Key Concepts Review

Now that you have completed your study of derivatives in Chapter 2, you should be familiar with such concepts as derivatives of polynomial functions, the product rule, the quotient rule, the power rule for rational exponents, and the chain rule. Consider the following summary to confirm your understanding of the key concepts.

- The derivative of f at a is given by $f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$ or, alternatively, by $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$.
- The derivative function of $f(x)$ with respect to x is $f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$.
- The derivative of a function at a point $(a, f(a))$ can be interpreted as
  - the slope of the tangent line at this point
  - the instantaneous rate of change at this point

Summary of Differentiation Techniques

Rule	Function Notation	Leibniz Notation
Constant	$f(x) = k,\ f'(x) = 0$	$\frac{d}{dx}(k) = 0$
Linear	$f(x) = x,\ f'(x) = 1$	$\frac{d}{dx}(x) = 1$
Power	$f(x) = x^n,\ f'(x) = nx^{n-1}$	$\frac{d}{dx}(x^n) = nx^{n-1}$
Constant Multiple	$f(x) = kg(x),\ f'(x) = kg'(x)$	$\frac{d}{dx}(ky) = k\frac{dy}{dx}$
Sum or Difference	$f(x) = p(x) \pm q(x)$, $f'(x) = p'(x) \pm q'(x)$	$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$
Product	$h(x) = f(x)g(x)$ $h'(x) = f'(x)g(x) + f(x)g'(x)$	$\frac{d}{dx}[f(x)g(x)] = \left[\frac{d}{dx}f(x)\right]g(x) + f(x)\left[\frac{d}{dx}g(x)\right]$
Quotient	$h(x) = \frac{f(x)}{g(x)}$ $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$	$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\left[\frac{d}{dx}f(x)\right]g(x) - f(x)\left[\frac{d}{dx}g(x)\right]}{[g(x)]^2}$
Chain	$h(x) = f(g(x)),\ h'(x) = f'(g(x))g'(x)$	$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$, where u is a function of x
Power of a Function	$f(x) = [g(x)]^n,\ f'(x) = n[g(x)]^{n-1}g'(x)$	$y = u^n,\ \frac{dy}{dx} = nu^{n-1}\frac{du}{dx}$, where u is a function of x