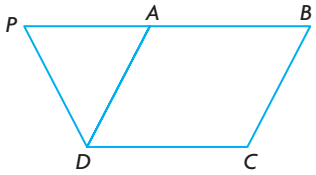
Mid-Chapter Review

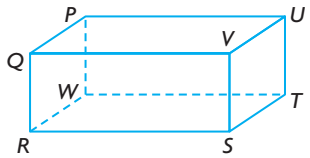
1. $ABCD$ is a parallelogram, and $|\overrightarrow{PD}| = |\overrightarrow{DA}|$.



   a. Determine which vectors (if any) are equal to $\overrightarrow{AB}$, $\overrightarrow{BA}$, $\overrightarrow{AD}$, $\overrightarrow{CB}$, and $\overrightarrow{AP}$.

   b. Explain why $|\overrightarrow{PD}| = |\overrightarrow{BC}|$.

2. The diagram below represents a rectangular prism. State a single vector equal to each of the following.



   a. $\overrightarrow{RQ} + \overrightarrow{RS}$
   b. $\overrightarrow{RQ} + \overrightarrow{QV}$
   c. $\overrightarrow{PW} + \overrightarrow{WS}$
   d. $(\overrightarrow{RQ} + \overrightarrow{RS}) + \overrightarrow{VU}$
   e. $\overrightarrow{PW} - \overrightarrow{VP}$
   f. $\overrightarrow{PW} + \overrightarrow{WR} + \overrightarrow{RQ}$

3. Two vectors, $\vec{a}$ and $\vec{b}$, have a common starting point with an angle of 120° between them. The vectors are such that $|\vec{a}| = 3$ and $|\vec{b}| = 4$.

   a. Calculate $|\vec{a} + \vec{b}|$.

   b. Calculate the angle between $\vec{a}$ and $\vec{a} + \vec{b}$.

4. Determine all possible values for t if the length of the vector $\vec{x} = t\vec{y}$ is $4|\vec{y}|$.

5. $PQRS$ is a quadrilateral where A, B, C, and D are the midpoints of SP, PQ, QR, and RS, respectively. Prove, using vector methods, that $ABCD$ is a parallelogram.

6. Given that $|\vec{u}| = 8$ and $|\vec{v}| = 10$ and the angle between vectors $\vec{u}$ and $\vec{v}$ is 60° determine:

   a. $|\vec{u} - \vec{v}|$
   b. the direction of $\vec{u} - \vec{v}$ relative to $\vec{u}$
   c. the unit vector in the direction of $\vec{u} + \vec{v}$
   d. $|5\vec{u} + 2\vec{v}|$

7. The vectors $\vec{p}$ and $\vec{q}$ are distinct unit vectors that are placed in a tail-to-tail position. If these two vectors have an angle of 60° between them, determine $|2\vec{p} - \vec{q}|$.

8. The vector $\vec{m}$ is collinear (parallel) to $\vec{n}$ but in the opposite direction. Express the magnitude of $\vec{m} + \vec{n}$ in terms of the magnitudes of $\vec{m}$ and $\vec{n}$.

9. $ABCD$ is a parallelogram. If $\overrightarrow{AB} = \vec{x}$ and $\overrightarrow{DA} = \vec{y}$, express $\overrightarrow{BC}$, $\overrightarrow{DC}$, $\overrightarrow{BD}$, and $\overrightarrow{AC}$ in terms of $\vec{x}$ and $\vec{y}$.

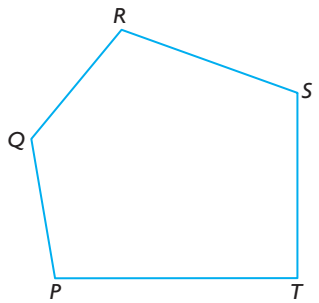
10. If A, B, and C are three collinear points with B at the midpoint of AC, and O is any point not on the line AC, prove that $\overrightarrow{OA} + \overrightarrow{OC} = 2\overrightarrow{OB}$. (*Hint*: $\overrightarrow{AB} = \overrightarrow{BC}$.)

11. $ABCD$ is a quadrilateral with $\overrightarrow{AB} = \vec{x}$, $\overrightarrow{CD} = 2\vec{y}$, and $\overrightarrow{AC} = 3\vec{x} - \vec{y}$. Express $\overrightarrow{BD}$ and $\overrightarrow{BC}$ in terms of $\vec{x}$ and $\vec{y}$.

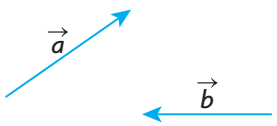
12. An airplane is heading due south at a speed of 500 km/h when it encounters a head wind from the south at 40 km/h. What is the resultant ground velocity of the airplane?

13. $PQRST$ is a pentagon. State a single vector that is equivalent to each of the following:

    a. $\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RT}$ b. $\overrightarrow{PQ} + \overrightarrow{QR} - \overrightarrow{TR}$ c. $\overrightarrow{PR} - (\overrightarrow{PT} - \overrightarrow{ST})$



14. The vectors $\vec{a}$ and $\vec{b}$ are given below. Use these vectors to sketch each of the following.



    a. $\frac{1}{3}\vec{a} + \vec{b}$ b. $\frac{3}{2}\vec{a} - 2\vec{b}$ c. $-\vec{b} + \vec{a}$ d. $\frac{\vec{b} + \vec{a}}{2}$

15. $PQRS$ is a quadrilateral with $\overrightarrow{PQ} = 2\vec{a}$, $\overrightarrow{QR} = 3\vec{b}$, and $\overrightarrow{QS} = 3\vec{b} - 3\vec{a}$. Express $\overrightarrow{PS}$ and $\overrightarrow{RS}$ in terms of $\vec{a}$ and $\vec{b}$.