Review Exercise

1. Determine f' and f'', if $f(x) = x^4 - \frac{1}{x^4}$.

2. For $y = x^9 - 7x^3 + 2$, find $\frac{d^2y}{dx^2}$.

3. Determine the velocity and acceleration of an object that moves along a straight line in such a way that its position is $s(t) = t^2 + (2t - 3)^{\frac{1}{2}}$.

4. Determine the velocity and acceleration as functions of time, t, for $s(t) = t - 7 + \frac{5}{t}$, $t \neq 0$.

5. A pellet is shot into the air. Its position above the ground at any time, t, is defined by $s(t) = 45t - 5t^2$ m. For what values of t, $t \geq 0$, is the upward velocity of the pellet positive? For what values of t is the upward velocity zero and negative? Draw a graph to represent the velocity of the pellet.

6. Determine the maximum and minimum of each function on the given interval.
   a. $f(x) = 2x^3 - 9x^2$, $-2 \leq x \leq 4$
   b. $f(x) = 12x - x^3$, $x \in [-3, 5]$
   c. $f(x) = 2x + \frac{18}{x}$, $1 \leq x \leq 5$

7. A motorist starts braking for a stop sign. After t seconds, the distance, in metres, from the front of the car to the sign is $s(t) = 62 - 16t + t^2$.
   a. How far was the front of the car from the sign when the driver started braking?
   b. Does the car go beyond the stop sign before stopping?
   c. Explain why it is unlikely that the car would hit another vehicle that is travelling perpendicular to the motorist's road when the car first comes to a stop at the intersection.

8. The position function of an object that moves in a straight line is $s(t) = 1 + 2t - \frac{8}{t^2 + 1}$, $0 \leq t \leq 2$. Calculate the maximum and minimum velocities of the object over the given time interval.

9. Suppose that the cost, in dollars, of manufacturing x items is approximated by $C(x) = 625 + 15x + 0.01x^2$, for $1 \leq x \leq 500$. The unit cost (the cost of manufacturing one item) would then be $U(x) = \frac{C(x)}{x}$. How many items should be manufactured to ensure that the unit cost is minimized?

10. For each of the following cost functions, determine

    i. the cost of producing 400 items

    ii. the average cost of each of the first 400 items produced

    iii. the marginal cost when $x = 400$, as well as the cost of producing the 401st item

    a. $C(x) = 3x + 1000$

    b. $C(x) = 0.004x^2 + 40x + 8000$

    c. $C(x) = \sqrt{x} + 5000$

    d. $C(x) = 100x^{-\frac{1}{2}} + 5x + 700$

11. Find the production level that minimizes the average cost per unit for the cost function $C(x) = 0.004x^2 + 40x + 16\ 000$. Show that it is a minimum by using a graphing calculator to sketch the graph of the average cost function.

12. a. The position of an object moving along a straight line is described by the function $s(t) = 3t^2 - 10$ for $t \geq 0$. Is the object moving toward or away from its starting position when $t = 3$?

    b. Repeat the problem using $s(t) = -t^3 + 4t^2 - 10$ for $t \geq 0$.

13. A particle moving along a straight line will be s centimetres from a fixed point at time t seconds, where $t > 0$ and $s = 27t^3 + \frac{16}{t} + 10$.

    a. Determine when the velocity will be zero.

    b. Is the particle accelerating? Explain.

14. A box with a square base and no top must have a volume of 10 000 cm^3. If the smallest dimension is 5 cm, determine the dimensions of the box that minimize the amount of material used.

15. An animal breeder wishes to create five adjacent rectangular pens, each with an area of 2400 m^2. To ensure that the pens are large enough for grazing, the minimum for either dimension must be 10 m. Find the dimensions required for the pens to keep the amount of fencing used to a minimum.

16. You are given a piece of sheet metal that is twice as long as it is wide and has an area of 800 m^2. Find the dimensions of the rectangular box that would contain a maximum volume if it were constructed from this piece of metal by cutting out squares of equal area at all four corners and folding up the sides. The box will not have a lid. Give your answer correct to one decimal place.

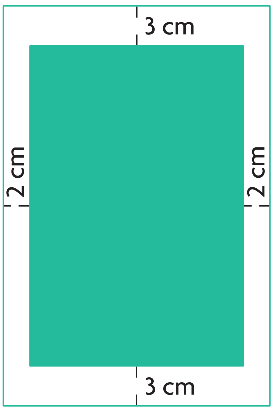
17. A cylindrical can needs to hold 500 cm^3 of apple juice. The height of the can must be between 6 cm and 15 cm, inclusive. How should the can be constructed so that a minimum amount of material will be used in the construction? (Assume that there will be no waste.)

