Chapter 3

DERIVATIVES AND THEIR APPLICATIONS

We live in a world that is always in flux. Sir Isaac Newton's name for calculus was "the method of fluxions." He recognized in the seventeenth century, as you probably recognize today, that understanding change is important. Newton was what we might call a "mathematical physicist." He developed his method of fluxions to gain a better understanding of the natural world, including motion and gravity. But change is not limited to the natural world, and, since Newton's time, the use of calculus has spread to include applications in the social sciences. Psychology, business, and economics are just a few of the areas in which calculus continues to be an effective problem-solving tool. As we shall see in this chapter, anywhere functions can be used as models, the derivative is certain to be meaningful and useful.

CHAPTER EXPECTATIONS

In this chapter, you will

- make connections between the concept of motion and the concept of derivatives, **Section 3.1**
- solve problems involving rates of change, **Section 3.1**
- determine second derivatives, **Section 3.1**
- determine the extreme values of a function, **Section 3.2**
- solve problems by applying mathematical models and their derivatives to determine, interpret, and communicate the mathematical results, **Section 3.3**
- solve problems by determining the maximum and minimum values of a mathematical model, **Career Link, Sections 3.3, 3.4**

Review of Prerequisite Skills

In Chapter 2, we developed an understanding of derivatives and differentiation. In this chapter, we will consider a variety of applications of derivatives. The following skills will be helpful:

- graphing polynomial and simple rational functions
- working with circles in standard position
- solving polynomial equations
- finding the equations of tangents and normals
- using the following formulas:
  circle: circumference: $C = 2\pi r$, area: $A = \pi r^2$
  right circular cylinder: surface area: $SA = 2\pi rh + 2\pi r^2$, volume: $V = \pi r^2 h$

Exercise

1. Sketch the graph of each function.

a. $2x + 3y - 6 = 0$

b. $3x - 4y = 12$

c. $y = \sqrt{x}$

d. $y = \sqrt{x - 2}$

e. $y = x^2 - 4$

f. $y = -x^2 + 9$

2. Solve each equation, where $x, t \in \mathbf{R}$.

a. $3(x - 2) + 2(x - 1) - 6 = 0$

b. $\frac{1}{3}(x - 2) + \frac{2}{5}(x + 3) = \frac{x - 5}{2}$

c. $t^2 - 4t + 3 = 0$

d. $2t^2 - 5t - 3 = 0$

e. $\frac{6}{t} + \frac{t}{2} = 4$

f. $x^3 + 2x^2 - 3x = 0$

g. $x^3 - 8x^2 + 16x = 0$

h. $4t^3 + 12t^2 - t - 3 = 0$

i. $4t^4 - 13t^2 + 9 = 0$

3. Solve each inequality, where $x \in \mathbf{R}$.

a. $3x - 2 > 7$

b. $x(x - 3) > 0$

c. $-x^2 + 4x > 0$

4. Determine the area of each figure. Leave your answers in terms of π, where applicable.

a. square: perimeter 20 cm

b. rectangle: length 8 cm, width 6 cm

c. circle: radius 7 cm

d. circle: circumference 12π cm

5. Two measures of each right circular cylinder are given. Calculate the two remaining measures.

	Radius, r	**Height, h**	**Surface Area, $S = 2\pi rh + 2\pi r^2$**	**Volume, $V = \pi r^2 h$**
a.	4 cm	3 cm		
b.	4 cm			96π cm^3
c.		6 cm		216π cm^3
d.	5 cm		120π cm^2	

6. Calculate total surface area and volume for cubes with the following side lengths:

a. 3 cm

b. $\sqrt{5}$ cm

c. $2\sqrt{3}$ cm

d. $2k$ cm

7. Express each set of numbers using interval notation.

a. $\{x \in \mathbf{R} | x > 3\}$

b. $\{x \in \mathbf{R} | x \leq -2\}$

c. $\{x \in \mathbf{R} | x < 0\}$

d. $\{x \in \mathbf{R} | x \geq -5\}$

e. $\{x \in \mathbf{R} | -2 < x \leq 8\}$

f. $\{x \in \mathbf{R} | -4 < x < 4\}$

8. Express each interval using set notation, where $x \in \mathbf{R}$.

a. $(5, \infty)$

b. $(-\infty, 1]$

c. $(-\infty, \infty)$

d. $[-10, 12]$

e. $(-1, 3)$

f. $[2, 20)$

9. Use graphing technology to graph each function and determine its maximum and/or minimum values.

a. $f(x) = x^2 - 5$

b. $f(x) = -x^2 - 10x$

c. $f(x) = 3x^2 - 30x + 82$

d. $f(x) = |x| - 1$

e. $f(x) = 3\sin x + 2$

f. $f(x) = -2\cos 2x - 5$

CAREER LINK Investigate

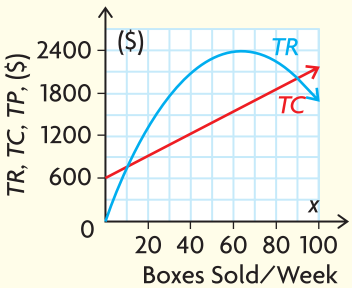
CHAPTER 3: MAXIMIZING PROFITS

We live in a world that demands we determine the best, the worst, the maximum, and the minimum. Through mathematical modelling, calculus can be used to establish optimum operating conditions for processes that seem to have competing variables. For example, minimizing transportation costs for a delivery vehicle would seem to require the driver to travel as fast as possible to reduce hourly wages. Higher rates of speed, however, increase gas consumption. With calculus, an optimal speed can be established to minimize the total cost of driving the delivery vehicle, considering both gas consumption and hourly wages. In this chapter, calculus tools will be used in realistic contexts to solve optimization problems—from business applications (such as minimizing cost) to psychology (such as maximizing learning).

Case Study—Entrepreneurship

In the last 10 years, the Canadian economy has seen a dramatic increase in the number of small businesses. An ability to use graphs to interpret the marginal profit (a calculus concept) will help an entrepreneur make good business decisions.

A person with an old family recipe for gourmet chocolates decides to open her own business. Her weekly total revenue (*TR*) and total cost (*TC*) curves are plotted on the set of axes shown.



DISCUSSION QUESTIONS

Make a rough sketch of the graph in your notes, and answer the following questions:

1. What sales interval would keep the company profitable? What do we call these values?
2. Superimpose the total profit (*TP*) curve over the *TR* and *TC* curves. What would the sales level have to be to obtain maximum profits? Estimate the slopes on the *TR* and *TC* curves at this level of sales. Should they be the same? Why or why not?
3. On a separate set of axes, sketch the marginal profit (the extra profit earned by selling one more box of chocolates), $MP = \frac{dTP}{dx}$. What can you say about the marginal profit as the level of sales progresses from just less than the maximum to the maximum, and then to just above the maximum? Does this make sense? Explain.