Mid-Chapter Review

1. Determine the second derivative of each of the following functions:
   a. $h(x) = 3x^4 - 4x^3 - 3x^2 - 5$
   b. $f(x) = (2x - 5)^3$
   c. $y = \dfrac{15}{x + 3}$
   d. $g(x) = \sqrt{x^2 + 1}$
2. The displacement of an object in motion is described by $s(t) = t^3 - 21t^2 + 90t$, where the horizontal displacement, s, is measured in metres at t seconds.
   a. Calculate the displacement at 3 s.
   b. Calculate the velocity at 5 s.
   c. Calculate the acceleration at 4 s.
3. A ball is thrown upward. Its motion can be described by $h(t) = -4.9t^2 + 6t + 2$, where the height, h, is measured in metres at t seconds.
   a. Determine the initial velocity.
   b. When does the ball reach its maximum height?
   c. When does the ball hit the ground?
   d. What is the velocity of the ball when it hits the ground?
   e. What is the acceleration of the ball on the way up? What is its acceleration on the way down?
4. An object is moving horizontally. The object's displacement, s, in metres at t seconds is described by $s(t) = 4t - 7t^2 + 2t^3$.
   a. Determine the velocity and acceleration at $t = 2$.
   b. When is the object stationary? Describe the motion immediately before and after these times.
   c. At what time, to the nearest tenth of a second, is the acceleration equal to 0? Describe the motion at this time.
5. Determine the absolute extreme values of each function on the given interval, using the algorithm for finding maximum and minimum values.
   a. $f(x) = x^3 + 3x^2 + 1,\ -2 \le x \le 2$
   b. $f(x) = (x + 2)^2,\ -3 \le x \le 3$
   c. $f(x) = \dfrac{1}{x} - \dfrac{1}{x^3},\ x \in [1, 5]$
6. The volume, V, of 1 kg of H_2O at temperature t between 0 °C and 30 °C can be modelled by $V(t) = -0.000\,067t^3 + 0.008\,504\,3t^2 - 0.064\,26t + 999.87$. Volume is measured in cubic centimetres. Determine the temperature at which the volume of water is the greatest in the given interval.

7. Evaluate each of the following:
   a. $f'(3)$ if $f(x) = x^4 - 3x$
   b. $f'(-2)$ if $f(x) = 2x^3 + 4x^2 - 5x + 8$
   c. $f''(1)$ if $f(x) = -3x^2 - 5x + 7$
   d. $f''(-3)$ if $f(x) = 4x^3 - 3x^2 + 2x - 6$
   e. $f'(0)$ if $f(x) = 14x^2 + 3x - 6$
   f. $f''(4)$ if $f(x) = x^4 + x^5 - x^3$
   g. $f''\left(\frac{1}{3}\right)$ if $f(x) = -2x^5 + 2x - 6 - 3x^3$
   h. $f'\left(\frac{3}{4}\right)$ if $f(x) = -3x^3 - 7x^2 + 4x - 11$
8. On the surface of the Moon, an astronaut can jump higher because the force of gravity is less than it is on Earth. When a certain astronaut jumps, his height, in metres above the Moon's surface, can be modelled by $s(t) = t\left(-\frac{5}{6}t + 1\right)$, where t is measured in seconds. What is the acceleration due to gravity on the Moon?
9. The forward motion of a space shuttle, t seconds after touchdown, is described by $s(t) = 189t - t^{\frac{7}{3}}$, where s is measured in metres.
   a. What is the velocity of the shuttle at touchdown?
   b. How much time is required for the shuttle to stop completely?
   c. How far does the shuttle travel from touchdown to a complete stop?
   d. What is the deceleration 8 s after touchdown?
10. In a curling game, one team's skip slides a stone toward the rings at the opposite end of the ice. The stone's position, s, in metres at t seconds, can be modelled by $s(t) = 12t - 4t^{\frac{3}{2}}$. How far does the stone travel before it stops? How long is it moving?
11. After a football is punted, its height, h, in metres above the ground at t seconds, can be modelled by $h(t) = -4.9t^2 + 21t + 0.45$.
    a. Determine the restricted domain of this model.
    b. When does the ball reach its maximum height?
    c. What is the ball's maximum height?