Advanced Functions

Teacher Notes

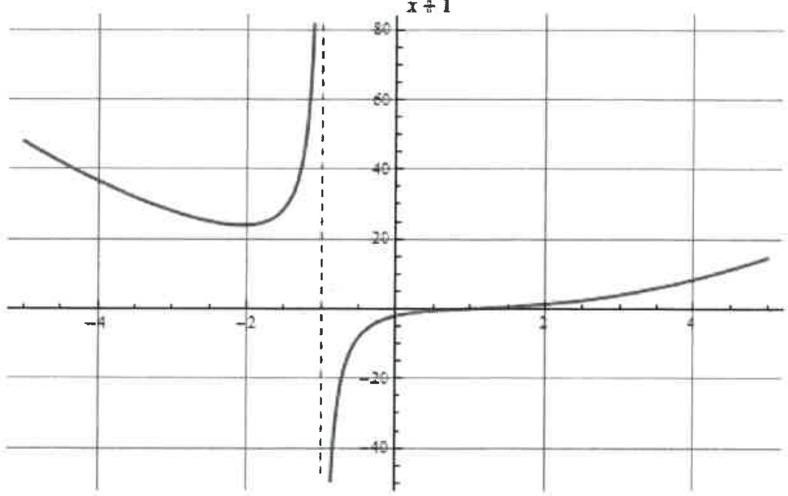
Course Notes

Unit 4 – Rational Functions, Equations and Inequalities

We are learning to

- *sketch the graphs of simple rational functions*
- *solve rational equations and inequalities with and without tech*
- *apply the techniques and concepts to solve problems involving rational models*

$$f(x) = \frac{x^3 - 2x^2 + 3x - 2}{x + 1}$$



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MATH@TD

Unit 4 – Rational Functions, Equations and Inequalities

Contents with suggested problems from the Nelson Textbook (Chapter 5)

4.1 Rational Functions, Domain and Asymptotes

Learning Goal: We are learning to identify the asymptotes of rational functions.

Definition 4.1.1

A **Rational Function** is of the form

$f(x) = \frac{p(x)}{q(x)}$, $q(x) \neq 0$ and both $p(x)$ and $q(x)$ are polynomials.

e.g. $f(x) = \frac{3x^2 - 5x + 1}{2x - 1}$ is a rational function.

$g(x) = \frac{\sqrt{2x+5}}{3x-2}$ not rational bc $\sqrt{2x+5}$ is not a polynomial

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Domain

Definition 4.1.2

Given a rational function $f(x) = \frac{p(x)}{q(x)}$, then the **natural domain** of $f(x)$ is given by

$D_f = \{x \in \mathbb{R} \mid q(x) \neq 0\}$

the zeros of $q(x)$

Example 4.1.1

Determine the natural domain of $f(x) = \frac{x^2 - 4}{x - 3}$.

3 makes this not work

$D_f: \{x \in \mathbb{R} \mid x \neq 3\}$

$x \in (-\infty, 3) \cup (3, \infty)$

Asymptotes

There are 3 possible types of **asymptotes**:

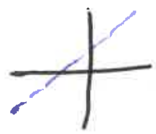
1) Vertical Asymptotes

A vertical line w/ equation $x = \#$

2) Horizontal Asymptotes

A horizontal equation w/ $y = \#$

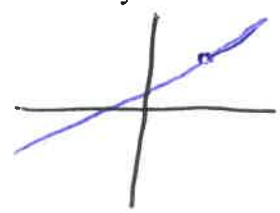
3) Oblique Asymptotes



equation is $y = mx + b$

Vertical Asymptotes

A rational function $f(x) = \dfrac{p(x)}{q(x)}$ MIGHT have a V.A. when $q(x) = 0$, but there may be a hole discontinuity instead. A quick bit of algebra will dispense the mystery.



Example 4.1.2

Determine the domain, and V.A., or hole discontinuities for:

a) $f(x) = \dfrac{5x}{x^2 - x - 6}$

$$f(x) = \frac{5x}{(x-3)(x+2)}$$

These factors stay. $\therefore$ V.A.

$$D_f: \{x \in \mathbb{R} \mid x \neq -2, x \neq 3\}$$

$$x \in (-\infty, -2) \cup (-2, 3) \cup (3, \infty)$$

b) $h(x) = \frac{x+3}{x^2-9}$

$h(x) = \frac{\cancel{x+3}}{\cancel{(x+3)}(x-3)}$

V.A. at $x = 3$

Hole at $x = -3$ because it disappeared

$D_h = \{x \in \mathbb{R} \mid x \neq -3, x \neq 3\}$

So... FACTOR EVERYTHING

c) $g(x) = \frac{x^2-4}{x+2}$

$g(x) = \frac{\cancel{(x+2)}(x-2)}{\cancel{x+2}}$

Hole at $x = -2$

What is the y-value of the hole?
well, we are left with...

$g(x) = x - 2$

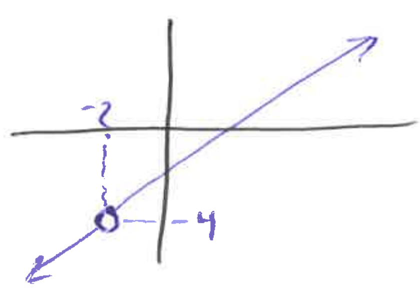
Sub in $x = -2$

$g(x) = (-2) - 2$
$= -4$

So hole at $(-2, -4)$

$D_g: \{x \in \mathbb{R} \mid x \neq -2\}$

$R_g: \{g(x) \in \mathbb{R} \mid g(x) \neq -4\}$



Horizontal Asymptotes

Here we are concerned with the END behaviour of the rational function.

i.e. We are asking, given a rational function $f(x)=\dfrac{p(x)}{q(x)}$, how is $f(x)$ behaving as $x \to \pm\infty$.