18. In oil pipeline construction, the cost of pipe to go underwater is 60% more than the cost of pipe used in dry-land situations. A pipeline comes to a river that is 1 km wide at point A and must be extended to a refinery, R, on the other side, 8 km down the river. Find the best way to cross the river (assuming it is straight) so that the total cost of the pipe is kept to a minimum. (Give your answer correct to one decimal place.)

19. A train leaves the station at 10:00 p.m. and travels due north at a speed of 100 km/h. Another train has been heading due west at 120 km/h and reaches the same station at 11:00 p.m. At what time were the two trains closest together?

20. A store sells portable MP3 players for \$100 each and, at this price, sells 120 MP3 players every month. The owner of the store wishes to increase his profit, and he estimates that, for every \$2 increase in the price of MP3 players, one less MP3 player will be sold each month. If each MP3 player costs the store \$70, at what price should the store sell the MP3 players to maximize profit?

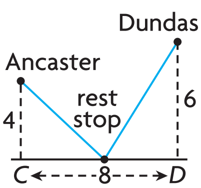
21. An offshore oil well, P, is located in the ocean 5 km from the nearest point on the shore, A. A pipeline is to be built to take oil from P to a refinery that is 20 km along the straight shoreline from A. If it costs \$100 000 per kilometre to lay pipe underwater and only \$75 000 per kilometre to lay pipe on land, what route from the well to the refinery will be the cheapest? (Give your answer correct to one decimal place.)



22. The printed area of a page in a book will be 81 cm^2. The margins at the top and bottom of the page will each be 3 cm deep. The margins at the sides of the page will each be 2 cm wide. What page dimensions will minimize the amount of paper?

23. A rectangular rose garden will be surrounded by a brick wall on three sides and by a fence on the fourth side. The area of the garden will be 1000 m^2. The cost of the brick wall is \$192/m. The cost of the fencing is \$48/m. Find the dimensions of the garden so that the cost of the materials will be as low as possible.

24. A boat leaves a dock at 2:00 p.m., heading west at 15 km/h. Another boat heads south at 12 km/h and reaches the same dock at 3:00 p.m. When were the boats closest to each other?



25. Two towns, Ancaster and Dundas, are 4 km and 6 km, respectively, from an old railroad line that has been made into a bike trail. Points C and D on the trail are the closest points to the two towns, respectively. These points are 8 km apart. Where should a rest stop be built to minimize the length of new trail that must be built from both towns to the rest stop?

26. Find the absolute maximum and minimum values.

a. $f(x) = x^2 - 2x + 6, -1 \le x \le 7$

b. $f(x) = x^3 + x^2, -3 \le x \le 3$

c. $f(x) = x^3 - 12x + 2, -5 \le x \le 5$

d. $f(x) = 3x^5 - 5x^3, -2 \le x \le 4$

27. Sam applies the brakes steadily to stop his car, which is travelling at 20 m/s. The position of the car, s, in metres at t seconds, is given by $s(t) = 20t - 0.3t^3$. Determine

a. the stopping distance b. the stopping time c. the deceleration at 2 s

28. Calculate each of the following:

a. $f''(2)$ if $f(x) = 5x^3 - x$

b. $f''(-1)$ if $f(x) = -2x^{-3} + x^2$

c. $f''(0)$ if $f(x) = (4x - 1)^4$

d. $f''(1)$ if $f(x) = \dfrac{2x}{x - 5}$

e. $f''(4)$ if $f(x) = \sqrt{x + 5}$

f. $f''(8)$ if $f(x) = \sqrt[3]{x^2}$

29. An object moves along a straight line. The object's position at time t is given by $s(t)$. Find the position, velocity, acceleration, and speed at the specified time.

a. $s(t) = \dfrac{2t}{t + 3}, t = 3$

b. $s(t) = t + \dfrac{5}{t + 2}, t = 1$

30. The function $s(t) = (t^2 + t)^{\frac{2}{3}}, t \ge 0$, represents the displacement, s, in metres, of a particle moving along a straight line after t seconds.

a. Determine $v(t)$ and $a(t)$.

b. Find the average velocity during the first 5 s.

c. Determine the velocity at exactly 5 s.

d. Find the average acceleration during the first 5 s.

e. Determine the acceleration at exactly 5 s.