Key Concepts Review

In Chapter 7, you were introduced to applications of geometric vectors involving force and velocity. You were also introduced to the dot product and cross product between two vectors and should be familiar with the differences in their formulas and applications. Consider the following summary of key concepts:

- When two or more forces are applied to an object, the net effect of the forces can be represented by the resultant vector determined by adding the vectors that represent the forces.

- A system is in a state of equilibrium when the net effect of all the forces acting on an object causes no movement of the object. If there are three forces, this implies that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$.

- The velocity of a moving object can be influenced by external forces, such as wind and the current of a river. The resultant velocity is determined by adding the vectors that represent the object in motion and the effect of the external force: $\vec{v}_r = \vec{v}_{object} + \vec{v}_{external\ force}$

- The dot product between two geometric vectors $\vec{a}$ and $\vec{b}$ is a scalar quantity defined as $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$, where θ is the angle between the two vectors.

- The dot product between two algebraic vectors $\vec{a}$ and $\vec{b}$ is:

  $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2$ in R^2 $\quad$ $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$ in R^3

- If $\vec{a} \cdot \vec{b} = 0$, then $\vec{a} \perp \vec{b}$.

- The cross product $\vec{a} \times \vec{b}$ between two vectors $\vec{a}$ and $\vec{b}$ results in a third vector that is perpendicular to the plane in which the given vectors lie:

  $\vec{a} \times \vec{b} = (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)$ and $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$.

  Geometrically, $|\vec{a} \times \vec{b}|$ is equivalent to the area of the parallelogram formed by vectors $\vec{a}$ and $\vec{b}$.

- Work is an application of the dot product, while torque is an application of the cross product.
  - $W = \vec{F} \cdot \vec{s}$, where $\vec{F}$ is the force applied to an object measured in newtons (N), $\vec{s}$ is the objects displacement measured in meters(m), and W is work measured in Joules (J).
  - Torque $= \vec{r} \times \vec{f} = |\vec{r}||\vec{f}|\sin\theta$, where $\vec{r}$ is the vector determined by the lever arm acting from the axis of rotation, $\vec{f}$ is the applied force and θ is the angle between the force and the lever arm.