Now, since $p(x)$ and $q(x)$ are both polynomials, they have an order (degree). We must consider **three possible situations regarding their order:**

1) Order of $p(x)$ (top) $>$ Order of $q(x)$ (bottom)

e.g. $f(x)=\dfrac{x^3-2}{x^2+1}$

Divide each term by the highest order term. Then "plug in ∞"

$$f(x)=\frac{\frac{x^3}{x^3}-\frac{2}{x^3}}{\frac{x^2}{x^3}+\frac{1}{x^3}}=\frac{1-\cancel{\frac{2}{x^3}}^{0}}{\cancel{\frac{x^2}{x^3}}^{0}+\cancel{\frac{1}{x^3}}^{0}}=\frac{1}{0}$$

$\therefore$ No Horizontal Asymptote

2) Order of numerator $=$ Order of denominator

e.g. $f(x)=\dfrac{2x^2-3x+1}{3x^2+4x-5}$

$$f(x)=\frac{\frac{2\cancel{x^2}}{\cancel{x^2}}-\cancel{\frac{3x}{x^2}}^{0}+\cancel{\frac{1}{x^2}}^{0}}{\frac{3\cancel{x^2}}{\cancel{x^2}}+\cancel{\frac{4x}{x^2}}^{0}-\cancel{\frac{5}{x^2}}^{0}}=\frac{2-0+0}{3+0-0}=\frac{2}{3}$$

$\boxed{\therefore\ y=\tfrac{2}{3} \text{ is the H.A.}}$

e.g. Determine the horizontal asymptote of $g(x)=\dfrac{3x-4x^5}{5x^5+2x-1}$

Shortcut: It is the coefficients of the highest degree term

$\therefore$ H.A. is $y=-\dfrac{4}{5}$

3) Order of numerator $p(x) <$ Order of denominator $q(x)$

e.g. $f(x) = \dfrac{x^2 - 5x + 6}{x^5 + 7}$

$$f(x) = \frac{\cancel{\frac{x^2}{x^5}}^{0} - \cancel{\frac{5x}{x^5}}^{0} + \cancel{\frac{6}{x^5}}^{0}}{\cancel{\frac{x^5}{x^5}}^{1} + \cancel{\frac{7}{x^5}}^{0}} = \frac{0 - 0 + 0}{1 + 0} = \frac{0}{1}$$

$$\boxed{\therefore \text{ H.A. is at } y = 0}$$

Oblique Asymptotes

These occur when the order of the top is EXACTLY ONE greater than the order of the bottom.

e.g. $f(x) = \dfrac{x^2 - 2x + 3}{x - 1}$ $\rightarrow$ $\dfrac{O2}{O1}$ Has an O.A.

With Oblique Asymptotes we are still dealing with end behaviours

O.A. have the form $y = mx + b$ (shocking, I know!) The question we have to face is this:

How do we find the line representing the O.A.?

Ans: By polynomial Division!

$\therefore$ The O.A. is the Quotient

for $f(x) = \dfrac{x^2 - 2x + 3}{x - 1}$

$$\begin{array}{c|ccc} 1 & 1 & -2 & 3 \\ & & 1 & -1 \\ \hline & \boxed{1 \quad -1} & & \boxed{2} \end{array}$$

O.A. Remainder

Equation is $y = 1x - 1$

Same example

(Rough) Sketch of $f(x)=\dfrac{x^2-2x+3}{x-1}$

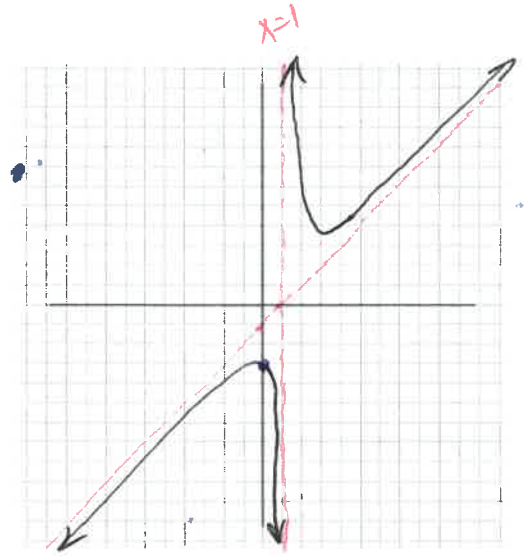
V.A. $x=1$

H.A. None

O.A. $y=x-1$

y-int @ $\dfrac{(0)^2-2(0)+3}{(0)-1}=\dfrac{3}{-1}$

$(0,-3)$



Example 4.1.3

Determine the equations of all asymptotes, and any hole discontinuities for:

a) $f(x)=\dfrac{x+2}{x^2+3x+2}$ → order 1 / → order 2

$f(x)=\dfrac{\cancel{x+2}}{(x+1)\cancel{(x+2)}}$

$f(x)=\dfrac{1}{x+1}$

Denom.

