Chapter 5 Test

1. Determine the derivative $\frac{dy}{dx}$ for each of the following:

   a. $y = e^{-2x^2}$

   b. $y = 3^{x^2+3x}$

   c. $y = \dfrac{e^{3x} + e^{-3x}}{2}$

   d. $y = 2 \sin x - 3 \cos 5x$

   e. $y = \sin^3 (x^2)$

   f. $y = \tan\sqrt{1 - x}$

2. Determine the equation of the tangent to the curve defined by $y = 2e^{3x}$ that is parallel to the line defined by $-6x + y = 2$.

3. Determine the equation of the tangent to $y = e^x + \sin x$ at $(0, 1)$.

4. The velocity of a certain particle that moves in a straight line under the influence of forces is given by $v(t) = 10e^{-kt}$, where k is a positive constant and $v(t)$ is in centimetres per second.

   a. Show that the acceleration of the particle is proportional to a constant multiple of its velocity. Explain what is happening to the particle.

   b. What is the initial velocity of the particle?

   c. At what time is the velocity equal to half the initial velocity? What is the acceleration at this time?

5. Determine $f''(x)$.

   a. $f(x) = \cos^2 x$

   b. $f(x) = \cos x \cot x$

6. Determine the absolute extreme values of $f(x) = \sin^2 x$, where $x \in [0, \pi]$.

7. Calculate the slope of the tangent line that passes through $y = 5^x$, where $x = 2$. Express your answer to two decimal places.

8. Determine all the maximum and minimum values of $y = xe^x + 3e^x$.

9. $f(x) = 2 \cos x - \sin 2x$ where $x \in [-\pi, \pi]$

   a. Determine all critical number for $f(x)$ on the given interval.

   b. Determine the intervals where $f(x)$ is increasing and where it is decreasing.

   c. Determine all local maximum and minimum values of $f(x)$ on the given interval.

   d. Use the information you found above to sketch the curve.