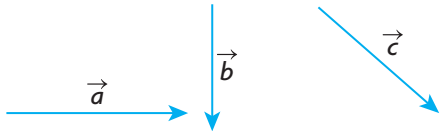
Chapter 6 Test

1. The vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are shown.



Using these three vectors, demonstrate that $\vec{a} + \left(\vec{b} + \vec{c}\right) = \left(\vec{a} + \vec{b}\right) + \vec{c}$. Name this property and explain how your answer shows this to be true.

2. $A(-2, 3, -5)$ and $B(6, 7, 3)$ are two points in R^3. Determine each of the following:

   a. $\overrightarrow{AB}$ b. $\left|\overrightarrow{AB}\right|$ c. a unit vector in the direction of $\overrightarrow{BA}$

3. The vectors $\vec{x}$ and $\vec{y}$ are each of length 3 units, i.e., $|\vec{x}| = |\vec{y}| = 3$. If $|\vec{x} + \vec{y}| = \sqrt{17}$, determine $|\vec{x} - \vec{y}|$.

4. a. If $3\vec{x} - 2\vec{y} = \vec{a}$ and $5\vec{x} - 3\vec{y} = \vec{b}$, express the vectors $\vec{x}$ and $\vec{y}$ in terms of $\vec{a}$ and $\vec{b}$.

   b. Solve for a, b, and c: $(2, -1, c) + (a, b, 1) - 3(2, a, 4) = (-3, 1, 2c)$.

5. a. Explain why the vectors $\vec{a} = (-2, 3)$ and $\vec{b} = (3, -1)$ span R^2.

   b. Determine the values of p and q in $p(-2, 3) + q(3, -1) = (13, -9)$.

6. a. Show that the vector $\vec{a} = (1, 12, -29)$ can be written as a linear combination of $\vec{b} = (3, 1, 4)$ and $\vec{c} = (1, 2, -3)$.

   b. Determine whether $\vec{r} = (16, 11, -24)$ can be written as a linear combination of $\vec{p} = (-2, 3, 4)$ and $\vec{q} = (4, 1, -6)$. Explain the significance of your result geometrically.

7. $\vec{x}$ and $\vec{y}$ are vectors of magnitude 1 and 2, respectively, with an angle of 120° between them. Determine $|3\vec{x} + 2\vec{y}|$ and the direction of $3\vec{x} + 2\vec{y}$.

8. In triangle ABC, point D is the midpoint of $\overrightarrow{BC}$ and point E is the midpoint of $\overrightarrow{AC}$. Vectors are marked as shown. Use vectors to prove that $\overrightarrow{DE} = \frac{1}{2}\overrightarrow{BA}$.