V.A.	$x=-1$
Hole	$x=-2$
H.A.	$y=0$
O.A.	N/A

b) $g(x)=\dfrac{4x^2-25}{x^2-9}$

$g(x)=\dfrac{(2x-5)(2x+5)}{(x+3)(x-3)}$

V.A	$x=-3,\ x=3$
Hole	N/A
H.A.	$y=4$
O.A.	N/A

will be oblique $\frac{O2}{O1}$

c) $h(x)=\dfrac{x^2+0x+0}{x+3}$

$$\begin{array}{r|rrr} -3 & 1 & 0 & 0 \\ & & -3 & 9 \\ \hline & 1 & -3 & 9 \end{array}$$

$y = x - 3$

V.A.	$x=-3$
Hole	N/A
H.A.	N/A
O.A	$y=x-3$

Cannot have both a H.A. and O.A.

Example 4.1.4

Determine an equation for a function with a vertical asymptote at $x=-3$, and a horizontal asymptote at $y=0$.

$x+3$ in denom

bottom bigger

$f(x) = \dfrac{8}{x+3}$ OR $g(x) = \dfrac{3x^2-2x+10,000}{(x+3)(x-8)(x+5)}$

Example 4.1.5

Determine an equation for a function with a hole discontinuity at $x=3$.

Common factor of $(x-3)$

$f(x) = \dfrac{(x-3)(x^2-8x+5)}{(x-3)(2x^3+8n-200)}$

Success Criteria:

- I can identify a hole when there is a common factor between *p(x)* and *q(x)*
- I can identify a vertical asymptote as the zeros of *q(x)*
- I can identify a horizontal asymptote by studying the degrees of *p(x)* and *q(x)*
- I can identify an oblique asymptote when the degree of *p(x)* is exactly 1 greater than *q(x)*

4.2 Graphs of Rational Functions

Learning Goal: We are learning to sketch the graphs of rational functions.

Note: In Advanced Functions we will only consider rational functions of the form

$$f(x)=\frac{ax+b}{cx+d}$$

Rational Functions of the form $f(x)=\frac{ax+b}{cx+d}$ will have:

1) One Vertical Asymptote

$cx+d=0$

$x=-\frac{d}{c}$, unless $c=0$ (no variable in denominator)

2) One Zero (unless $a=0$ (no variable in numerator)

$0=\frac{ax+b}{\cancel{cx+d}}$ $=ax+b$ $\therefore x=-\frac{b}{a}$

3) Functional Intercept

"y"

when $x=0$

$f(0)=\frac{a(0)+b}{c(0)+d}=\frac{b}{d}$ So $\left(0,\frac{b}{d}\right)$

4) A Horizontal Asymptote

option 1: $y=\frac{a}{c}$

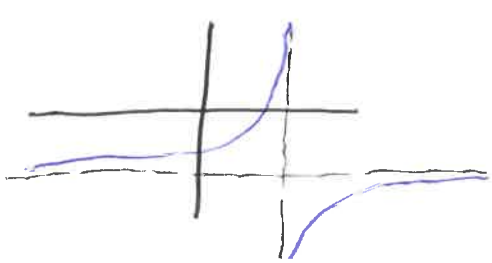
option 2: $y=0$ (if $a=0$)

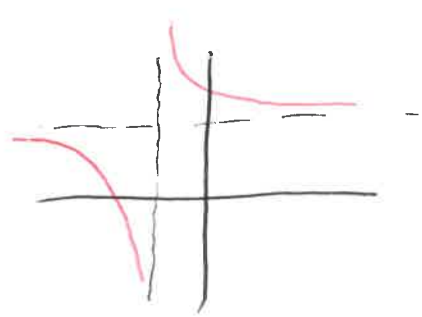
Degree of numerator is smaller

5) These functions will always be either

Always increasing OR Always decreasing.

meaning NO turning points

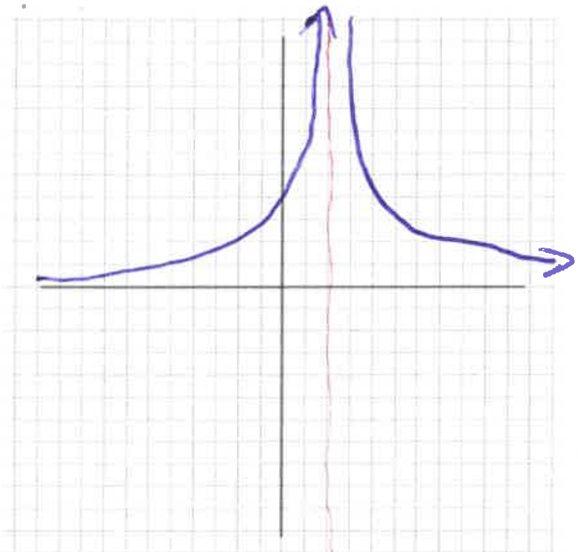




Functional Behaviour Near A Vertical Asymptote

There are **FOUR** possible functional behaviours near a V.A.:

1



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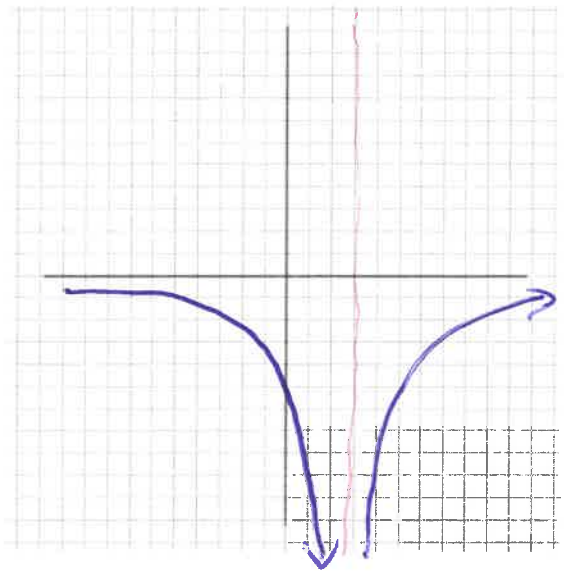

