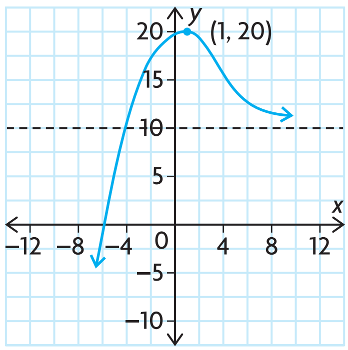
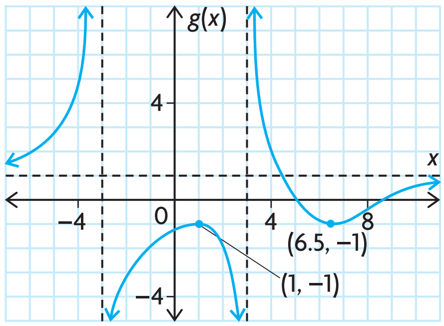
Review Exercise

1. For each of the following graphs, state
   i. the intervals where the function is increasing
   ii. the intervals where the function is decreasing
   iii. the points where the tangent to the function is horizontal

a.



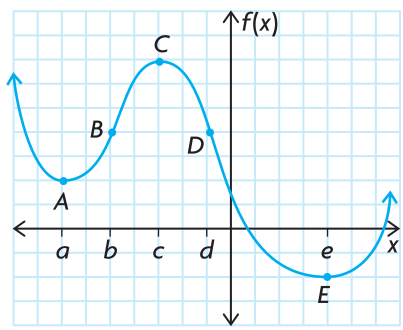
b.



2. Is it always true that an increasing function is concave up in shape? Explain.

3. Determine the critical points for each function. Determine whether the critical point is a local maximum or local minimum and whether or not the tangent is parallel to the x-axis.
   a. $f(x) = -2x^3 + 9x^2 + 20$
   b. $f(x) = x^4 - 8x^3 + 18x^2 + 6$
   c. $h(x) = \dfrac{x - 3}{x^2 + 7}$
   d. $g(x) = (x - 1)^{\frac{1}{3}}$

4. The graph of the function $y = f(x)$ has local extrema at points A, C, and E and points of inflection at B and D. If a, b, c, d, and e are the x-coordinates of the points, state the intervals on which the following conditions are true:
   a. $f'(x) > 0$ and $f''(x) > 0$
   b. $f'(x) > 0$ and $f''(x) < 0$
   c. $f'(x) < 0$ and $f''(x) > 0$
   d. $f'(x) < 0$ and $f''(x) < 0$



5. For each of the following, check for discontinuities and state the equation of any vertical asymptotes. Conduct a limit test to determine the behaviour of the curve on either side of the asymptote.

a. $y = \dfrac{2x}{x - 3}$

b. $g(x) = \dfrac{x - 5}{x + 5}$

c. $f(x) = \dfrac{x^2 - 2x - 15}{x + 3}$

d. $g(x) = \dfrac{5}{x^2 - x - 20}$

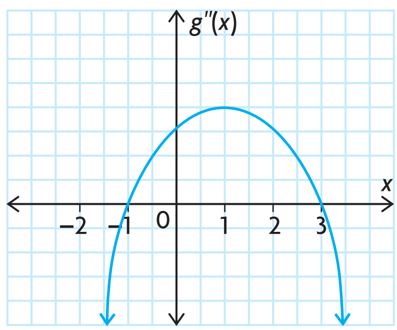
6. Determine the point of inflection on the curve defined by $y = x^3 + 5$. Show that the tangent line at this point crosses the curve.

7. Sketch a graph of a function that is differentiable on the interval $-3 \leq x \leq 5$ and satisfies the following conditions:

- There are local maxima at $(-2, 10)$ and $(3, 4)$.
- The function f is decreasing on the intervals $-2 < x < 1$ and $3 \leq x \leq 5$.
- The derivative $f'(x)$ is positive for $-3 \leq x < -2$ and for $1 < x < 3$.
- $f(1) = -6$

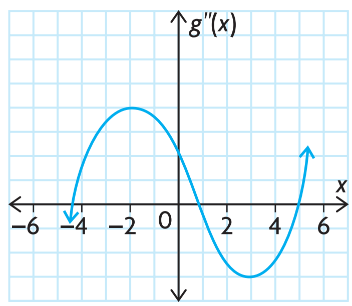
8. Each of the following graphs represents the second derivative, $g''(x)$, of a function $g(x)$:

a.



