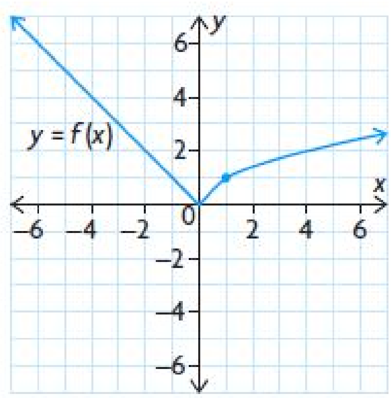
2. State whether each function in question 1 is continuous or not. If not, state where it is discontinuous.

3. Write the algebraic representation of each piecewise function, using function notation.

a)



b)



4. State the domain of each piecewise function in question 3, and comment on the continuity of the function.

5. Graph the following piecewise functions. Determine whether each **K** function is continuous or not, and state the domain and range of the function.

a) $f(x) = \begin{cases} 2, \text{ if } x < -1 \\ 3, \text{ if } x \geq -1 \end{cases}$

b) $f(x) = \begin{cases} -x, \text{ if } x \leq 0 \\ x, \text{ if } x > 0 \end{cases}$

c) $f(x) = \begin{cases} x^2 + 1, \text{ if } x < 2 \\ 2x + 1, \text{ if } x \geq 2 \end{cases}$

d) $f(x) = \begin{cases} 1, \text{ if } x < -1 \\ x + 2, \text{ if } -1 \leq x \leq 3 \\ 5, \text{ if } x > 3 \end{cases}$

7. Many income tax systems are calculated using a tiered method. Under a certain tax law, the first \$100 000 of earnings are subject to a 35% tax; earnings greater than \$100 000 and up to \$500 000 are subject to a 45% tax. Any earnings greater than \$500 000 are taxed at 55%. Write a piecewise function that models this situation.

8. Find the value of k that makes the following function continuous. **T** Graph the function.

$$f(x) = \begin{cases} x^2 - k, \text{ if } x < -1 \\ 2x - 1, \text{ if } x \geq -1 \end{cases}$$

9. The fish population, in thousands, in a lake at any time, x, in years is modelled by the following function:

$$f(x) = \begin{cases} 2^x, \text{ if } 0 \leq x \leq 6 \\ 4x + 8, \text{ if } x > 6 \end{cases}$$

This function describes a sudden change in the population at time $x = 6$, due to a chemical spill.

a) Sketch the graph of the piecewise function.

b) Describe the continuity of the function.

c) How many fish were killed by the chemical spill?

d) At what time did the population recover to the level it was before the chemical spill?

e) Describe other events relating to fish populations in a lake that might result in piecewise functions.