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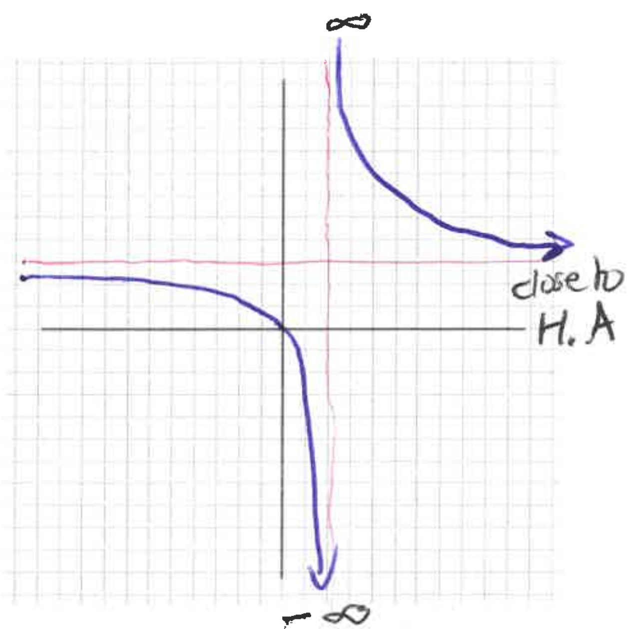

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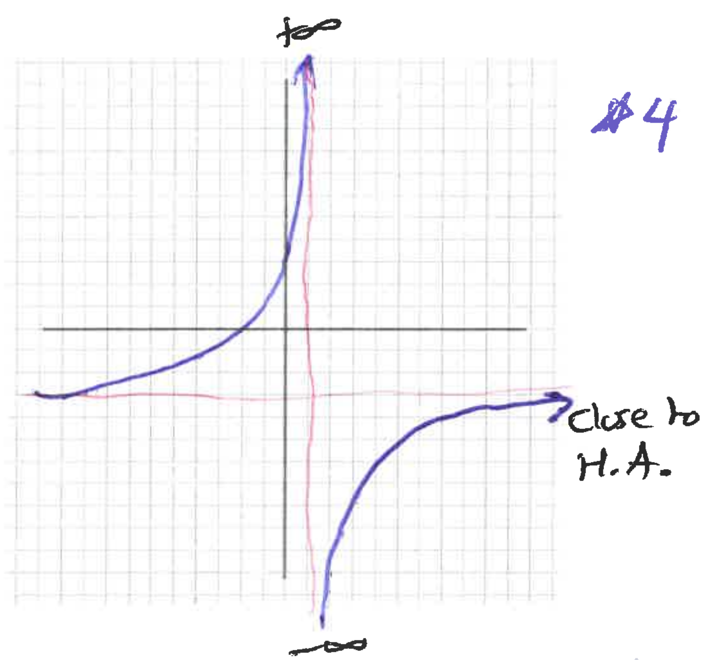


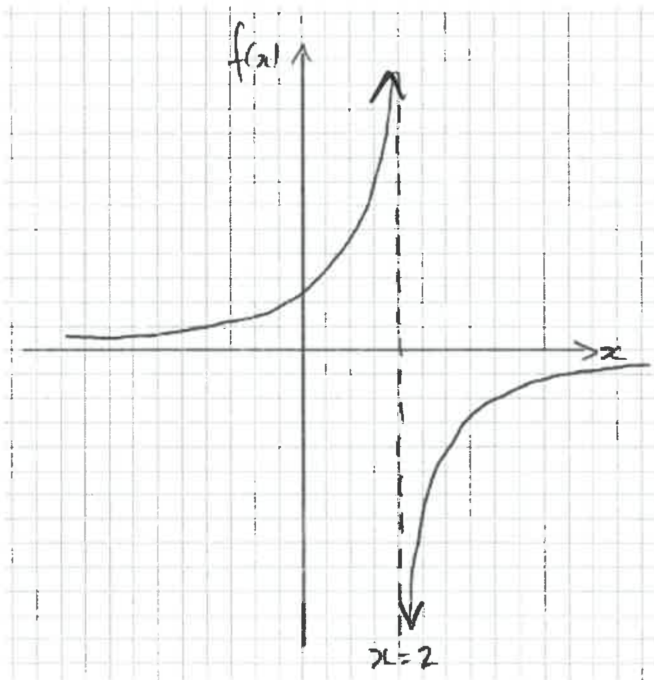
For functions of the form $f(x)=\dfrac{ax+b}{cx+d}$ we will see behaviours like ③ and ④

The questions is, **how do we know which?**

We need to **analyze the function near the V.A.**

We need to become familiar with some **Notation.**

Consider some rational function with a sketch of its graph which looks like:



As $x \to 2^{-}$; $f(x) \to \infty$

"left" or "below"

As $x \to 2^{+}$, $f(x) \to -\infty$

"right" or "above"

Example 4.2.1

Determine the functional behaviour of $f(x) = \dfrac{2x+1}{x-3}$ near its V.A.

V.A. is $x = 3$

Approach from left	Approach from right
strategy: pick a # really close to 3 and less than 3.	Pick a # really close to 3 and above 3.
$x = 2.99$ $\quad f(2.99) = -698$	$x = 3.01$ $\quad f(3.01) = 702$
$x = 2.999$ $\quad f(2.999) = -6998$	$x = 3.001$ $\quad f(3.001) = 7002$
∴ getting smaller	∴ getting bigger
So, as $x \to 3^{-}$, $f(x) \to -\infty$	So, as $x \to 3^{+}$, $f(x) \to +\infty$

We now have the tools to sketch some graphs!

