

Key Concepts Review

We began our introduction to calculus by considering the slope of a tangent and the related concept of rate of change. This led us to the study of limits and has laid the groundwork for Chapter 2 and the concept of the derivative of a function. Consider the following brief summary to confirm your understanding of the key concepts covered in Chapter 1:

- slope of the tangent as the limit of the slope of the secant as Q approaches P along the curve
- slope of a tangent at an arbitrary point
- average and instantaneous rates of change, average velocity, and (instantaneous) velocity
- the limit of a function at a value of the independent variable, which exists when the limiting value from the left equals the limiting value from the right
- properties of limits and the indeterminate form $\frac{0}{0}$
- continuity as a property of a graph “without breaks or jumps or gaps”

Formulas

- The slope of the tangent to the graph $y = f(x)$ at point $P(a, f(a))$ is

$$m = \lim_{x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h}.$$

- Average velocity = $\frac{\text{change in position}}{\text{change in time}}$
- The (instantaneous) velocity of an object, represented by position function $s(t)$, at time $t = a$, is $v(a) = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \lim_{h \rightarrow 0} \frac{s(a + h) - s(a)}{h}$.
- If f is a polynomial function, then $\lim_{x \rightarrow a} f(x) = f(a)$.
- The function $f(x)$ is continuous at $x = a$ if $f(a)$ is defined and if $\lim_{x \rightarrow a} f(x) = f(a)$.