Review Exercise

1. Describe the process of finding a derivative using the definition of $f'(x)$.

2. Use the definition of the derivative to find $f'(x)$ for each of the following functions:

   a. $y = 2x^2 - 5x$ b. $y = \sqrt{x - 6}$ c. $y = \dfrac{x}{4 - x}$

3. Differentiate each of the following functions:

   a. $y = x^2 - 5x + 4$
   b. $f(x) = x^{\frac{3}{4}}$
   c. $y = \dfrac{7}{3x^4}$
   d. $y = \dfrac{1}{x^2 + 5}$
   e. $y = \dfrac{3}{(3 - x^2)^2}$
   f. $y = \sqrt{7x^2 + 4x + 1}$

4. Determine the derivative of the given function. In some cases, it will save time if you rearrange the function before differentiating.

   a. $f(x) = \dfrac{2x^3 - 1}{x^2}$
   b. $g(x) = \sqrt{x}(x^3 - x)$
   c. $y = \dfrac{x}{3x - 5}$
   d. $y = \sqrt{x - 1}(x + 1)$
   e. $f(x) = (\sqrt{x} + 2)^{-\frac{2}{3}}$
   f. $y = \dfrac{x^2 + 5x + 4}{x + 4}$

5. Determine the derivative, and give your answer in a simplified form.

   a. $y = x^4(2x - 5)^6$
   b. $y = x\sqrt{x^2 + 1}$
   c. $y = \dfrac{(2x - 5)^4}{(x + 1)^3}$
   d. $y = \left(\dfrac{10x - 1}{3x + 5}\right)^6$
   e. $y = (x - 2)^3(x^2 + 9)^4$
   f. $y = (1 - x^2)^3(6 + 2x)^{-3}$

6. If f is a differentiable function, find an expression for the derivative of each of the following functions:

   a. $g(x) = f(x^2)$ b. $h(x) = 2xf(x)$

7. a. If $y = 5u^2 + 3u - 1$ and $u = \dfrac{18}{x^2 + 5}$, find $\dfrac{dy}{dx}$ when $x = 2$.

   b. If $y = \dfrac{u + 4}{u - 4}$ and $u = \dfrac{\sqrt{x} + x}{10}$, find $\dfrac{dy}{dx}$ when $x = 4$.

   c. If $y = f(\sqrt{x^2 + 9})$ and $f'(5) = -2$, find $\dfrac{dy}{dx}$ when $x = 4$.

8. Determine the slope of the tangent at point $(1, 4)$ on the graph of $f(x) = (9 - x^2)^{\frac{2}{3}}$.

9. For what values of x does the curve $y = -x^3 + 6x^2$ have a slope of -12? For what values of x does the curve $y = -x^3 + 6x^2$ have a slope of -15? Use a graphing calculator to graph the function and confirm your results.

10. a. Determine the values of x where the graph of each function has a horizontal tangent.

    i. $y = (x^2 - 4)^5$ ii. $y = (x^3 - x)^2$

    b. Use a graphing calculator to graph each function and its tangent at the values you found from part a. to confirm your result.

11. Determine the equation of the tangent to each function at the given point.

    a. $y = (x^2 + 5x + 2)^4$, $(0, 16)$ b. $y = (3x^{-2} - 2x^3)^5$, $(1, 1)$

12. A tangent to the parabola $y = 3x^2 - 7x + 5$ is perpendicular to $x + 5y - 10 = 0$. Determine the equation of the tangent.

13. The line $y = 8x + b$ is tangent to the curve $y = 2x^2$. Determine the point of tangency and the value of b.

14. a. Using a graphing calculator, graph the function $f(x) = \frac{x^3}{x^2 - 6}$.

    b. Using the draw function or an equivalent function on your calculator or graphing software, find the equations of the tangents where the slope is zero.

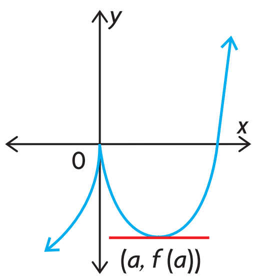
    c. Setting $f'(x) = 0$, find the coordinates of the points where the slope is zero.

    d. Determine the slope of the tangent to the graph at $(2, -4)$. Use the graph to verify that your answer is reasonable.

15. Consider the function $f(x) = 2x^{\frac{5}{3}} - 5x^{\frac{2}{3}}$.

    a. Determine the slope of the tangent at the point where the graph crosses the x-axis.

    b. Determine the value of a shown in the graph of $f(x)$ given below.