Example 4.2.2

Sketch the graph of the given function. State the domain, range, intervals of increase/decrease and where the function is positive and negative.

a) $f(x)=\dfrac{2x+1}{x-1}$

V.A: $x=1$

H.A: $y=2$

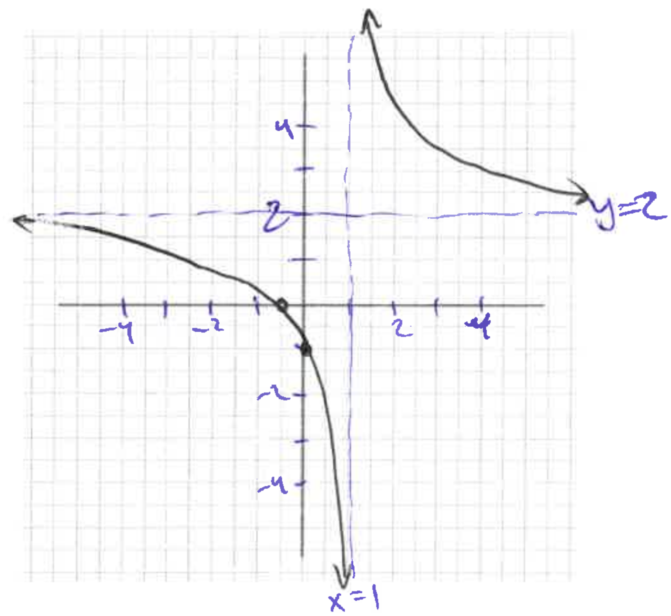
x-int: $x=-\frac{1}{2}$

y-int: $y=-1$

$D_f: \{x \in \mathbb{R} \mid x \neq 1\}$

$R_f: \{f(x) \in \mathbb{R} \mid f(x) \neq 2\}$

$f(x)$ is decreasing on $(-\infty, 1) \cup (1, \infty)$



$f(x) > 0$ on $(-\infty, -\frac{1}{2}) \cup (1, \infty)$

$f(x) < 0$ on $(-\frac{1}{2}, 1)$

b) $g(x)=\dfrac{3x-2}{2x+5}$

V.A: $x=-\frac{5}{2}$

H.A: $y=\frac{3}{2}$

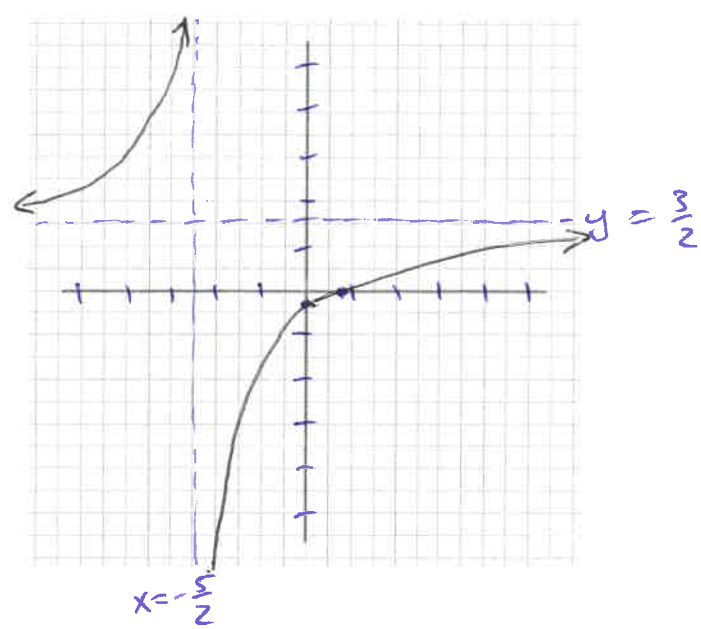
x-int: $x=\frac{2}{3}$

y-int: $y=-\frac{2}{5}$

$D_g: \{x \in \mathbb{R} \mid x \neq -\frac{5}{2}\}$

$R_g: \{g(x) \in \mathbb{R} \mid g(x) \neq \frac{3}{2}\}$

$g(x)$ is increasing on $(-\infty, -\frac{5}{2}) \cup (-\frac{5}{2}, \infty)$



$g(x) > 0$ on $(-\infty, -\frac{5}{2}) \cup (\frac{2}{3}, \infty)$

$g(x) < 0$ on $(-\frac{5}{2}, \frac{2}{3})$

Example 4.2.3

Consider question #9 on page 274:

