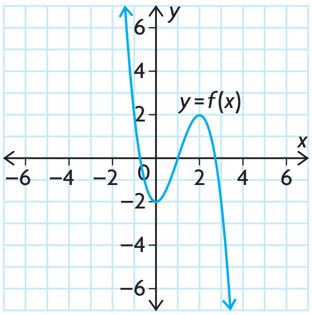
Mid-Chapter Review

1. Use a graphing calculator or graphing software to graph each of the following functions. Inspect the graph to determine where the function is increasing and where it is decreasing.
   a. $y = 3x^2 - 12x + 7$
   b. $y = 4x^3 - 12x^2 + 8$
   c. $f(x) = \frac{x + 2}{x + 3}$
   d. $f(x) = \frac{x^2 - 1}{x^2 + 3}$
2. Determine where $g(x) = 2x^3 - 3x^2 - 12x + 15$ is increasing and where it is decreasing.
3. Graph $f(x)$ if $f'(x) < 0$ when $x < -2$ and $x > 3$, $f'(x) > 0$ when $-2 < x < 3$, $f(-2) = 0$, and $f(3) = 5$.
4. Find all the critical numbers of each function.
   a. $y = -2x^2 + 16x - 31$
   b. $y = x^3 - 27x$
   c. $y = x^4 - 4x^2$
   d. $y = 3x^5 - 25x^3 + 60x$
   e. $y = \frac{x^2 - 1}{x^2 + 1}$
   f. $y = \frac{x}{x^2 + 2}$
5. For each function, find the critical numbers. Use the first derivative test to identify the local maximum and minimum values.
   a. $g(x) = 2x^3 - 9x^2 + 12x$
   b. $g(x) = x^3 - 2x^2 - 4x$
6. Find a value of k that gives $f(x) = x^2 + kx + 2$ a local minimum value of 1.
7. For $f(x) = x^4 - 32x + 4$, find the critical numbers, the intervals on which the function increases and decreases, and all the local extrema. Use graphing technology to verify your results.
8. Find the vertical asymptote(s) of the graph of each function. Describe the behaviour of $f(x)$ to the left and right of each asymptote.
   a. $f(x) = \frac{x - 1}{x + 2}$
   b. $f(x) = \frac{1}{9 - x^2}$
   c. $f(x) = \frac{x^2 - 4}{3x + 9}$
   d. $f(x) = \frac{2 - x}{3x^2 - 13x - 10}$
9. For each of the following, determine the equations of any horizontal asymptotes. Then state whether the curve approaches the asymptote from above or below.
   a. $y = \frac{3x - 1}{x + 5}$
   b. $f(x) = \frac{x^2 + 3x - 2}{(x - 1)^2}$
10. For each of the following, check for discontinuities and state the equation of any vertical asymptotes. Conduct a limit test to determine the behaviour of the curve on either side of the asymptote.
    a. $f(x) = \frac{x}{(x - 5)^2}$
    b. $f(x) = \frac{5}{x^2 + 9}$
    c. $f(x) = \frac{x - 2}{x^2 - 12x + 12}$

11. a. What does $f'(x) > 0$ imply about $f(x)$?
b. What does $f'(x) < 0$ imply about $f(x)$?

12. A diver dives from the 3 m springboard. The diver's height above the water, in metres, at t seconds is $h(t) = -4.9t^2 + 9.5t + 2.2$.
a. When is the height of the diver increasing? When is it decreasing?
b. When is the velocity of the diver increasing? When is it decreasing?

13. The concentration, C, of a drug injected into the bloodstream t hours after injection can be modelled by $C(t) = \frac{t}{4} + 2t^{-2}$. Determine when the concentration of the drug is increasing and when it is decreasing.



14. Graph $y = f'(x)$ for the function shown at the left.

15. For each function $f(x)$,
i. find the critical numbers
ii. determine where the function increases and decreases
iii. determine whether each critical number is at a local maximum, a local minimum, or neither
iv. use all the information to sketch the graph

a. $f(x) = x^2 - 7x - 18$
b. $f(x) = -2x^3 + 9x^2 + 3$
c. $f(x) = 2x^4 - 4x^2 + 2$
d. $f(x) = x^5 - 5x$

16. Determine the equations of any vertical or horizontal asymptotes for each function. Describe the behaviour of the function on each side of any vertical or horizontal asymptote.

a. $f(x) = \dfrac{x - 5}{2x + 1}$
b. $g(x) = \dfrac{x^2 - 4x - 5}{(x + 2)^2}$
c. $h(x) = \dfrac{x^2 + 2x - 15}{9 - x^2}$
d. $m(x) = \dfrac{2x^2 + x + 1}{x + 4}$

17. Find each limit.

a. $\lim\limits_{x \to \infty} \dfrac{3 - 2x}{3x}$
b. $\lim\limits_{x \to \infty} \dfrac{x^2 - 2x + 5}{6x^2 + 2x - 1}$
c. $\lim\limits_{x \to \infty} \dfrac{7 + 2x^2 - 3x^3}{x^3 - 4x^2 + 3x}$
d. $\lim\limits_{x \to \infty} \dfrac{5 - 2x^3}{x^4 - 4x}$
e. $\lim\limits_{x \to \infty} \dfrac{2x^5 - 1}{3x^4 - x^2 - 2}$
f. $\lim\limits_{x \to \infty} \dfrac{x^2 + 3x - 18}{(x - 3)^2}$
g. $\lim\limits_{x \to \infty} \dfrac{x^2 - 4x - 5}{x^2 - 1}$
h. $\lim\limits_{x \to \infty} \left(5x + 4 - \dfrac{7}{x + 3}\right)$