6.1 Basic Trigonometric Equivalencies

Learning Goal: We are learning to identify equivalent trigonometric relationships.

We have already seen a very basic trigonometric equivalency when we considered angles of rotation. For example, consider the angle of rotation for $\theta = \frac{2\pi}{3}$:

Q2

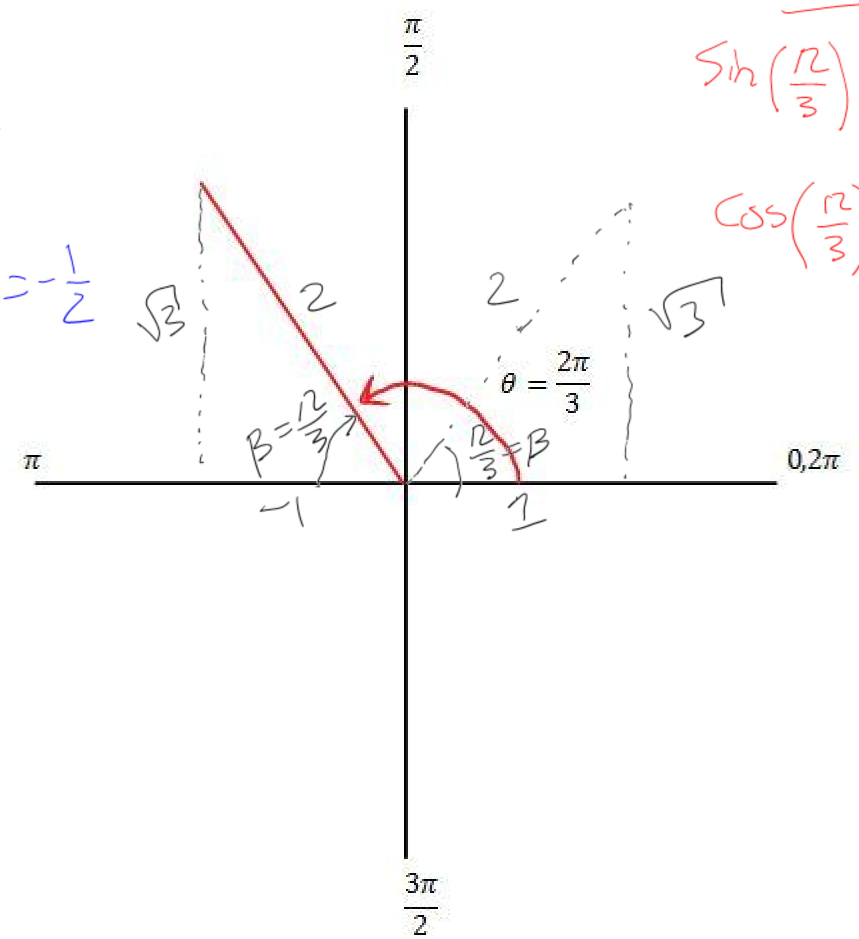
$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\cos\left(\frac{2\pi}{3}\right) = \frac{-1}{2} = -\frac{1}{2}$

Q1

$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$



therefore,

$\sin\left(\frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)$

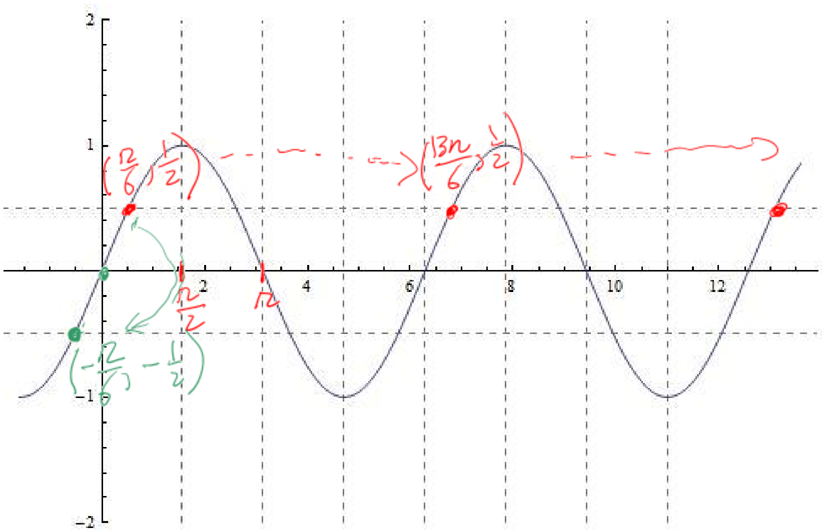
$\cos\left(\frac{2\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right)$

Periodic Equivalencies

Example 6.1.1

Consider the sketch of the function $f(\theta) = \sin(\theta)$

period is 2π.
$\therefore$ $\sin\theta$ is 2π periodic.



