Review Exercise

1. Differentiate each of the following:
   a. $y = 6 - e^x$
   b. $y = 2x + 3e^x$
   c. $y = e^{2x+3}$
   d. $y = e^{-3x^2+5x}$
   e. $y = xe^x$
   f. $s = \dfrac{e^t - 1}{e^t + 1}$

2. Determine $\frac{dy}{dx}$ for each of the following:
   a. $y = 10^x$
   b. $y = 4^{3x^2}$
   c. $y = (5x)(5^x)$
   d. $y = (x^4)\,2^x$
   e. $y = \dfrac{4x}{4^x}$
   f. $y = \dfrac{5^{\sqrt{x}}}{x}$

3. Differentiate each of the following:
   a. $y = 3 \sin 2x - 4 \cos 2x$
   b. $y = \tan 3x$
   c. $y = \dfrac{1}{2 - \cos x}$
   d. $y = x \tan 2x$
   e. $y = (\sin 2x)e^{3x}$
   f. $y = \cos^2 2x$

4. a. Given the function $f(x) = \frac{e^x}{x}$, solve the equation $f'(x) = 0$.
   b. Discuss the significance of the solution you found in part a.

5. a. If $f(x) = xe^{-2x}$, find $f'\left(\frac{1}{2}\right)$.
   b. Explain what this number represents.

6. Determine the second derivative of each of the following:
   a. $y = xe^x - e^x$
   b. $y = xe^{10x}$

7. If $y = \frac{e^{2x} - 1}{e^{2x} + 1}$, prove that $\frac{dy}{dx} = 1 - y^2$.

8. Determine the equation of the tangent to the curve defined by $y = x - e^{-x}$ that is parallel to the line represented by $3x - y - 9 = 0$.

9. Determine the equation of the tangent to the curve $y = x \sin x$ at the point where $x = \frac{\pi}{2}$.

10. An object moves along a line so that, at time t, its position is $s = \frac{\sin t}{3 + \cos 2t}$, where s is the displacement in metres. Calculate the object's velocity at $t = \frac{\pi}{4}$.

11. The number of bacteria in a culture, N, at time t is given by $N(t) = 2000\left[30 + te^{-\frac{t}{20}}\right]$.

   a. When is the rate of change of the number of bacteria equal to zero?

   b. If the bacterial culture is placed in a colony of mice, the number of mice that become infected, M, is related to the number of bacteria present by the equation $M(t) = \sqrt[3]{N + 1000}$. After 10 days, how many mice are infected per day?

12. The concentrations of two medicines in the bloodstream t hours after injection are $c_1(t) = te^{-t}$ and $c_2(t) = t^2e^{-t}$.

   a. Which medicine has the larger maximum concentration?

   b. Within the first half hour, which medicine has the larger maximum concentration?

13. Differentiate.

   a. $y = (2 + 3e^{-x})^3$
   b. $y = x^e$
   c. $y = e^{e^x}$
   d. $y = (1 - e^{5x})^5$

14. Differentiate.

   a. $y = 5^x$
   b. $y = (0.47)^x$
   c. $y = (52)^{2x}$
   d. $y = 5(2)^x$
   e. $y = 4(e)^x$
   f. $y = -2(10)^{3x}$

15. Determine y'.

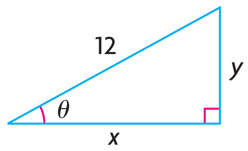
   a. $y = \sin 2^x$
   b. $y = x^2 \sin x$
   c. $y = \sin\left(\frac{\pi}{2} - x\right)$
   d. $y = \cos x \sin x$
   e. $y = \cos^2 x$
   f. $y = \cos x \sin^2 x$

16. Determine the equation of the tangent to the curve $y = \cos x$ at $\left(\frac{\pi}{2}, 0\right)$.

17. An object is suspended from the end of a spring. Its displacement from the equilibrium position is $s = 8 \sin(10\pi t)$ at time t. Calculate the velocity and acceleration of the object at any time t, and show that $\frac{d^2s}{dt^2} + 100\pi^2 s = 0$.

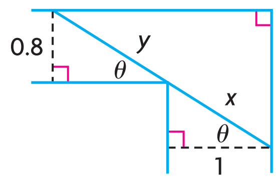
18. The position of a particle is given by $s = 5\cos\left(2t + \frac{\pi}{4}\right)$ at time t. What are the maximum values of the displacement, the velocity, and the acceleration?

19. The hypotenuse of a right triangle is 12 cm in length. Calculate the measures of the unknown angles in the triangle that will maximize its perimeter.

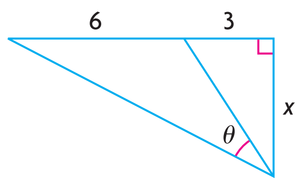


20. A fence is 1.5 m high and is 1 m from a wall. A ladder must start from the ground, touch the top of the fence, and rest somewhere on the wall. Calculate the minimum length of the ladder.

21. A thin rigid pole needs to be carried horizontally around a corner joining two corridors, which are 1 m and 0.8 m wide. Calculate the length of the longest pole that can be carried around this corner.



22. When the rules of hockey were developed, Canada did not use the metric system. Thus, the distance between the goal posts was designated to be six feet (slightly less than 2 m). If Sidney Crosby is on the goal line, three feet outside one of the goal posts, how far should he go out (perpendicular to the goal line) to maximize the angle in which he can shoot at the goal?
Hint: Determine the values of x that maximize θ in the following diagram.



23. Determine $f''(x)$

a. $f(x) = 4\sin^2(x - 2)$

b. $f(x) = 2(\cos x)(\sec^2 x)$

Chapter 5 Test

1. Determine the derivative $\frac{dy}{dx}$ for each of the following:

   a. $y = e^{-2x^2}$

   b. $y = 3^{x^2+3x}$

   c. $y = \frac{e^{3x} + e^{-3x}}{2}$

   d. $y = 2 \sin x - 3 \cos 5x$

   e. $y = \sin^3 (x^2)$

   f. $y = \tan\sqrt{1 - x}$

2. Determine the equation of the tangent to the curve defined by $y = 2e^{3x}$ that is parallel to the line defined by $-6x + y = 2$.

3. Determine the equation of the tangent to $y = e^x + \sin x$ at $(0, 1)$.

4. The velocity of a certain particle that moves in a straight line under the influence of forces is given by $v(t) = 10e^{-kt}$, where k is a positive constant and $v(t)$ is in centimetres per second.

   a. Show that the acceleration of the particle is proportional to a constant multiple of its velocity. Explain what is happening to the particle.

   b. What is the initial velocity of the particle?

   c. At what time is the velocity equal to half the initial velocity? What is the acceleration at this time?

5. Determine $f''(x)$.

   a. $f(x) = \cos^2 x$

   b. $f(x) = \cos x \cot x$

6. Determine the absolute extreme values of $f(x) = \sin^2 x$, where $x \in [0, \pi]$.

7. Calculate the slope of the tangent line that passes through $y = 5^x$, where $x = 2$. Express your answer to two decimal places.

8. Determine all the maximum and minimum values of $y = xe^x + 3e^x$.

9. $f(x) = 2 \cos x - \sin 2x$ where $x \in [-\pi, \pi]$

   a. Determine all critical number for $f(x)$ on the given interval.

   b. Determine the intervals where $f(x)$ is increasing and where it is decreasing.

   c. Determine all local maximum and minimum values of $f(x)$ on the given interval.

   d. Use the information you found above to sketch the curve.