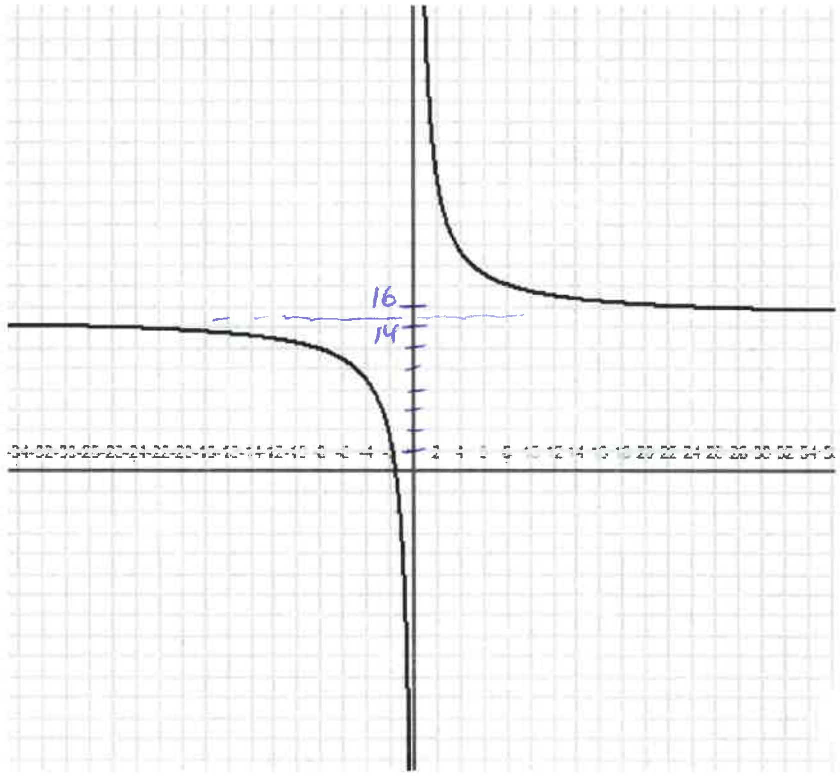
$$I(t)=\frac{15t+25}{t}$$

What is the value of the investment after 2 years?

$$I_{(t)}=\frac{15(2)+25}{2}$$

$$=27.5$$

or \$27,500



What is the value after 6 months?

$t=0.5$ $\quad I_{(0.5)}=\frac{15(0.5)+25}{0.5}$

$$=65$$

or \$65,000

There is an asymptote at $t=0$

Does this make sense?

No, the value @ $t=0$ should be the original amount invested.

Is the function accurate near $t=0$?

NO! As $t\to 0^+$, $I_{(t)}\to +\infty$

As time passes, what will the value of the investment approach?

$I_{(1000)}=\frac{15(1000)+25}{1000}=15.025$ $\quad I_{(100,000)}=15.00025$

Approaches \$15,000.

Success Criteria:

- I can identify the horizontal asymptote as $\frac{a}{c}$
- I can identify the vertical asymptote as $-\frac{d}{c}$
- I can identify the y-intercept as $\frac{b}{d}$
- I can identify the x-intercept as $-\frac{b}{a}$

4.4 Solving Rational Equations

Learning Goal: We are learning to solve rational equations. Think rationally!

Solving a Rational Equation is VERY MUCH like solving a Polynomial Equation. Thus, this stuff is so much fun it should be illegal. But it isn't illegal unless you break a rule of algebra. Math Safe!

KEY (*this is a major key for you music buffs*)

Multiplying by the Multiplicative Inverse of the Common Denominator is wonderful to use **WHEN YOU HAVE** something like:

$$\text{RATIONAL}_1 + \text{RATIONAL}_2 = \text{RATIONAL}_3$$

e.g. $\dfrac{3}{x-2} = \dfrac{4(x+5)}{x} + \dfrac{3}{2}$ C.D. is $(2)(x)(x-2)$

$$2x\cancel{(x-2)}\left(\frac{3}{\cancel{x-2}}\right) = 2\cancel{x}(x-2)\left(\frac{4(x+5)}{\cancel{x}}\right) + \cancel{2}x(x-2)\left(\frac{3}{\cancel{2}}\right)$$

$$2x(3) = 2(x-2)\cdot 4(x+5) + x(x-2)(3)$$

$$6x = 8(x-2)(x+5) + 3x(x-2)$$

No more fractions! Solve normally

Make Sure To Keep **RESTRICTIONS ON X** In Mind

Means that restrictions cannot be solutions.

So, $x \neq 2$

$x \neq 0$

① Expand

② Get "stuff $= 0$"

③ Solve

Example 4.4.1

a) Solve $\frac{x}{5} = \frac{9}{18}$ Cross multiply (This is the result of our earlier technique)

$$\frac{18x}{18} = \frac{45}{18}$$

$$x = \frac{5}{2}$$

No restrictions!

b) Solve $\frac{1}{x} - \frac{5x}{3} = \frac{2}{5}$ C.D. $(5)(3)(x)$

RESTRICTIONS

$x \neq 0$

$$15x\left(\frac{1}{x}\right) - 15x\left(\frac{5x}{3}\right) = 15x\left(\frac{2}{5}\right)$$

$$15 - 5x(5x) = 3x(2)$$

$$15 - 25x^2 = 6x$$

$$0 = 25x^2 + 6x - 15$$

Does not factor. Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(6) \pm \sqrt{6^2 - 4(25)(-15)}}{2(25)}$$

work. Show this work!

$x = -0.9$ and $x = 0.66$

c) Solve $\frac{3}{x}+\frac{4}{x+1}=2$

RESTRICTIONS

$x \neq -1, 0$

L.C.D: $x(x+1)$

$$\cancel{x}(x+1)\left(\frac{3}{\cancel{x}}\right) + x\cancel{(x+1)}\left(\frac{4}{\cancel{(x+1)}}\right) = x(x+1)(2)$$

$$3(x+1) + 4x = 2x(x+1)$$

$$3x+3 + 4x = 2x^2+2x$$

$$0 = 2x^2-5x-3$$

x−6

+−5

−6 1

$$0 = 2x^2-6x+x-3$$

$$0 = 2x(x-3)+1(x-3)$$

$$0 = (2x+1)(x-3)$$

$\therefore x = -\frac{1}{2}, x = 3$

d) Solve $\frac{10}{x^2-2x}+\frac{4}{x}=\frac{5}{x-2}$

RESTRICTIONS

C.D. $x(x-2)$

$x \neq 0, 2$

$$\cancel{x(x-2)}\left(\frac{10}{\cancel{x^2-2x}}\right) + \cancel{x}(x-2)\left(\frac{4}{\cancel{x}}\right) = x\cancel{(x-2)}\left(\frac{5}{\cancel{x-2}}\right)$$

$$10 + 4(x-2) = 5x$$

$$10 + 4x - 8 = 5x$$

$$2 = x$$

wait!! $x = 2$ is a restriction!

