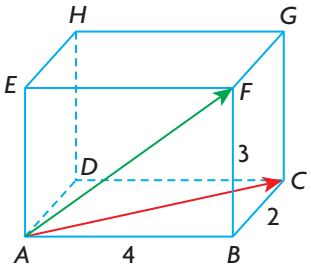
Review Exercise

1. Given that $\vec{a} = (-1, 2, 1)$, $\vec{b} = (-1, 0, 1)$, and $\vec{c} = (-5, 4, 5)$, determine each of the following:
   a. $\vec{a} \times \vec{b}$
   b. $\vec{b} \times \vec{c}$
   c. $|\vec{a} \times \vec{b}| \times |\vec{b} \times \vec{c}|$
   d. Why is it possible to conclude that the vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are coplanar?
2. Given that $\vec{i}$, $\vec{j}$, and $\vec{k}$ represent the standard basis vectors, $\vec{a} = 2\vec{i} - \vec{j} + 2\vec{k}$ and $\vec{b} = 6\vec{i} + 3\vec{j} - 2\vec{k}$, determine each of the following:
   a. $|\vec{a}|$
   b. $|\vec{b}|$
   c. $|\vec{a} - \vec{b}|$
   d. $|\vec{a} + \vec{b}|$
   e. $\vec{a} \cdot \vec{b}$
   f. $\vec{a} \cdot (\vec{a} - 2\vec{b})$
3. a. For what value(s) of a are the vectors $\vec{x} = (3, a, 9)$ and $\vec{y} = (a, 12, 18)$ collinear?
   b. For what value(s) of a are these vectors perpendicular?
4. Determine the angle between the vectors $\vec{x} = (4, 5, 20)$ and $\vec{y} = (-3, 6, 22)$.
5. A parallelogram has its sides determined by $\overrightarrow{OA} = (5, 1)$ and $\overrightarrow{OB} = (-1, 4)$.
   a. Draw a sketch of the parallelogram.
   b. Determine the angle between the two diagonals of this parallelogram.
6. An object of mass 10 kg is suspended by two pieces of rope that make an angle of 30° and 45° with the horizontal. Determine the tension in each of the two pieces of rope.
7. An airplane has a speed of 300 km/h and is headed due west. A wind is blowing from the south at 50 km/h. Determine the resultant velocity of the airplane.
8. The diagonals of a parallelogram are determined by the vectors $\vec{x} = (3, -3, 5)$ and $\vec{y} = (-1, 7, 5)$.
   a. Construct x, y, and z coordinate axes and draw the two given vectors. In addition, draw the parallelogram formed by these vectors.
   b. Determine the area of the parallelogram.
9. Determine the components of a unit vector perpendicular to $(0, 3, -5)$ and to $(2, 3, 1)$.
10. A triangle has vertices $A(2, 3, 7)$, $B(0, -3, 4)$, and $C(5, 2, -4)$.
    a. Determine the largest angle in the triangle.
    b. Determine the area of $\triangle ABC$.

11. A mass of 10 kg is suspended by two pieces of string, 30 cm and 40 cm long, from two points that are 50 cm apart and at the same level. Find the tension in each piece of string.

12. A particle is acted upon by the following four forces: 25 N pulling east, 30 N pulling west, 54 N pulling north, and 42 N pulling south.
    a. Draw a diagram showing these four forces.
    b. Calculate the resultant and equilibrant of these forces.



13. A rectangular box is drawn as shown in the diagram at the left. The lengths of the edges of the box are $AB = 4$, $BC = 2$, and $BF = 3$.
    a. Select an appropriate origin, and then determine coordinates for the other vertices.
    b. Determine the angle between $\overrightarrow{AF}$ and $\overrightarrow{AC}$.
    c. Determine the scalar projection of $\overrightarrow{AF}$ on $\overrightarrow{AC}$.

14. If $\vec{a}$ and $\vec{b}$ are unit vectors, and $|\vec{a} + \vec{b}| = \sqrt{3}$, determine $(2\vec{a} - 5\vec{b}) \cdot (\vec{b} + 3\vec{a})$.

15. Kayla wishes to swim from one side of a river, which has a current speed of 2 km/h, to a point on the other side directly opposite from her starting point. She can swim at a speed of 3 km/h in still water.
    a. At what angle to the bank should Kayla swim if she wishes to swim directly across?
    b. If the river has a width of 300 m, how long will it take for her to cross the river?
    c. If Kayla's speed and the river's speed had been reversed, explain why it would not have been possible for her to swim across the river.

16. A parallelogram has its sides determined by the vectors $\overrightarrow{OA} = (3, 2, -6)$ and $\overrightarrow{OB} = (-6, 6, -2)$.
    a. Determine the coordinates of vectors representing the diagonals.
    b. Determine the angle between the sides of the parallelogram.

17. You are given the vectors $\vec{p} = (2, -2, -3)$ and $\vec{q} = (a, b, 6)$.
    a. Determine values of a and b if $\vec{q}$ is collinear with $\vec{p}$.
    b. Determine an algebraic condition for $\vec{p}$ and $\vec{q}$ to be perpendicular.
    c. Using the answer from part b., determine the components of a unit vector that is perpendicular to $\vec{p}$.