$\sin\theta = \sin(\theta + 2\pi)$

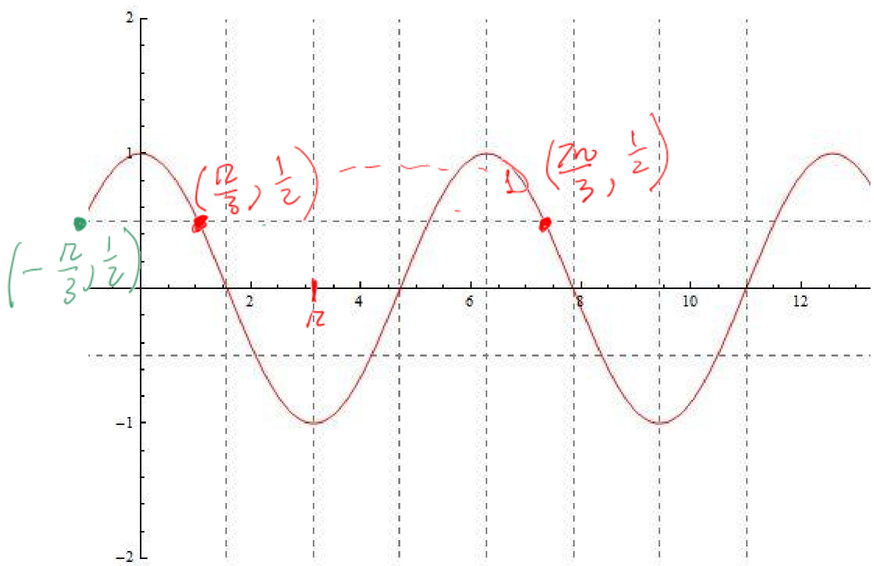
$\sin\theta = -\sin(-\theta)$

Sine is an odd function. (rotational symmetry)

Example 6.1.2

Consider $g(x) = \cos(x)$

cosine is 2π periodic.



$\cos\theta = \cos(\theta + 2\pi)$

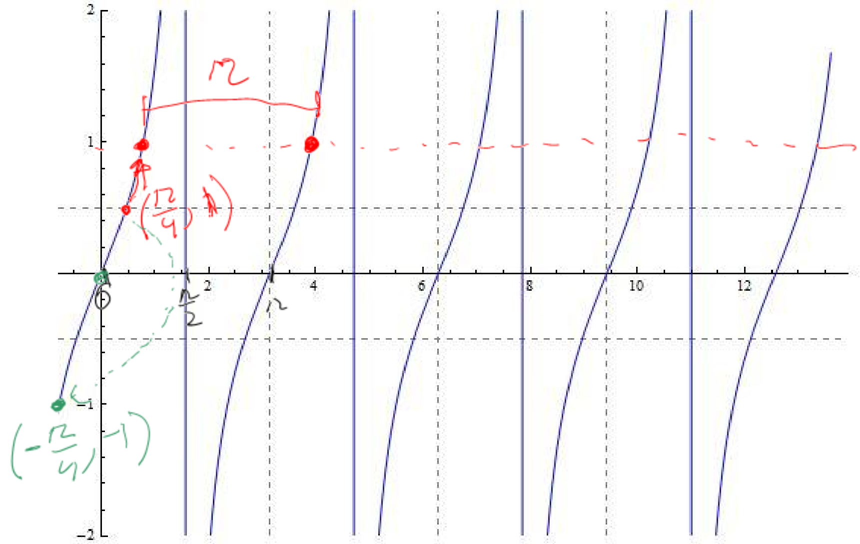
$\cos\theta = \cos(-\theta)$

$\therefore$ Cosine is an even function.

Example 6.1.3

tan is π periodic.

Consider $h(\theta)=\tan(\theta)$



$\tan\theta = \tan(\theta+\pi)$

$\tan\theta = -\tan(-\theta)$

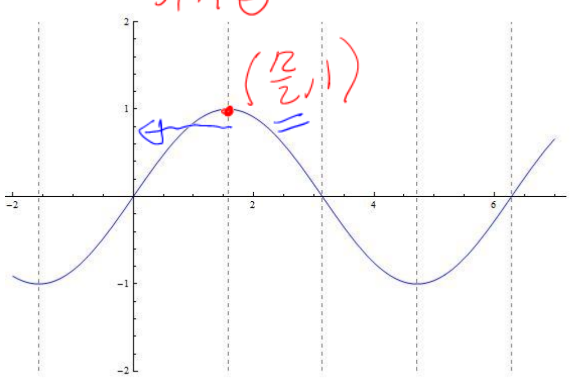
$\therefore$ tan is an odd function.