So, no solutions.

L.C.D. $(x+2)(x-2)$

Rest: $x \neq -2, 2$

$(x+2)(x-2)$

e) Solve $\left[16x - \frac{5}{x+2} = \frac{15}{x-2} - \frac{60}{(x-2)(x+2)}\right]$

$16x\underbrace{(x+2)(x-2)}_{x^2-4} - 5(x-2) = 15(x+2) - 60$

$16x^3 - 64x - 5x + 10 = 15x + 30 - 60$

$\frac{16x^3 - 84x + 40}{4} = \frac{0}{4}$

$4x^3 - 21x + 10 = 0$

$\pm 1, \pm 2, \pm 5 \pm 10$

Let $f(x) = 4x^3 - 21x + 10$

Try $f(2) = 4(2)^3 - 21(2) + 10$

$= 32 - 42 + 10$

$= 0$ ✓

2	4	0	-21	10
	↓	8	16	-10
	4	8	-5	0

$x - 20$

$+ 8$

$+10$
-2

$\therefore (x-2)(4x^2 + 8x - 5) = 0$

$\therefore (x-2)(4x^2 - 2x + 10x - 5) = 0$

$2x(2x-1) + 5(2x-1)$

$\therefore (x-2)(2x+5)(2x-1) = 0$

$\boxed{\cancel{\therefore x = 2}}$ Inadmissible

$x = \frac{1}{2}$

$x = -\frac{5}{2}$

Example 4.4.2

From your Text: Pg. 285 #10

The Turtledove Chocolate factory has two chocolate machines. Machine A takes *m* minutes to fill a case with chocolates, and machine B takes *m + 10* minutes to fill a case. Working together, the two machines take 15 min to fill a case. Approximately how long does each machine take to fill a case?

Work/Rate problem: $\frac{\text{\# cases filled}}{\text{\# of minutes}}$

A: $\frac{1 \text{ case}}{m \text{ minutes}}$ B: $\frac{1 \text{ case}}{m+10 \text{ minutes}}$

LCD: $(15m)(m+10)$

Rest: $m \neq 0$, $m \neq -10$

Together: $\left(\frac{1}{m} + \frac{1}{m+10} = \frac{1}{15}\right)(15m)(m+10)$ ← 1 case per ← 15 minutes

$\therefore 15(m+10) + 15m = m(m+10)$

$15m + 150 + 15m = m^2 + 10m$

$0 = m^2 - 20m - 150$

D.N.F. use Q.F.

Result: $m = 25.8$

$\therefore$ Machine A is 25.8 min

Machine B is 35.8 min

Success Criteria:

- I can recognize that the zeros of a rational function are the zeros of the numerator
- I can solve rational equations by multiplying each term by the lowest common denominator, then solving the resulting polynomial equation
- I can identify inadmissible solutions based on the context of the problem

4.5 Solving Rational Inequalities

Learning Goal: We are learning to solve rational inequalities using algebraic and graphical approaches.

The joy, wonder and peace these bring is really quite amazing

e.g. Solve $\frac{x-2}{7} \geq 0$ $\times 7$

$x-2 \geq 0$

$x \geq 2$

> Note: For Rational Inequalities, with a variable in the denominator, you **CANNOT** multiply by the multiplicative inverse of the common denominator!!!!
>
> Why? If the factor $(x+3)$ is negative, cross multiplying would change the direction of the inequality.

Example 4.5.1

Solve $\frac{x-2}{x+3} \geq 0$

We solve by using an Interval Chart

Rest: $x \neq -3$

So $x = 2$

For the intervals, we split $(-\infty, \infty)$ at all zeros **(where the numerator is zero)**, and all restrictions **(where the denominator is zero)** of the (SINGLE) rational expression. Keep in mind that it may take a good deal of algebraic manipulation to get a SINGLE rational expression…

Intervals:	$(-\infty, -3)$ R	$(-3, 2)$ R	$(2, \infty)$
Test values	-4	0	3
$x-2$	$-$	$-$	$+$
$x+3$	$-$	$+$	$+$
Ratio	$+$	$-$	$+$

$\therefore \frac{x-2}{x+3} \geq 0$ when $x \in (-\infty, -3) \cup [2, \infty)$

Restriction, so).