$g''(x)$ is a quadratic function.

b.



$g''(x)$ is a cubic function.

i. On what intervals is the graph of $g(x)$ concave up? On what intervals is the graph concave down?

ii. List the x-coordinates of the points of inflection.

iii. Make a rough sketch of a possible graph for $g(x)$, assuming that $g(0) = -3$.

9. a. If the graph of the function $g(x) = \frac{ax + b}{(x - 1)(x - 4)}$ has a horizontal tangent at point $(2, -1)$, determine the values of a and b.
   b. Sketch the function g.

10. Sketch each function using suitable techniques.
   a. $y = x^4 - 8x^2 + 7$
   b. $f(x) = \frac{3x - 1}{x + 1}$
   c. $g(x) = \frac{x^2 + 1}{4x^2 - 9}$
   d. $y = x(x - 4)^3$
   e. $h(x) = \frac{x}{x^2 - 4x + 4}$
   f. $f(t) = \frac{t^2 - 3t + 2}{t - 3}$

11. a. Determine the conditions on parameter k such that the function $f(x) = \frac{2x + 4}{x^2 - k^2}$ will have critical points.
   b. Select a value for k that satisfies the constraint established in part a, and sketch the section of the curve that lies in the domain $|x| \leq k$.

12. Determine the equation of the oblique asymptote in the form $y = mx + b$ for each function, and then show that $\lim_{x \to +\infty} [y - f(x)] = 0$.
   a. $f(x) = \frac{2x^2 - 7x + 5}{2x - 1}$
   b. $f(x) = \frac{4x^3 - x^2 - 15x - 50}{x^2 - 3x}$

13. Determine the critical numbers and the intervals on which $g(x) = (x^2 - 4)^2$ is increasing or decreasing.

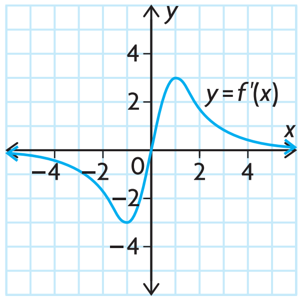
14. Use the second derivative test to identify all maximum and minimum values of $f(x) = x^3 + \frac{3}{2}x^2 - 7x + 5$ on the interval $-4 \leq x \leq 3$.

15. Use the y-intercept, local extrema, intervals of concavity, and points of inflection to graph $f(x) = 4x^3 + 6x^2 - 24x - 2$.

16. Let $p(x) = \frac{3x^3 - 5}{4x^2 + 1}$, $q(x) = \frac{3x - 1}{x^2 - 2x - 3}$, $r(x) = \frac{x^2 - 2x - 8}{x^2 - 1}$, and $s(x) = \frac{x^3 + 2x}{x - 2}$.
   a. Determine the asymptotes for each function, and identify their type (vertical, horizontal, or oblique).
   b. Graph $y = r(x)$, showing clearly the asymptotes and the intercepts.

17. If $f(x) = \frac{x^3 + 8}{x}$, determine the domain, intercepts, asymptotes, intervals of increase and decrease, and concavity. Locate any critical points and points of inflection. Use this information to sketch the graph of $f(x)$.

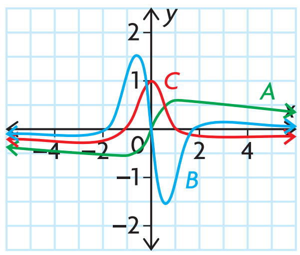
18. Explain how you can use this graph of $y = f'(x)$ to sketch a possible graph of the original function, $y = f(x)$.



19. For $f(x) = \frac{5x}{(x-1)^2}$, show that $f'(x) = \frac{-5(x+1)}{(x-1)^3}$ and $f''(x) = \frac{100(x+2)}{(x-1)^4}$. Use the function and its derivatives to determine the domain, intercepts, asymptotes, intervals of increase and decrease, and concavity, and to locate any local extrema and points of inflection. Use this information to sketch the graph of f.

20. The graphs of a function and its derivatives, $y = f(x)$, $y = f'(x)$, and $y = f''(x)$, are shown on each pair of axes. Which is which? Explain how you can tell.

a.



b.