16. A rested student is able to memorize M words after t minutes, where $M = 0.1t^2 - 0.001t^3$, $0 \leq t \leq \frac{200}{3}$.

    a. How many words are memorized in the first 10 min? How many words are memorized in the first 15 min?

    b. What is the memory rate at $t = 10$? What is the memory rate at $t = 15$?

17. A grocery store determines that, after t hours on the job, a new cashier can scan $N(t) = 20 - \frac{30}{\sqrt{9 + t^2}}$ items per minute.

   a. Find $N'(t)$, the rate at which the cashier's productivity is changing.

   b. According to this model, does the cashier ever stop improving? Why?

18. An athletic-equipment supplier experiences weekly costs of $C(x) = \frac{1}{3}x^3 + 40x + 700$ in producing x baseball gloves per week.

   a. Find the marginal cost, $C'(x)$.

   b. Find the production level x at which the marginal cost is \$76 per glove.

19. A manufacturer of kitchen appliances experiences revenue of $R(x) = 750x - \frac{x^2}{6} - \frac{2}{3}x^3$ dollars from the sale of x refrigerators per month.

   a. Find the marginal revenue, $R'(x)$.

   b. Find the marginal revenue when 10 refrigerators per month are sold.

20. An economist has found that the demand function for a particular new product is given by $D(p) = \frac{20}{\sqrt{p - 1}}$, $p > 1$. Find the slope of the demand curve at the point $(5, 10)$.

21. Kathy has diabetes. Her blood sugar level, B, one hour after an insulin injection, depends on the amount of insulin, x, in milligrams injected.

$$B(x) = -0.2x^2 + 500, 0 \le x \le 40$$

   a. Find $B(0)$ and $B(30)$.

   b. Find $B'(0)$ and $B'(30)$.

   c. Interpret your results.

   d. Consider the values of $B'(50)$ and $B(50)$. Comment on the significance of these values. Why are restrictions given for the original function?

22. Determine which functions are differentiable at $x = 1$. Give reasons for your choices.

   a. $f(x) = \frac{3x}{1 - x^2}$

   b. $g(x) = \frac{x - 1}{x^2 + 5x - 6}$

   c. $h(x) = \sqrt[3]{(x - 2)^2}$

   d. $m(x) = |3x - 3| - 1$

23. At what x-values is each function *not* differentiable? Explain.

   a. $f(x) = \frac{3}{4x^2 - x}$

   b. $f(x) = \frac{x^2 - x - 6}{x^2 - 9}$

   c. $f(x) = \sqrt{x^2 - 7x + 6}$

24. At a manufacturing plant, productivity is measured by the number of items, p, produced per employee per day over the previous 10 years. Productivity is modelled by $p(t) = \frac{25t}{t + 1}$, where t is the number of years measured from 10 years ago. Determine the rate of change of p with respect to t.

25. Choose a simple polynomial function in the form $f(x) = ax + b$. Use the quotient rule to find the derivative of the reciprocal function $\frac{1}{ax + b}$. Repeat for other polynomial functions, and devise a rule for finding the derivative of $\frac{1}{f(x)}$. Confirm your rule using first principles.

26. Given $f(x) = \frac{(2x - 3)^2 + 5}{2x - 3}$,
    a. Express f as the composition of two simpler functions.
    b. Use this composition to determine $f'(x)$.

27. Given $g(x) = \sqrt{2x - 3} + 5(2x - 3)$,
    a. Express g as the composition of two simpler functions.
    b. Use this composition to determine $g'(x)$.

28. Determine the derivative of each function.
    a. $f(x) = (2x - 5)^3(3x^2 + 4)^5$
    b. $g(x) = (8x^3)(4x^2 + 2x - 3)^5$
    c. $y = (5 + x)^2(4 - 7x^3)^6$
    d. $h(x) = \dfrac{6x - 1}{(3x + 5)^4}$
    e. $y = \dfrac{(2x^2 - 5)^3}{(x + 8)^2}$
    f. $f(x) = \dfrac{-3x^4}{\sqrt{4x - 8}}$
    g. $g(x) = \left(\dfrac{2x + 5}{6 - x^2}\right)^4$
    h. $y = \left[\dfrac{1}{(4x + x^2)^3}\right]^3$

29. Find numbers a, b, and c so that the graph of $f(x) = ax^2 + bx + c$ has x-intercepts at $(0, 0)$ and $(8, 0)$, and a tangent with slope 16 where $x = 2$.

30. An ant colony was treated with an insecticide and the number of survivors, A, in hundreds at t hours is $A(t) = -t^3 + 5t + 750$.
    a. Find $A'(t)$.
    b. Find the rate of change of the number of living ants in the colony at 5 h.
    c. How many ants were in the colony before it was treated with the insecticide?
    d. How many hours after the insecticide was applied were no ants remaining in the colony?