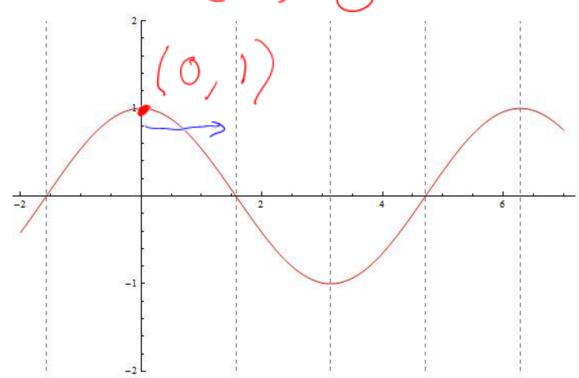
Shift Equivalencies

Example 6.1.4

Consider the sketches of the graphs for $f(\theta)=\sin(\theta)$ and $g(\theta)=\cos(\theta)$

$\sin\theta$ $\qquad$ $\cos\theta$





$\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right)$ shift right

$\cos\theta = \sin\left(\theta + \frac{\pi}{2}\right)$ shift left.

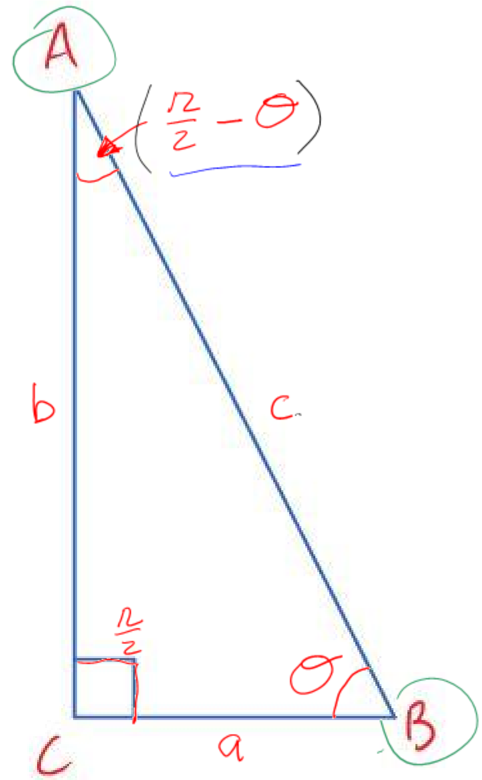
Cofunction Identities

Consider the right angle triangle

$$\sin\theta = \boxed{\frac{b}{c}} = \cos\left(\frac{\pi}{2}-\theta\right)$$

$$\cos\theta = \frac{a}{c} = \sin\left(\frac{\pi}{2}-\theta\right)$$

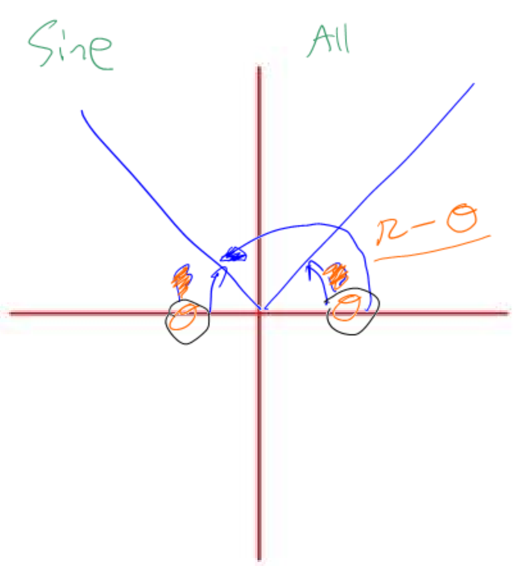
$$\tan\theta = \frac{b}{a} = \cot\left(\frac{\pi}{2}-\theta\right)$$



