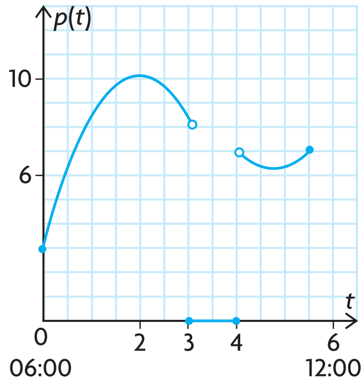
Review Exercise

1. Consider the graph of the function $f(x) = 5x^2 - 8x$.
   a. Find the slope of the secant that joins the points on the graph given by $x = -2$ and $x = 3$.
   b. Determine the average rate of change as x changes from -1 to 4.
   c. Find an equation for the line that is tangent to the graph of the function at $x = 1$.
2. Calculate the slope of the tangent to the given function at the given point or value of x.
   a. $f(x) = \dfrac{3}{x + 1}$, $P(2, 1)$
   b. $g(x) = \sqrt{x + 2}$, $P(-1, 1)$
   c. $h(x) = \dfrac{2}{\sqrt{x + 5}}$, $P\left(4, \dfrac{2}{3}\right)$
   d. $f(x) = \dfrac{5}{x - 2}$, $P\left(4, \dfrac{5}{2}\right)$
3. Calculate the slope of the graph of $f(x) = \begin{cases} 4 - x^2, \text{ if } x \le 1 \\ 2x + 1, \text{ if } x > 1 \end{cases}$ at each of the following points:
   a. $P(-1, 3)$
   b. $P(2, 5)$
4. The height, in metres, of an object that has fallen from a height of 180 m is given by the position function $s(t) = -5t^2 + 180$, where $t \ge 0$ and t is in seconds.
   a. Find the average velocity during each of the first two seconds.
   b. Find the velocity of the object when $t = 4$.
   c. At what velocity will the object hit the ground?
5. After t minutes of growth, a certain bacterial culture has a mass, in grams, of $M(t) = t^2$.
   a. How much does the bacterial culture grow during the time $3 \le t \le 3.01$?
   b. What is its average rate of growth during the time interval $3 \le t \le 3.01$?
   c. What is its rate of growth when $t = 3$?
6. It is estimated that, t years from now, the amount of waste accumulated Q, in tonnes, will be $Q(t) = 10^4(t^2 + 15t + 70)$, $0 \le t \le 10$.
   a. How much waste has been accumulated up to now?
   b. What will be the average rate of change in this quantity over the next three years?

c. What is the present rate of change in this quantity?

d. When will the rate of change reach 3.0×10^5 per year?

7. The electrical power $p(t)$, in kilowatts, being used by a household as a function of time t, in hours, is modelled by a graph where $t = 0$ corresponds to 06:00. The graph indicates peak use at 08:00 and a power failure between 09:00 and 10:00.



a. Determine $\lim_{t \to 2} p(t)$.

b. Determine $\lim_{t \to 4^+} p(t)$ and $\lim_{t \to 4^-} p(t)$.

c. For what values of t is $p(t)$ discontinuous?

8. Sketch a graph of any function that satisfies the given conditions.

a. $\lim_{x \to -1} f(x) = 0.5$, f is discontinuous at $x = -1$

b. $f(x) = -4$ if $x < 3$, f is an increasing function when $x > 3$, $\lim_{x \to 3^+} f(x) = 1$

9. a. Sketch the graph of the following function:

$$f(x) = \begin{cases} x + 1, \text{ if } x < -1 \\ -x + 1, \text{ if } -1 \leq x < 1 \\ x - 2, \text{ if } x > 1 \end{cases}$$

b. Find all values at which the function is discontinuous.

c. Find the limits at those values, if they exist.

10. Determine whether $f(x) = \frac{x^2 + 2x - 8}{x + 4}$ is continuous at $x = -4$.

11. Consider the function $f(x) = \frac{2x - 2}{x^2 + x - 2}$.

a. For what values of x is f discontinuous?

b. At each point where f is discontinuous, determine the limit of $f(x)$, if it exists.

