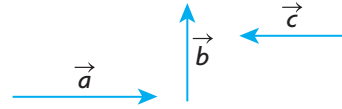


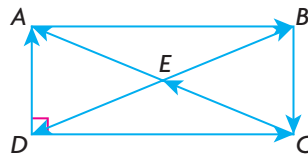
Review Exercise

- Determine whether each of the following statements is true or false. Provide a brief explanation for each answer.
 - $|\vec{a} + \vec{b}| \geq |\vec{a}|$
 - $|\vec{a} + \vec{b}| = |\vec{a} + \vec{c}|$ implies $|\vec{b}| = |\vec{c}|$
 - $\vec{a} + \vec{b} = \vec{a} + \vec{c}$ implies $\vec{b} = \vec{c}$
 - $\overrightarrow{RF} = \overrightarrow{SW}$ implies $\overrightarrow{RS} = \overrightarrow{FW}$
 - $m\vec{a} + n\vec{a} = (m + n)\vec{a}$
 - If $|\vec{a}| = |\vec{b}|$ and $|\vec{c}| = |\vec{d}|$, then $|\vec{a} + \vec{b}| = |\vec{c} + \vec{d}|$.
- If $\vec{x} = 2\vec{a} - 3\vec{b} - 4\vec{c}$, $\vec{y} = -2\vec{a} + 3\vec{b} + 3\vec{c}$, and $\vec{z} = 2\vec{a} - 3\vec{b} + 5\vec{c}$, determine simplified expressions for each of the following:
 - $2\vec{x} - 3\vec{y} + 5\vec{z}$
 - $3(-2\vec{x} - 4\vec{y} + \vec{z}) - (2\vec{x} - \vec{y} + \vec{z}) - 2(-4\vec{x} - 5\vec{y} + \vec{z})$
- If $X(-2, 1, 2)$ and $Y(-4, 4, 8)$ are two points in R^3 , determine the following:
 - $\overrightarrow{XY}$ and $|\overrightarrow{XY}|$
 - The coordinates of a unit vector in the same direction as $\overrightarrow{XY}$.
- $X(-1, 2, 6)$ and $Y(5, 5, 12)$ are two points in R^3 .
 - Determine the components of a position vector equivalent to $\overrightarrow{YX}$.
 - Determine the components of a *unit* vector that is in the same direction as $\overrightarrow{YX}$.
- Find the components of the unit vector with the opposite direction to that of the vector from $M(2, 3, 5)$ to $N(8, 1, 2)$.
- A parallelogram has its sides determined by the vectors $\overrightarrow{OA} = (3, 2, -6)$ and $\overrightarrow{OB} = (-6, 6, -2)$.
 - Determine the components of the vectors representing the diagonals.
 - Determine the angles between the sides of the parallelogram.
- The points $A(-1, 1, 1)$, $B(2, 0, 3)$, and $C(3, 3, -4)$ are vertices of a triangle.
 - Show that this triangle is a right triangle.
 - Calculate the area of triangle ABC .
 - Calculate the perimeter of triangle ABC .
 - Calculate the coordinates of the fourth vertex D that completes the rectangle of which A , B , and C are the other three vertices.

8. The vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are as shown.

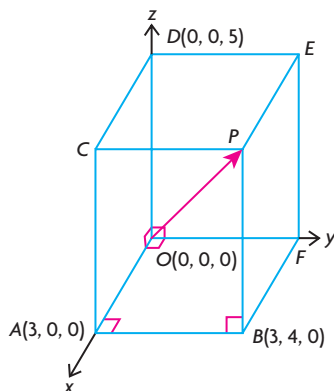


- a. Construct the vector $\vec{a} - \vec{b} + \vec{c}$.
 - b. If the vectors $\vec{a}$ and $\vec{b}$ are perpendicular, and if $|\vec{a}| = 4$ and $|\vec{b}| = 3$, determine $|\vec{a} + \vec{b}|$.
9. Given $\vec{p} = (-11, 7)$, $\vec{q} = (-3, 1)$, and $\vec{r} = (-1, 2)$, express each vector as a linear combination of the other two.
10. a. Find an equation to describe the set of points equidistant from $A(2, -1, 3)$ and $B(1, 2, -3)$.
- b. Find the coordinates of two points that are equidistant from A and B .
11. Calculate the values of a , b , and c in each of the following:
- a. $2(a, b, 4) + \frac{1}{2}(6, 8, c) - 3(7, c, -4) = (-24, 3, 25)$
 - b. $2\left(a, a, \frac{1}{2}a\right) + (3b, 0, -5c) + 2\left(c, \frac{3}{2}c, 0\right) = (3, -22, 54)$
12. a. Determine whether the points $A(1, -1, 1)$, $B(2, 2, 2)$, and $C(4, -2, 1)$ represent the vertices of a right triangle.
- b. Determine whether the points $P(1, 2, 3)$, $Q(2, 4, 6)$, and $R(-1, -2, -3)$ are collinear.
13. a. Show that the points $A(3, 0, 4)$, $B(1, 2, 5)$, and $C(2, 1, 3)$ represent the vertices of a right triangle.
- b. Determine $\cos \angle ABC$.
14. In the following rectangle, vectors are indicated by the direction of the arrows.