Related Acute Angle Equivalencies

Using CAST, relating angles of rotation to π and 2π

Compare Q1 and Q2

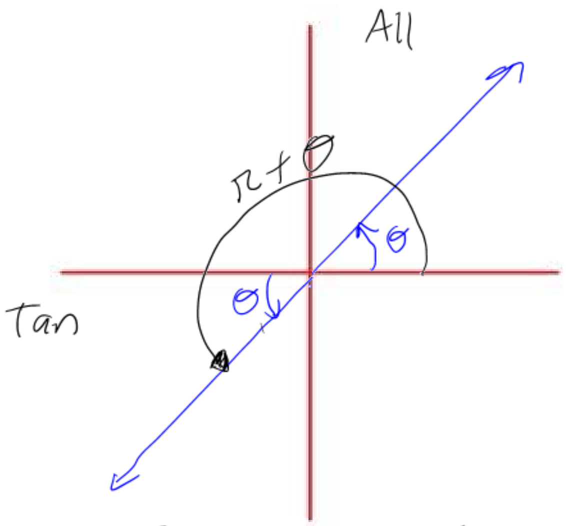


$$\sin\theta = \sin(\pi-\theta)$$

$$\cos\theta = -\cos(\pi-\theta)$$

$$\tan\theta = -\tan(\pi-\theta)$$

Compare Q1 and Q3

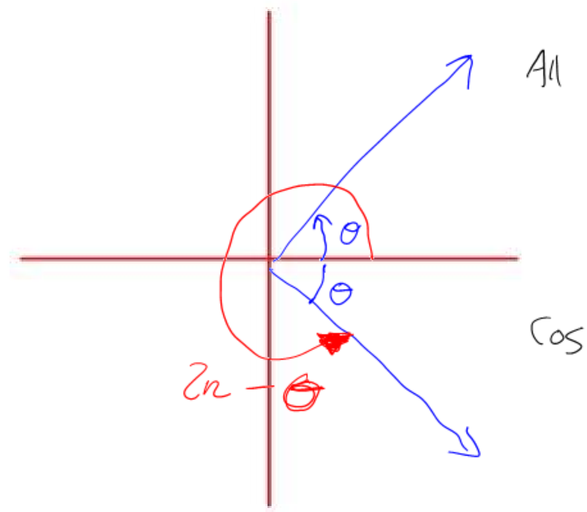


$\sin\theta = -\sin(\pi+\theta)$

$\cos\theta = -\cos(\pi+\theta)$

$\tan\theta = \tan(\pi+\theta)$

Compare Q1 and Q4



$\sin\theta = -\sin(2\pi-\theta)$

$\cos\theta = \cos(2\pi-\theta)$

$\tan\theta = -\tan(2\pi-\theta)$

Example 6.1.5

From your text: Pg. 392 #3

Use a cofunction identity to find an equivalency:

a) $\sin\left(\dfrac{\pi}{6}\right)$ θ

$$\sin\theta = \cos\left(\frac{\pi}{2}-\theta\right)$$

$$\sin\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{2}-\frac{\pi}{6}\right)$$

$$= \cos\left(\frac{3\pi}{6}-\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{3}\right)$$

d) $\cos\left(\dfrac{5\pi}{16}\right)$ θ

$$= \sin\left(\frac{\pi}{2}-\theta\right)$$

$$= \sin\left(\frac{\pi}{2}-\frac{5\pi}{16}\right)$$

$$= \sin\left(\frac{8\pi}{16}-\frac{5\pi}{16}\right)$$

$$= \sin\left(\frac{3\pi}{16}\right)$$

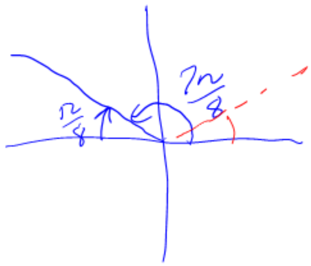
Example 6.1.6

From your text: Pg. 393 #5

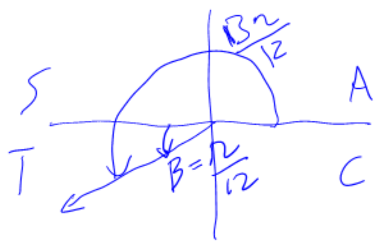
These depend on your quadrant!

Using the related acute angle, find an equivalent expression:

a) $\sin\left(\frac{7\pi}{8}\right) = \sin\left(\frac{\pi}{8}\right)$



b) $\cos\left(\frac{13\pi}{12}\right) = -\cos\left(\frac{\pi}{12}\right)$



Success Criteria:

- I can recognize that there are many equivalent trigonometric expressions due to their periodic nature
- I can recognize several types of equivalencies
  - Shifting sin/cos by 2π OR π/2
  - Cos has even symmetry, Sin and Tan have odd symmetry
  - Cofunction equivalencies
  - Equivalencies based on the quadrant a principal angle is in