12. Use a graphing calculator to graph each function and estimate the limits, if they exist.

a. $f(x) = \frac{1}{x^2}, \lim_{x\to 0} f(x)$

b. $g(x) = x(x - 5), \lim_{x\to 0} g(x)$

c. $h(x) = \frac{x^3 - 27}{x^2 - 9}, \lim_{x\to 4} h(x)$ and $\lim_{x\to -3} h(x)$

13. Copy and complete each table, and use your results to estimate the limit. Use a graphing calculator to graph the function to confirm your result.

a. $\lim_{x\to 2} \frac{x - 2}{x^2 - x - 2}$

x	1.9	1.99	1.999	2.001	2.01	2.1
$\frac{x - 2}{x^2 - x - 2}$						

b. $\lim_{x\to 1} \frac{x - 1}{x^2 - 1}$

x	0.9	0.99	0.999	1.001	1.01	1.1
$\frac{x - 1}{x^2 - 1}$						

14. Copy and complete the table, and use your results to estimate the limit.

$\lim_{x\to 0} \frac{\sqrt{x + 3} - \sqrt{3}}{x}$

x	−0.1	−0.01	−0.001	0.001	0.01	0.1
$\frac{\sqrt{x + 3} - \sqrt{3}}{x}$						

Then determine the limit using an algebraic technique, and compare your answer with your estimate.

15. a. Copy and complete the table to approximate the limit of $f(x) = \frac{\sqrt{x + 2} - 2}{x - 2}$ as $x \to 2$.

x	2.1	2.01	2.001	2.0001
$f(x) = \frac{\sqrt{x + 2} - 2}{x - 2}$				

b. Use a graphing calculator to graph f, and use the graph to approximate the limit.

c. Use the technique of rationalizing the numerator to find $\lim_{x\to 2} \frac{\sqrt{x + 2} - 2}{x - 2}$.

16. Evaluate the limit of each difference quotient. Interpret the limit as the slope of the tangent to a curve at a specific point.

a. $\lim_{h \to 0} \frac{(5 + h)^2 - 25}{h}$

b. $\lim_{h \to 0} \frac{\sqrt{4 + h} - 2}{h}$

c. $\lim_{h \to 0} \frac{\frac{1}{(4 + h)} - \frac{1}{4}}{h}$

17. Evaluate each limit using one of the algebraic methods discussed in this chapter, if the limit exists.

a. $\lim_{x \to -4} \frac{x^2 + 12x + 32}{x + 4}$

b. $\lim_{x \to a} \frac{(x + 4a)^2 - 25a^2}{x - a}$

c. $\lim_{x \to 0} \frac{\sqrt{x + 5} - \sqrt{5 - x}}{x}$

d. $\lim_{x \to 2} \frac{x^2 - 4}{x^3 - 8}$

e. $\lim_{x \to 4} \frac{4 - \sqrt{12 + x}}{x - 4}$

f. $\lim_{x \to 0} \frac{1}{x}\left(\frac{1}{2 + x} - \frac{1}{2}\right)$

18. Explain why the given limit does not exist.

a. $\lim_{x \to 3} \sqrt{x - 3}$

b. $\lim_{x \to 2} \frac{x^2 - 4}{x^2 - 4x + 4}$

c. $f(x) = \begin{cases} -5, \text{ if } x < 1 \\ 2, \text{ if } x \geq 1 \end{cases}; \lim_{x \to 1} f(x)$

d. $\lim_{x \to 2} \frac{1}{\sqrt{x - 2}}$

e. $\lim_{x \to 0} \frac{|x|}{x}$

f. $f(x) = \begin{cases} 5x^2, \text{ if } x < -1 \\ 2x + 1, \text{ if } x \geq -1 \end{cases}; \lim_{x \to -1} f(x)$

19. Determine the equation of the tangent to the curve of each function at the given value of x.

a. $y = -3x^2 + 6x + 4$ where $x = 1$

b. $y = x^2 - x - 1$ where $x = -2$

c. $f(x) = 6x^3 - 3$ where $x = -1$

d. $f(x) = -2x^4$ where $x = 3$

20. The estimated population of a bacteria colony is $P(t) = 20 + 61t + 3t^2$, where the population, P, is measured in thousands at t hours.

a. What is the estimated population of the colony at 8 h?

b. At what rate is the population changing at 8 h?