- a. Name two pairs of vectors that are opposites.
- b. Name two pairs of identical vectors.
- c. Explain why $|\overrightarrow{AD}|^2 + |\overrightarrow{DC}|^2 = |\overrightarrow{DB}|^2$.

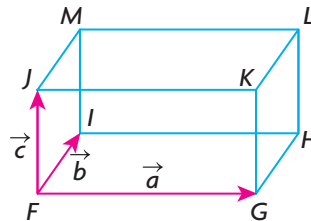
15. A rectangular prism measuring 3 by 4 by 5 is drawn on a coordinate axis as shown in the diagram.



- Determine the coordinates of points C , P , E , and F .
 - Determine position vectors for $\overrightarrow{DB}$ and $\overrightarrow{CF}$.
 - By drawing the rectangle containing $\overrightarrow{DB}$ and $\overrightarrow{OP}$, determine the acute angle between these vectors.
 - Determine the angle between $\overrightarrow{OP}$ and $\overrightarrow{AE}$.
16. The vectors $\vec{d}$ and $\vec{e}$ are such that $|\vec{d}| = 3$ and $|\vec{e}| = 5$, and the angle between them is 30° . Determine each of the following:
- $|\vec{d} + \vec{e}|$
 - $|\vec{d} - \vec{e}|$
 - $|\vec{e} - \vec{d}|$
17. An airplane is headed south at speed 400 km/h. The airplane encounters a wind from the east blowing at 100 km/h.
- How far will the airplane travel in 3 h?
 - What is the direction of the airplane?
18. a. Explain why the set of vectors: $\{(2, 3), (3, 5)\}$ spans R^2 .
b. Find m and n in the following: $m(2, 3) + n(3, 5) = (323, 795)$.
19. a. Show that the vector $\vec{a} = (5, 9, 14)$ can be written as a linear combination of the vectors $\vec{b}$ and $\vec{c}$, where $\vec{b} = (-2, 3, 1)$ and $\vec{c} = (3, 1, 4)$. Explain why $\vec{a}$ lies in the plane determined by $\vec{b}$ and $\vec{c}$.
b. Is the vector $\vec{a} = (-13, 36, 23)$ in the span of $\vec{b} = (-2, 3, 1)$ and $\vec{c} = (3, 1, 4)$? Explain your answer.

20. A cube is placed so that it has three of its edges located along the positive x -, y -, and z -axes (one edge along each axis) and one of its vertices at the origin.
- If the cube has a side length of 4, draw a sketch of this cube and write the coordinates of its vertices on your sketch.
 - Write the coordinates of the vector with its head at the origin and its tail at the opposite vertex.
 - Write the coordinates of a vector that starts at $(4, 4, 4)$ and is a diagonal in the plane parallel to the xz -plane.
 - What vector starts at the origin and is a diagonal in the xy -plane?
21. If $\vec{a} = \vec{i} + \vec{j} - \vec{k}$, $\vec{b} = 2\vec{i} - \vec{j} + 3\vec{k}$, and $\vec{c} = 2\vec{i} + 13\vec{k}$, determine $\left| 2(\vec{a} + \vec{b} - \vec{c}) - (\vec{a} + 2\vec{b}) + 3(\vec{a} - \vec{b} + \vec{c}) \right|$.
22. The three points $A(-3, 4)$, $B(3, -4)$, and $C(5, 0)$ are on a circle with radius 5 and centre at the origin. Points A and B are the endpoints of a diameter, and point C is on the circle.
- Calculate $|\vec{AB}|$, $|\vec{AC}|$, and $|\vec{BC}|$.
 - Show that A , B , and C are the vertices of a right triangle.
23. In terms of $\vec{a}$, $\vec{b}$, $\vec{c}$, and $\vec{0}$, find a vector expression for each of the following:

- $\vec{FL}$
- $\vec{MK}$
- $\vec{HJ}$
- $\vec{IH} + \vec{KJ}$
- $\vec{IK} - \vec{IH}$



24. Draw a diagram showing the vectors $\vec{a}$ and $\vec{b}$, where $|\vec{a}| = 2|\vec{b}|$ and $|\vec{b}| = |\vec{a} + \vec{b}|$ are both true. (Make sure to indicate the direction of the vectors.)
25. If the vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other, express each of the following in terms of $|\vec{a}|$ and $|\vec{b}|$:
- $|\vec{a} + \vec{b}|$
 - $|\vec{a} - \vec{b}|$
 - $|2\vec{a} + 3\vec{b}|$
26. Show that if $\vec{a}$ is perpendicular to each of the vectors $\vec{b}$ and $\vec{c}$, then $\vec{a}$ is perpendicular to $2\vec{b} + 4\vec{c}$.