Mid-Chapter Review

1. Determine the derivative of each function.

   a. $y = 5e^{-3x}$
   b. $y = 7e^{\frac{1}{7}x}$
   c. $y = x^3e^{-2x}$
   d. $y = (x - 1)^2e^x$
   e. $y = (x - e^{-x})^2$
   f. $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

2. A certain radioactive substance decays exponentially over time. The amount of a sample of the substance that remains, P, after t years is given by $P(t) = 100e^{-5t}$, where P is expressed as a percent.

   a. Determine the rate of change of the function, $\frac{dP}{dt}$.
   b. What is the rate of decay when 50% of the original sample has decayed?

3. Determine the equation of the tangent to the curve $y = 2 - xe^x$ at the point where $x = 0$.

4. Determine the first and second derivatives of each function.

   a. $y = -3e^x$
   b. $y = xe^{2x}$
   c. $y = e^x(4 - x)$

5. Determine the derivative of each function.

   a. $y = 8^{2x+5}$
   b. $y = 3.2(10)^{0.2x}$
   c. $f(x) = x^22^x$
   d. $H(x) = 300(5)^{3x-1}$
   e. $q(x) = 1.9^x + x^{1.9}$
   f. $f(x) = (x - 2)^2 \times 4^x$

6. The number of rabbits in a forest at time t, in months, is $R(t) = 500[10 + e^{-\frac{t}{10}}]$.

   a. What is the initial number of rabbits in the forest?
   b. Determine the rate of change of the number of rabbits at time t.
   c. How fast is the number of rabbits changing after one year?
   d. Determine the largest number of rabbits in the forest during the first three years.
   e. Use graphing technology to graph R versus t. Give physical reasons why the population of rabbits might behave this way.

7. A drug is injected into the body in such a way that the concentration, C, in the blood at time t hours is given by the function $C(t) = 10(e^{-2t} - e^{-3t})$. At what time does the highest concentration occur within the first 5 h?

8. Given $y = c(e^{kx})$, for what values of k does the function represent growth? For what values of k does the function represent decay?

9. The rapid growth in the number of a species of insect is given by $P(t) = 5000e^{0.02t}$, where t is the number of days.
   a. What is the initial population $(t = 0)$?
   b. How many insects will there be after a week?
   c. How many insects will there be after a month (30 days)?
10. If you have ever travelled in an airplane, you probably noticed that the air pressure in the airplane varied. The atmospheric pressure, y, varies with the altitude, x kilometres, above Earth. For altitudes up to 10 km, the pressure in millimetres of mercury (mm Hg) is given by $y = 760e^{-0.125x}$. What is the atmospheric pressure at each distance above Earth?
    a. 5 km b. 7 km c. 9 km
11. A radioactive substance decays in such a way that the amount left after t years is given by $A = 100e^{-0.3t}$. The amount, A, is expressed as a percent. Find the function, A', that describes the rate of decay. What is the rate of decay when 50% of the substance is gone?
12. Given $f(x) = xe^x$, find all the x values for which $f'(x) > 0$. What is the significance of this?
13. Find the equation of the tangent to the curve $y = 5^{-x^2}$ at the point on the curve where $x = 1$. Graph the curve and the tangent at this point.
14. a. Determine an equation for $A(t)$, the amount of money in the account at any time t.
    b. Find the derivative $A'(t)$ of the function.
    c. At what rate is the amount growing at the end of two years? At what rate is it growing at the end of five years and at the end of 10 years?
    d. Is the rate constant?
    e. Determine the ratio of $\frac{A'(t)}{A(t)}$ for each value that you determined for $A'(t)$.
    f. What do you notice?
15. The function $y = e^x$ is its own derivative. It is not the only function, however, that has this property. Show that for every value of c, $y = c\,(e^x)$ has the same property.