18. For the vectors $\vec{m} = (\sqrt{3}, -2, -3)$ and $\vec{n} = (2, \sqrt{3}, -1)$, determine the following:
    a. the angle between these two vectors, to the nearest degree
    b. the scalar projection of $\vec{n}$ on $\vec{m}$
    c. the vector projection of $\vec{n}$ on $\vec{m}$
    d. the angle that $\vec{m}$ makes with the z-axis

19. A number of unit vectors, each of which is perpendicular to the other vectors in the set, is said to form a *special* set. Determine which of the following sets are special.
    a. $(1, 0, 0), (0, 0, -1), (0, 1, 0)$
    b. $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), (0, 0, -1)$

20. If $\vec{p} = \vec{i} - 2\vec{j} + \vec{k}$, $\vec{q} = 2\vec{i} - \vec{j} + \vec{k}$, and $\vec{r} = \vec{j} - 2\vec{k}$, determine each of the following:
    a. $\vec{p} \times \vec{q}$
    b. $(\vec{p} - \vec{q}) \times (\vec{p} + \vec{q})$
    c. $(\vec{p} \times \vec{r}) \cdot \vec{r}$
    d. $(\vec{p} \times \vec{q}) \times \vec{r}$

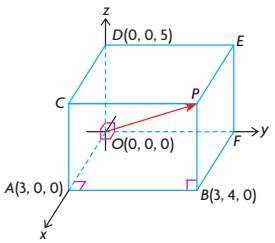
21. Two forces of equal magnitude act on an object so that the angle between their directions is 60°. If their resultant has a magnitude of 20 N, find the magnitude of the equal forces.

22. Determine the components of a vector that is perpendicular to the vectors $\vec{a} = (3, 2, -1)$ and $\vec{b} = (5, 0, 1)$.

23. If $|\vec{x}| = 2$ and $|\vec{y}| = 5$, determine the dot product between $\vec{x} - 2\vec{y}$ and $\vec{x} + 3\vec{y}$ if the angle between $\vec{x}$ and $\vec{y}$ is 60°.

24. The magnitude of the scalar projection of $(1, m, 0)$ on $(2, 2, 1)$ is 4. Determine the value of m.

25. Determine the angle that the vector $\vec{a} = (12, -3, 4)$ makes with the y-axis.



26. A rectangular solid measuring 3 by 4 by 5 is placed on a coordinate axis as shown in the diagram at the left.
    a. Determine the coordinates of points C and F.
    b. Determine $\overrightarrow{CF}$.
    c. Determine the angle between the vectors $\overrightarrow{CF}$ and $\overrightarrow{OP}$.

27. The vectors $\vec{d}$ and $\vec{e}$ are such that $|\vec{d}| = 3$ and $|\vec{e}| = 5$, where the angle between the two given vectors is 50°. Determine each of the following:
    a. $|\vec{d} + \vec{e}|$ b. $|\vec{d} - \vec{e}|$ c. $|\vec{e} - \vec{d}|$

28. Find the scalar and vector projections of $\vec{i} + \vec{j}$ on each of the following vectors:
    a. $\vec{i}$ b. $\vec{j}$ c. $\vec{k} + \vec{j}$

29. a. Determine which of the following are unit vectors:
    $\vec{a} = \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{6}\right)$, $\vec{b} = \left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$, $\vec{c} = \left(\frac{1}{2}, \frac{-1}{\sqrt{2}}, \frac{1}{2}\right)$, and
    $\vec{d} = (-1, 1, 1)$
    b. Which one of vectors $\vec{a}$, $\vec{b}$, or $\vec{c}$ is perpendicular to vector $\vec{d}$? Explain.

30. A 25 N force is applied at the end of a 60 cm wrench. If the force makes a 30° angle with the wrench, calculate the magnitude of the torque.

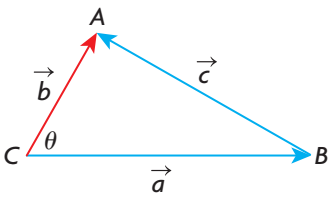
31. a. Verify that the vectors $\vec{a} = (2, 5, -1)$ and $\vec{b} = (3, -1, 1)$ are perpendicular.
    b. Find the direction cosines for each vector.
    c. If $\overrightarrow{m_1} = (\cos \alpha_a, \cos \beta_a, \cos \gamma_a)$, the direction cosines for $\vec{a}$, and if $\overrightarrow{m_2} = (\cos \alpha_b, \cos \beta_b, \cos \gamma_b)$, the direction cosines for $\overrightarrow{b_2}$, verify that $\overrightarrow{m_1} \cdot \overrightarrow{m_2} = 0$.

32. The diagonals of quadrilateral *ABCD* are $3\vec{i} + 3\vec{j} + 10\vec{k}$ and $-\vec{i} + 9\vec{j} - 6\vec{k}$. Show that quadrilateral *ABCD* is a rectangle.

33. The vector $\vec{v}$ makes an angle of 30° with the *x*-axis and equal angles with both the *y*-axis and *z*-axis.
    a. Determine the direction cosines for $\vec{v}$.
    b. Determine the angle that $\vec{v}$ makes with the *z*-axis.

34. The vectors $\vec{a}$ and $\vec{b}$ are unit vectors that make an angle of 60° with each other. If $\vec{a} - 3\vec{b}$ and $m\vec{a} + \vec{b}$ are perpendicular, determine the value of *m*.

35. If $\vec{a} = (0, 4, -6)$ and $\vec{b} = (-1, -5, -2)$, verify that
    $\vec{a} \cdot \vec{b} = \frac{1}{4}|\vec{a} + \vec{b}|^2 - \frac{1}{4}|\vec{a} - \vec{b}|^2$.



36. Use the fact that $|\vec{c}|^2 = \vec{c} \cdot \vec{c}$ to prove the cosine law for the triangle shown in the diagram with sides $\vec{a}$, $\vec{b}$, and $\vec{c}$.

37. Find the lengths of the sides, the cosines of the angles, and the area of the triangle whose vertices are $A(1, -2, 1)$, $B(3, -2, 5)$, and $C(2, -2, 3)$.