Chapter 7

APPLICATIONS OF VECTORS

In Chapter 6, we discussed some of the basic ideas about vectors. In this chapter, we will use vectors in both mathematical and physical situations to calculate quantities that would otherwise be difficult to determine. You will discover how vectors enable calculations in situations involving the velocity at which a plane flies under windy conditions and the force at which two other people must pull to balance the force created by two others in a game of tug-o-war. In addition, we will introduce the concept of vector multiplication and show how vectors can be applied in a variety of contexts.

CHAPTER EXPECTATIONS

In this chapter, you will

- use vectors to model and solve problems arising from real-world applications involving velocity and force, **Sections 7.1, 7.2**
- perform the operation of the dot product on two vectors, **Sections 7.3, 7.4**
- determine properties of the dot product, **Sections 7.3, 7.4**
- determine the scalar and vector projections of a vector, **Section 7.5**
- perform the operation of cross product on two algebraic vectors in three-dimensional space, **Section 7.6**
- determine properties of the cross product, **Section 7.6**
- solve problems involving the dot product and cross product, **Section 7.7**

Review of Prerequisite Skills

In this chapter, you will use vectors in applications involving elementary force and velocity problems. As well, you will be introduced to the study of scalar and vector products. You will find it helpful to be able to

- find the magnitude and the direction of vectors using trigonometry
- plot points and find coordinates of points in two- and three-dimensional systems

Exercise

1. The velocity of an airplane is 800 km/h north. A wind is blowing due east at 100 km/h. Determine the velocity of the airplane relative to the ground.

2. A particle is displaced 5 units to the west and then displaced 12 units in a direction $N45°W$. Find the magnitude and direction of the resultant displacement.

3. Draw the x-axis, y-axis, and z-axis, and plot the following points:

a. $A(0, 1, 0)$

b. $B(-3, 2, 0)$

c. $C(-2, 0, 1)$

d. $D(0, 2, -3)$

4. Express each of the following vectors in component form (a, b, c). Then determine its magnitude.

a. $3\vec{i} - 2\vec{j} + 7\vec{k}$

b. $-9\vec{i} + 3\vec{j} + 14\vec{k}$

c. $\vec{i} + \vec{j}$

d. $2\vec{i} - 9\vec{k}$

5. Describe where the following general points are located.

a. $A(x, y, 0)$

b. $B(x, 0, z)$

c. $C(0, y, z)$

6. Find a single vector that is equivalent to each linear combination.

a. $(-6, 0) + 7(1, -1)$

b. $(4, -1, 3) - (-2, 1, 3)$

c. $2(-1, 1, 3) + 3(-2, 3, -1)$

d. $-\frac{1}{2}(4, -6, 8) + \frac{3}{2}(4, -6, 8)$

7. If $\vec{a} = 3\vec{i} + 2\vec{j} - \vec{k}$ and $\vec{b} = -2\vec{i} + \vec{j}$, determine a single vector that is equivalent to each linear combination.

a. $\vec{a} + \vec{b}$

b. $\vec{a} - \vec{b}$

c. $2\vec{a} - 3\vec{b}$

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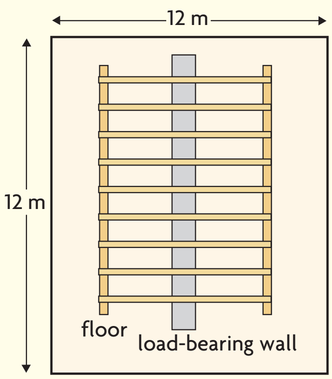
CHAPTER 7: FORCES IN ARCHITECTURE: STRUCTURAL ENGINEERING

Structural engineers are a specific kind of architect: they help in the design of large-scale structures, such as bridges and skyscrapers. The role of structural engineers is to make sure that the structure being built will be stable and not collapse. To do this, they need to calculate all the forces acting on the structure, including the weight of building materials, occupants, and any furniture or items that might be stored in the building. They also need to account for the forces of wind, water, and seismic activity, including hurricanes and earthquakes. To design a safe structure effectively, the composition of forces must be calculated to find the resultant force. The strength and structure of the materials must exert a force greater than the equilibrant force. A simple example of this is a load-bearing wall inside a house. A structural engineer must calculate the total weight of the floor above, which is considered a dead load. Then the engineer has to factor in the probable weight of the occupants and their furniture, which is considered a live load. The load-bearing wall must be built to exert an opposing force that is greater than the force created by the live load.

Type of Load	Load (kg/m^2)
live	90
dead	150



Case Study—Replacing a Load-Bearing Wall with a Steel Support Beam

The table at the left shows the normal loads created by a timber floor and a non-load-bearing wall above. A homeowner wants to make one large room out of two. This will require removing a wall that is bearing the load of the floor above and replacing the wall with a steel support beam. The horizontal and vertical yellow segments represent the framing for the area of the upper floor that is currently being supported by the wall, without help from the walls at the edges of the rooms. Complete the discussion questions to determine what size of beam will be required to bear the load.

DISCUSSION QUESTIONS

1. Find the area of the floor that is currently being supported by the load-bearing wall. Use the information in the table to calculate the live and dead loads for this area.
2. Find the resultant downward force created by the weight of the floor above including an estimate for the expected weight of four occupants and their furniture.
3. Determine the equilibrant force required by a steel support beam that would support the force you calculated in question 2. Explain why, for safety reasons, a beam that supports a greater force is used.