Example 4.5.2

Solve $\frac{1}{x+5} < 5$ -5

DO NOT CROSS MULTIPLY (or else)

- Get everything on one side
- Simplify into a single Rational Expression using a common denominator
- Interval Chart it up

$$\frac{1}{x+5} - \frac{5}{1}\frac{(x+5)}{(x+5)} < 0$$

$$\frac{1-5x-25}{x+5} < 0$$

$$\frac{-5x-24}{x+5} < 0$$

Zeros: $x = \frac{24}{-5}$ or -4.8

Restriction: $x \neq -5$

Interval	$(-\infty, -5)$	$(-5, -4.8)$	$(-4.8, \infty)$
T.V.	-6	-4.9	0
$-5x-24$	$+$	$+$	$-$
$x+5$	$-$	$+$	$+$
Ratio	$-$	$+$	$-$

$\therefore \frac{1}{x+5} < 5$ for $x \in (-\infty, -5) \cup (-4.8, \infty)$

Example 4.5.3

Solve $\frac{x^2+3x+2}{x^2-16} \geq 0$

FACTORED FORM IS YOUR FRIEND

$$\frac{(x+2)(x+1)}{(x-4)(x+4)} \geq 0$$

Zeros: $x=-2,\ x=-1$

Restrictions $x\neq-4,\ x\neq 4$

Intervals:	R $(-\infty,-4)$	R $(-4,-2)$	$(-2,-1)$	R $(-1,4)$	R $(4,\infty)$
Test values:	-5	-3	-1.5	0	5
$(x+1)$	$-$	$-$	$-$	$+$	$+$
$(x+2)$	$-$	$-$	$+$	$+$	$+$
$(x-4)$	$-$	$-$	$-$	$-$	$+$
$(x+4)$	$-$	$+$	$+$	$+$	$+$
Ratio	$+$ ✓	$-$	$+$ ✓	$-$	$+$ ✓

$\therefore \frac{x^2+3x+2}{x^2-16} \geq 0$ for $x \in (-\infty,-4) \cup [-2,-1] \cup (4,\infty)$

Example 4.5.4

Solve $\frac{3}{x+2} \le x$ $-x$

$$\frac{3}{x+2} - \frac{x}{1} \le 0 \quad \frac{(x+2)}{(x+2)}$$

$$\frac{3 - x^2 - 2x}{x+2} \le 0$$

$$\left(\frac{-1}{-1}\right)\frac{-x^2 - 2x + 3}{x+2} \le 0 \left(\frac{-1}{-1}\right)$$

Zeros: $x = -3$, $x = 1$

Restr. $x \neq -2$

$$\frac{x^2 + 2x - 3}{x+2} \ge 0$$

this flipped!

$$\frac{(x+3)(x-1)}{(x+2)} \ge 0$$

Intervals	$(-\infty, -3)$	$(-3, -2)$ R	$(-2, 1)$ R	$(1, \infty)$
T.V.	-4	-2.5	0	2
$(x+3)$	$-$	$+$	$+$	$+$
$(x-1)$	$-$	$-$	$-$	$+$
$(x+2)$	$-$	$-$	$+$	$+$
Ratio	$-$	$+$	$-$	$+$

$\therefore \frac{3}{x+2} \le x$ for $x \in [-3, -2) \cup [1, \infty)$

Example 4.5.5

From your Text: Pg. 296 #6a

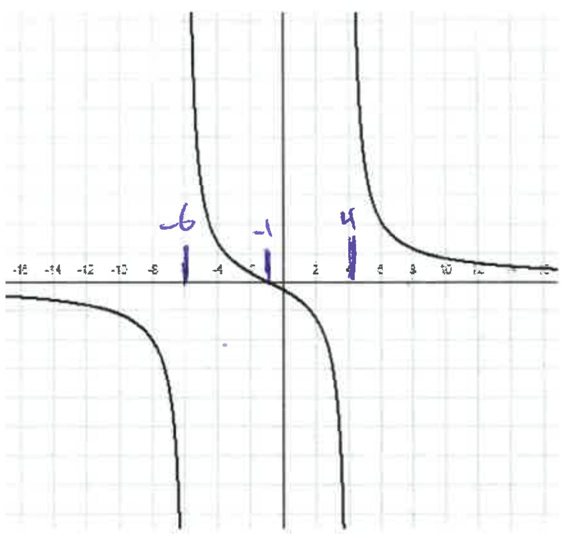
Using **Graphing Tech**

Solve $\dfrac{x+3}{x-4} \ge \dfrac{x-1}{x+6}$

Note: There are **TWO** methods, both of which require a **FUNCTION** (let $f(x) = \ldots$ returns)

1) Get a Single Function (on one side of the inequality)

$\left(\dfrac{x+3}{x-4}\right) - \left(\dfrac{x-1}{x+6}\right) \ge 0$

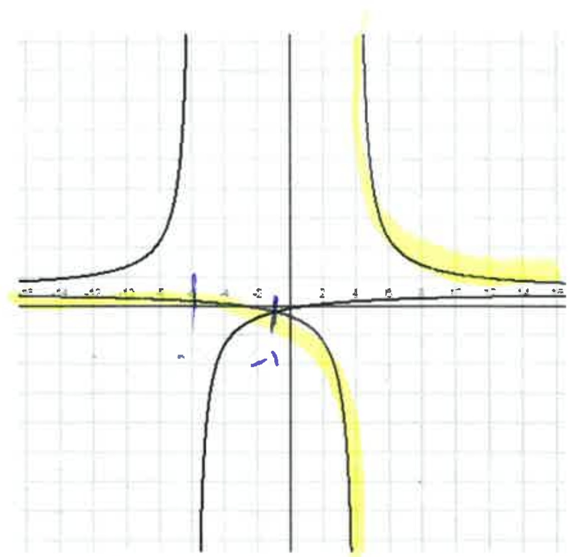


$x \in (-6,-1] \cup (4,\infty)$

2) Use Two Functions (one for each side)

let $f(x) = \dfrac{x+3}{x-4}$ $g(x) = \dfrac{x-1}{x+6}$

So: $f(x) \ge g(x)$



$x \in (-6,-1] \cup (4,\infty)$

Success Criteria:

- I can recognize that an inequality has many possible intervals of solutions
- I can solve an inequality algebraically, using an interval chart
- I can solve an inequality graphically