Mid-Chapter Review

1. a. Sketch the graph of $f(x) = x^2 - 5x$.
   b. Calculate the slopes of the tangents to $f(x) = x^2 - 5x$ at points with x-coordinates 0, 1, 2, ..., 5.
   c. Sketch the graph of the derivative function $f'(x)$.
   d. Compare the graphs of $f(x)$ and $f'(x)$.
2. Use the definition of the derivative to find $f'(x)$ for each function.
   a. $f(x) = 6x + 15$
   b. $f(x) = 2x^2 - 4$
   c. $f(x) = \frac{5}{x + 5}$
   d. $f(x) = \sqrt{x - 2}$
3. a. Determine the equation of the tangent to the curve $y = x^2 - 4x + 3$ at $x = 1$.
   b. Sketch the graph of the function and the tangent.
4. Differentiate each of the following functions:
   a. $y = 6x^4$
   b. $y = 10x^{\frac{1}{2}}$
   c. $g(x) = \frac{2}{x^3}$
   d. $y = 5x + \frac{3}{x^2}$
   e. $y = (11t + 1)^2$
   f. $y = \frac{x - 1}{x}$
5. Determine the equation of the tangent to the graph of $f(x) = 2x^4$ that has slope 1.
6. Determine $f'(x)$ for each of the following functions:
   a. $f(x) = 4x^2 - 7x + 8$
   b. $f(x) = -2x^3 + 4x^2 + 5x - 6$
   c. $f(x) = \frac{5}{x^2} - \frac{3}{x^3}$
   d. $f(x) = \sqrt{x} + \sqrt[3]{x}$
   e. $f(x) = 7x^{-2} - 3\sqrt{x}$
   f. $f(x) = -4x^{-1} + 5x - 1$
7. Determine the equation of the tangent to the graph of each function.
   a. $y = -3x^2 + 6x + 4$ when $x = 1$
   b. $y = 3 - 2\sqrt{x}$ when $x = 9$
   c. $f(x) = -2x^4 + 4x^3 - 2x^2 - 8x + 9$ when $x = 3$
8. Determine the derivative using the product rule.
   a. $f(x) = (4x^2 - 9x)(3x^2 + 5)$
   b. $f(t) = (-3t^2 - 7t + 8)(4t - 1)$
   c. $y = (3x^2 + 4x - 6)(2x^2 - 9)$
   d. $y = (3 - 2x^3)^3$

9. Determine the equation of the tangent to $y = (5x^2 + 9x - 2)(-x^2 + 2x + 3)$ at $(1, 48)$.

10. Determine the point(s) where the tangent to the curve $y = 2(x - 1)(5 - x)$ is horizontal.

11. If $y = 5x^2 - 8x + 4$, determine $\frac{dy}{dx}$ from first principles.

12. A tank holds 500 L of liquid, which takes 90 min to drain from a hole in the bottom of the tank. The volume, V, remaining in the tank after t minutes is

$$V(t) = 500\left(1 - \frac{t}{90}\right)^2, \text{ where } 0 \leq t \leq 90$$

a. How much liquid remains in the tank at 1 h?

b. What is the average rate of change of volume with respect to time from 0 min to 60 min?

c. How fast is the liquid draining at 30 min?

13. The volume of a sphere is given by $V(r) = \frac{4}{3}\pi r^3$.

a. Determine the average rate of change of volume with respect to radius as the radius changes from 10 cm to 15 cm.

b. Determine the rate of change of volume when the radius is 8 cm.

14. A classmate says, "The derivative of a cubic polynomial function is a quadratic polynomial function." Is the statement always true, sometimes true, or never true? Defend your choice in words, and provide two examples to support your argument.

15. Show that $\frac{dy}{dx} = (a + 4b)x^{a+4b-1}$ if $y = \frac{x^{2a+3b}}{x^{a-b}}$ and a and b are integers.

16. a. Determine $f'(3)$, where $f(x) = -6x^3 + 4x - 5x^2 + 10$.

b. Give two interpretations of the meaning of $f'(3)$.

17. The population, P, of a bacteria colony at t hours can be modelled by

$$P(t) = 100 + 120t + 10t^2 + 2t^3$$

a. What is the initial population of the bacteria colony?

b. What is the population of the colony at 5 h?

c. What is the growth rate of the colony at 5 h?

18. The relative percent of carbon dioxide, C, in a carbonated soft drink at t minutes can be modelled by $C(t) = \frac{100}{t}$, where $t > 2$. Determine $C'(t)$ and interpret the results at 5 min, 50 min, and 100 min. Explain what is happening.