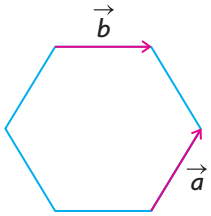
Mid-Chapter Review

1. a. If $|\vec{a}| = 3$ and $|\vec{b}| = 2$, and the angle between these two vectors is 60°, determine $\vec{a} \cdot \vec{b}$.
   b. Determine the numerical value of $(3\vec{a} + 2\vec{b}) \cdot (4\vec{a} - 3\vec{b})$.
2. A mass of 15 kg is suspended by two cords from a ceiling. The cords have lengths of 15 cm and 20 cm, and the distance between the points where they are attached on the ceiling is 25 cm. Determine the tension in each of the two cords.
3. In a square that has side lengths of 10 cm, what is the dot product of the vectors representing the diagonals?
4. An airplane is travelling at 500 km/h due south when it encounters a wind from $W45°N$ at 100 km/h.
   a. What is the resultant velocity of the airplane?
   b. How long will it take for the airplane to travel 1000 km?
5. A 15 kg block lies on a smooth ramp that is inclined at 40° to the ground.
   a. Determine the force that this block exerts in a direction perpendicular to the ramp.
   b. What is the force, parallel to the inclined plane, needed to prevent the block from slipping?
6. A regular hexagon, with sides of 3 cm, is shown below. Determine $\vec{a} \cdot \vec{b}$.



7. Given the vectors $\vec{a} = (4, -5, 20)$ and $\vec{b} = (1, 2, 2)$, determine the following:
   a. $\vec{a} \cdot \vec{b}$
   b. the cosine of the angle between the two vectors
8. Given the vectors $\vec{a} = \vec{i} + 2\vec{j} + \vec{k}, \vec{b} = 2\vec{i} - 3\vec{j} + 4\vec{k}$, and $\vec{c} = 3\vec{i} - \vec{j} - \vec{k}$, determine the following:
   a. $\vec{a} \cdot \vec{b}$
   b. $\vec{b} \cdot \vec{c}$
   c. $\vec{b} + \vec{c}$
   d. $\vec{a} \cdot (\vec{b} + \vec{c})$
   e. $(\vec{a} + \vec{b}) \cdot (\vec{b} + \vec{c})$
   f. $(2\vec{a} - 3\vec{b}) \cdot (2\vec{a} + \vec{c})$

9. Given the vectors $\vec{p} = x\vec{i} + \vec{j} + 3\vec{k}$ and $\vec{q} = 3x\vec{i} + 10x\vec{j} + \vec{k}$, determine the following:
   a. the value(s) of x that make these vectors perpendicular
   b. the values(s) of x that make these vectors parallel
10. If $\vec{x} = \vec{i} - 2\vec{j} - \vec{k}$ and $\vec{y} = \vec{i} - \vec{j} - \vec{k}$, determine the value of each of the following:
   a. $3\vec{x} - 2\vec{y}$
   b. $3\vec{x} \cdot 2\vec{y}$
   c. $|\vec{x} - 2\vec{y}|$
   d. $(2\vec{x} - 3\vec{y}) \cdot (\vec{x} + 4\vec{y})$
   e. $2\vec{x} \cdot \vec{y} - 5\vec{y} \cdot \vec{x}$
11. Three forces of 3 N, 4 N, and 5 N act on an object so that the object is in equilibrium. Determine the angle between the largest and smallest forces.
12. A force of 3 N and a force of 4 N act on an object. If these two forces make an angle of 60° to each other, find the resultant and equilibrant of these two forces.
13. The sides of a parallelogram are determined by the vectors $\vec{m} = (2, -3, 5)$ and $\vec{n} = (-1, 7, 5)$. Determine
   a. the larger angle between the diagonals of this parallelogram
   b. the smaller angle between the sides
14. Martina is planning to fly to a town 1000 km due north of her present location. There is a 45 km/h wind blowing from $N30°E$.
   a. If her plane travels at 500 km/h, what direction should the pilot head to reach the destination?
   b. How long will the trip take?
15. Determine the coordinates of a unit vector that is perpendicular to $\vec{a} = (-1, 2, 5)$ and $\vec{b} = (1, 3, 5)$.
16. Clarence leaves a dock, paddling a canoe at 3 m/s. He heads downstream at an angle of 45° to the current, which is flowing at 4 m/s.
   a. How far downstream does he travel in 10 s?
   b. What is the length of time required to cross the river if it is 180 m wide?
17. a. Under what conditions does $(\vec{x} + \vec{y}) \cdot (\vec{x} - \vec{y}) = 0$?
   b. Give a geometrical interpretation of the vectors $\vec{a}$, $\vec{b}$, $\vec{a} + \vec{b}$, and $\vec{a} - \vec{b}$.
18. A lawn roller with a mass of 60 kg is being pulled with a force of 350 N. If the handle of the roller makes an angle of 40° with the ground, what horizontal component of the force is causing the roller to move forward?